

# Computer algebra independent integration tests

3-Logarithms/3.1.2-d-x-<sup>m</sup>-a+b-log-c-x<sup>n</sup>-<sup>p</sup>

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 193 ]. This is test number [ 56 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 193 )	% 0.00 ( 0 )
Mathematica	% 100.00 ( 193 )	% 0.00 ( 0 )
Maple	% 50.78 ( 98 )	% 49.22 ( 95 )
Maxima	% 54.92 ( 106 )	% 45.08 ( 87 )
Fricas	% 63.73 ( 123 )	% 36.27 ( 70 )
Sympy	% 39.38 ( 76 )	% 60.62 ( 117 )
Giac	% 52.85 ( 102 )	% 47.15 ( 91 )
Mupad	% 31.09 ( 60 )	% 68.91 ( 133 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

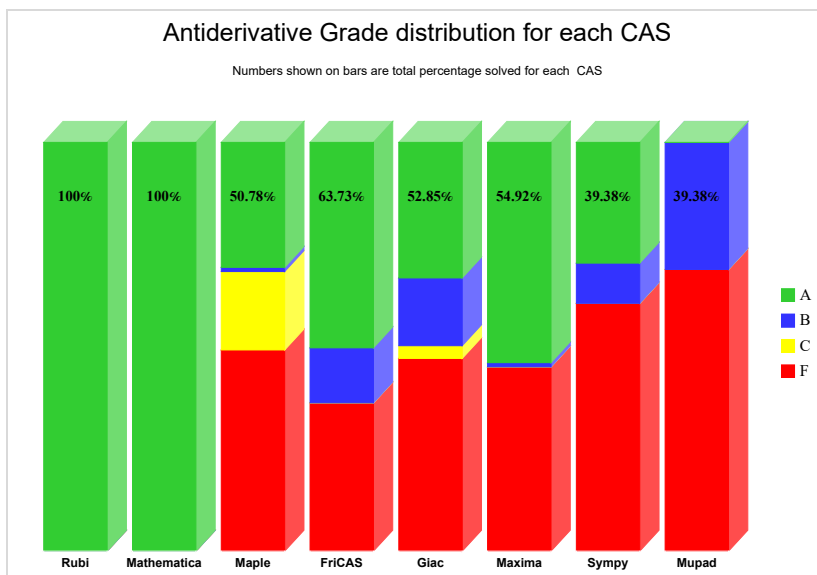
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

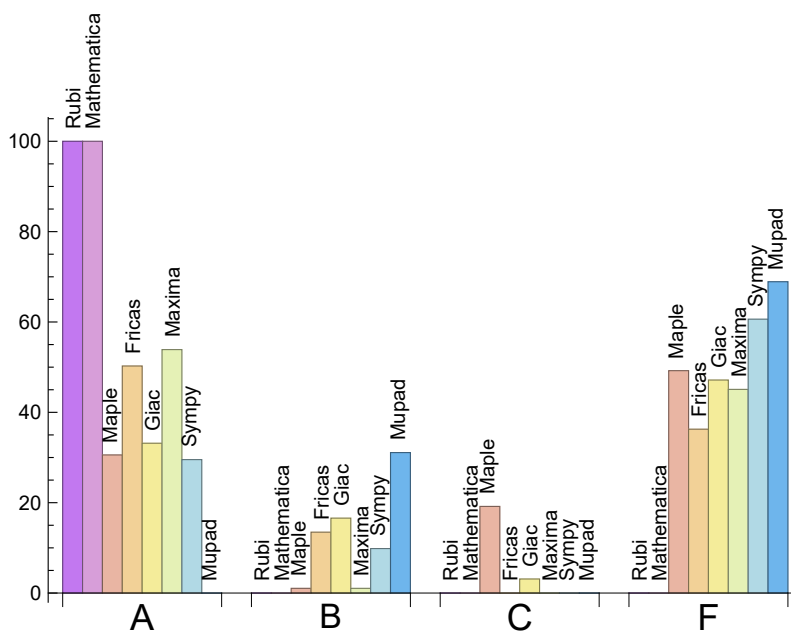
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	100.00	0.00	0.00	0.00
Maple	30.57	1.04	19.17	49.22
Maxima	53.89	1.04	0.00	45.08
Fricas	50.26	13.47	0.00	36.27
Sympy	29.53	9.84	0.00	60.62
Giac	33.16	16.58	3.11	47.15
Mupad	0.00	31.09	0.00	68.91

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	95	100.00 %	0.00 %	0.00 %
Maxima	87	90.80 %	0.00 %	9.20 %
Fricas	70	55.71 %	0.00 %	44.29 %
Sympy	117	95.73 %	3.42 %	0.85 %
Giac	91	100.00 %	0.00 %	0.00 %
Mupad	133	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

## 1.3 Performance

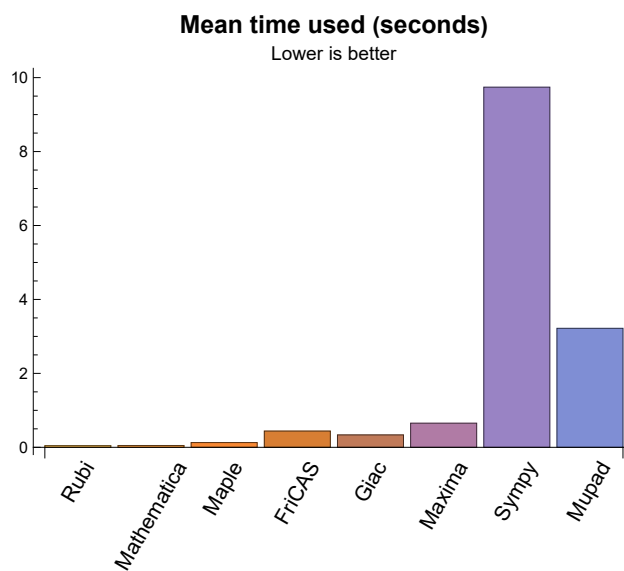
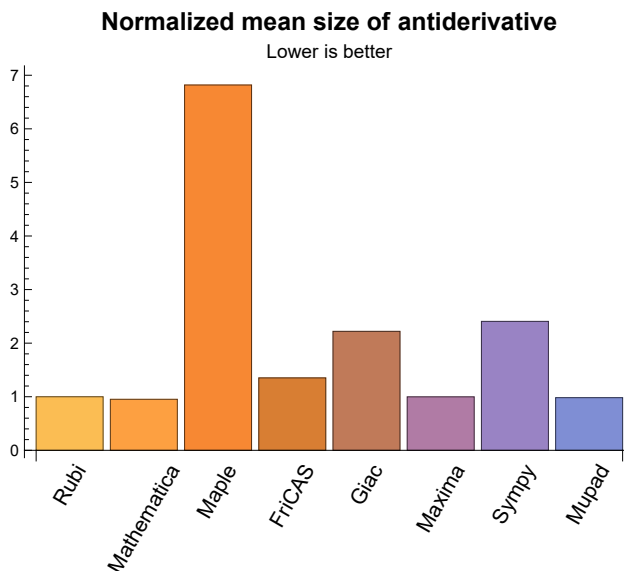
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	56.56	1.00	58.00	1.00
Mathematica	0.05	53.03	0.95	54.00	1.00
Maple	0.13	461.78	6.82	37.00	1.17
Maxima	0.65	42.14	1.00	26.00	0.92
Fricas	0.44	70.10	1.35	37.00	1.11
Sympy	9.74	133.91	2.41	42.00	1.71
Giac	0.34	132.74	2.22	37.00	1.08
Mupad	3.22	33.55	0.98	22.00	0.92

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

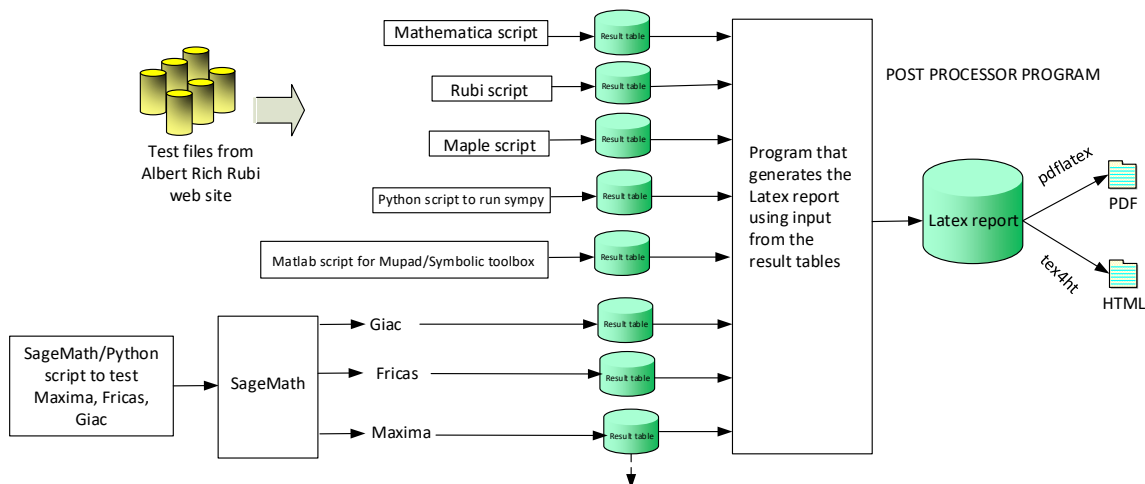
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,..}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171,

172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 53, 69, 77, 85, 92, 98, 118, 125, 132, 139, 146, 171, 179, 187 }

B grade: { 54, 61 }

C grade: { 43, 44, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 62, 63, 64, 68, 76, 84, 89, 90, 91, 93, 94, 95, 96, 97, 99, 100, 110, 149, 150, 151, 152, 156, 157, 158 }

F grade: { 65, 66, 67, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 77, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 118, 125, 132, 139, 146, 151, 152, 156, 157, 158, 171, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190 }

B grade: { 149, 150 }

C grade: { }

F grade: { 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 183, 191, 192, 193 }



## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 55, 56, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 118, 132, 139, 149, 152, 153, 154, 156, 157, 158, 159, 160, 161, 170, 171, 178, 179, 187 }

B grade: { 50, 51, 52, 54, 57, 58, 59, 60, 61, 62, 63, 64, 81, 82, 83, 84, 85, 86, 87, 88, 95, 125, 146, 150, 151, 155 }

C grade: { }

F grade: { 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 32, 33, 39, 40, 43, 44, 45, 46, 47, 48, 49, 54, 69, 77, 85, 90, 91, 92, 93, 94, 118, 125, 132, 139, 149, 150, 151, 152, 156, 157, 158, 171, 179, 187 }

B grade: { 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 96, 97, 98, 99, 100 }

C grade: { }

F grade: { 22, 23, 24, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 41, 42, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 55, 56, 65, 66, 67, 68, 77, 85, 92, 93, 98, 105, 118, 132, 139, 146, 157, 158, 171, 179, 187 }

B grade: { 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 69, 73, 74, 75, 76, 81, 82, 83, 84, 94, 99, 100, 111, 125, 149, 150, 151, 152, 156 }

C grade: { 89, 90, 91, 95, 96, 97 }

F grade: { 27, 28, 34, 35, 41, 42, 70, 71, 72, 78, 79, 80, 86, 87, 88, 101, 102, 103, 104, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163,

164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 32, 33, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 77, 85, 118, 125, 132, 139, 146, 171, 179, 187 }

C grade: { }

F grade: { 22, 23, 24, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 41, 42, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	15	11
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.79	0.58
time (sec)	N/A	0.008	0.001	0.028	0.494	0.413	0.171	0.302	0.041
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	15	11
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.79	0.58
time (sec)	N/A	0.007	0.001	0.031	0.613	0.433	0.105	0.264	0.029
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	15	11
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.79	0.58
time (sec)	N/A	0.004	0.001	0.030	0.564	0.416	0.101	0.216	0.032

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	10	7	16	8
normalized size	1	1.00	1.00	1.10	1.60	1.00	0.70	1.60	0.80
time (sec)	N/A	0.001	0.001	0.023	0.560	0.400	0.088	0.233	0.020

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.006	0.001	0.023	0.679	0.406	0.090	0.198	3.601

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	11	10	15	11
normalized size	1	1.00	1.00	1.07	1.00	0.73	0.67	1.00	0.73
time (sec)	N/A	0.007	0.001	0.033	0.533	0.416	0.097	0.209	3.476

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	13	17	15	11
normalized size	1	1.00	1.00	0.84	0.79	0.68	0.89	0.79	0.58
time (sec)	N/A	0.007	0.001	0.030	0.542	0.396	0.105	0.201	0.024

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	26	26	26	21
normalized size	1	1.00	1.00	0.84	0.66	0.81	0.81	0.81	0.66
time (sec)	N/A	0.020	0.001	0.030	0.601	0.395	0.117	0.211	3.605

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	26	29	26	21
normalized size	1	1.00	1.00	0.84	0.66	0.81	0.91	0.81	0.66
time (sec)	N/A	0.020	0.002	0.028	0.591	0.413	0.114	0.198	3.546

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	26	26	26	21
normalized size	1	1.00	1.00	0.84	0.66	0.81	0.81	0.81	0.66
time (sec)	N/A	0.011	0.001	0.029	0.513	0.425	0.108	0.211	3.491

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	16	19	19	19	16
normalized size	1	1.00	1.00	1.05	0.84	1.00	1.00	1.00	0.84
time (sec)	N/A	0.005	0.001	0.031	0.479	0.401	0.098	0.177	3.589

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.012	0.001	0.024	0.549	0.420	0.092	0.196	3.396

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	19	19	20	26	19
normalized size	1	1.00	1.00	1.04	0.73	0.73	0.77	1.00	0.73
time (sec)	N/A	0.020	0.001	0.032	0.530	0.450	0.118	0.187	3.586

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	21	29	26	21
normalized size	1	1.00	1.00	0.84	0.66	0.66	0.91	0.81	0.66
time (sec)	N/A	0.019	0.001	0.031	0.500	0.430	0.126	0.208	3.380

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	37	42	37	29
normalized size	1	1.00	1.00	0.84	0.64	0.82	0.93	0.82	0.64
time (sec)	N/A	0.036	0.002	0.030	0.545	0.416	0.131	0.212	3.289

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	37	41	37	29
normalized size	1	1.00	1.00	0.84	0.64	0.82	0.91	0.82	0.64
time (sec)	N/A	0.031	0.002	0.028	0.506	0.395	0.133	0.174	3.559

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	37	42	37	29
normalized size	1	1.00	1.00	0.84	0.64	0.82	0.93	0.82	0.64
time (sec)	N/A	0.018	0.001	0.031	0.500	0.414	0.127	0.216	3.475

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	24	28	29	28	24
normalized size	1	1.00	1.00	1.04	0.86	1.00	1.04	1.00	0.86
time (sec)	N/A	0.008	0.001	0.029	0.480	0.397	0.110	0.203	3.556

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.011	0.001	0.025	0.487	0.436	0.091	0.200	3.527

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	27	27	31	37	27
normalized size	1	1.00	1.00	1.03	0.73	0.73	0.84	1.00	0.73
time (sec)	N/A	0.034	0.002	0.030	0.518	0.411	0.134	0.197	3.459

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	29	44	37	29
normalized size	1	1.00	1.00	0.84	0.64	0.64	0.98	0.82	0.64
time (sec)	N/A	0.033	0.002	0.033	0.572	0.427	0.144	0.216	3.600

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	11	-1
normalized size	1	1.00	1.00	1.27	1.00	1.09	0.00	1.00	-0.09
time (sec)	N/A	0.024	0.015	0.037	0.958	0.455	0.000	0.203	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	11	-1
normalized size	1	1.00	1.00	1.27	1.00	1.09	0.00	1.00	-0.09
time (sec)	N/A	0.023	0.015	0.035	0.872	0.416	0.000	0.243	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	11	-1
normalized size	1	1.00	1.00	1.27	1.00	1.09	0.00	1.00	-0.09
time (sec)	N/A	0.017	0.013	0.036	0.932	0.402	0.000	0.162	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	14	9	8	5	9	8
normalized size	1	1.00	1.00	1.75	1.12	1.00	0.62	1.12	1.00
time (sec)	N/A	0.003	0.005	0.034	0.892	0.401	0.479	0.184	3.428

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.012	0.005	0.025	0.685	0.422	0.105	0.214	3.538

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	10	0	0	-1
normalized size	1	1.00	1.00	1.11	1.00	1.11	0.00	0.00	-0.11
time (sec)	N/A	0.024	0.014	0.036	0.709	0.415	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	0	-1
normalized size	1	1.00	1.00	1.27	1.00	1.09	0.00	0.00	-0.09
time (sec)	N/A	0.024	0.015	0.037	0.897	0.422	0.000	0.000	0.000



Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	24	-1
normalized size	1	1.00	1.00	1.08	0.54	1.38	0.00	1.00	-0.04
time (sec)	N/A	0.038	0.015	0.033	0.768	0.416	0.000	0.194	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	24	-1
normalized size	1	1.00	1.00	1.08	0.54	1.38	0.00	1.00	-0.04
time (sec)	N/A	0.037	0.017	0.030	0.957	0.402	0.000	0.188	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	24	-1
normalized size	1	1.00	1.00	1.08	0.54	1.38	0.00	1.00	-0.04
time (sec)	N/A	0.026	0.014	0.033	0.819	0.429	0.000	0.203	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	12	25	12	19	18
normalized size	1	1.00	1.00	1.33	0.67	1.39	0.67	1.06	1.00
time (sec)	N/A	0.005	0.004	0.032	0.909	0.410	0.504	0.203	3.343

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.012	0.001	0.025	0.644	0.421	0.089	0.200	3.557

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	9	28	0	0	-1
normalized size	1	1.00	1.00	0.95	0.41	1.27	0.00	0.00	-0.05
time (sec)	N/A	0.035	0.016	0.033	0.835	0.399	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	0	-1
normalized size	1	1.00	1.00	1.08	0.54	1.38	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.015	0.032	0.835	0.399	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	37	13	47	0	35	-1
normalized size	1	1.00	1.00	1.00	0.35	1.27	0.00	0.95	-0.03
time (sec)	N/A	0.052	0.018	0.030	0.839	0.422	0.000	0.232	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	37	13	47	0	35	-1
normalized size	1	1.00	1.00	0.90	0.32	1.15	0.00	0.85	-0.02
time (sec)	N/A	0.053	0.015	0.032	0.874	0.595	0.000	0.219	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	37	13	47	0	35	-1
normalized size	1	1.00	1.00	1.00	0.35	1.27	0.00	0.95	-0.03
time (sec)	N/A	0.031	0.005	0.031	0.851	0.418	0.000	0.216	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	13	34	26	29	29
normalized size	1	1.00	1.00	0.97	0.38	1.00	0.76	0.85	0.85
time (sec)	N/A	0.009	0.006	0.031	0.788	0.395	0.503	0.223	3.534

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.012	0.001	0.025	0.508	0.408	0.092	0.198	3.450

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	9	34	0	0	-1
normalized size	1	1.00	1.00	0.85	0.23	0.87	0.00	0.00	-0.03
time (sec)	N/A	0.050	0.015	0.030	0.950	0.404	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	13	41	0	0	-1
normalized size	1	1.00	1.00	1.00	0.36	1.14	0.00	0.00	-0.03
time (sec)	N/A	0.049	0.018	0.032	0.809	0.414	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	32	112	26	30	36	31	25
normalized size	1	1.00	1.19	4.15	0.96	1.11	1.33	1.15	0.93
time (sec)	N/A	0.013	0.002	0.224	0.496	0.413	1.365	0.232	3.588

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	32	112	26	30	36	31	25
normalized size	1	1.00	1.19	4.15	0.96	1.11	1.33	1.15	0.93
time (sec)	N/A	0.012	0.001	0.175	0.547	0.421	0.780	0.288	3.361

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	32	29	26	30	36	31	25
normalized size	1	1.00	1.19	1.07	0.96	1.11	1.33	1.15	0.93
time (sec)	N/A	0.007	0.001	0.048	0.608	0.449	0.465	0.251	3.264

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	22	19	20	18
normalized size	1	1.00	1.00	1.06	1.00	1.22	1.06	1.11	1.00
time (sec)	N/A	0.006	0.001	0.031	0.585	0.435	0.249	0.272	3.497

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	27	20	18	34	19	19
normalized size	1	1.00	0.95	1.23	0.91	0.82	1.55	0.86	0.86
time (sec)	N/A	0.012	0.001	0.026	0.441	0.451	8.603	0.218	3.407

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	112	26	19	24	24	23
normalized size	1	1.00	1.13	4.87	1.13	0.83	1.04	1.04	1.00
time (sec)	N/A	0.013	0.001	0.118	0.545	0.462	0.456	0.253	3.558

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	32	111	26	23	37	27	26
normalized size	1	1.00	1.19	4.11	0.96	0.85	1.37	1.00	0.96
time (sec)	N/A	0.013	0.001	0.114	0.533	0.430	0.938	0.253	3.490

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	691	71	102	131	111	61
normalized size	1	1.00	0.83	13.29	1.37	1.96	2.52	2.13	1.17
time (sec)	N/A	0.036	0.018	0.209	0.507	0.421	2.403	0.395	3.605

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	692	71	103	143	111	62
normalized size	1	1.00	0.88	13.31	1.37	1.98	2.75	2.13	1.19
time (sec)	N/A	0.037	0.021	0.209	0.675	0.415	1.518	0.347	3.562

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	692	70	102	126	108	60
normalized size	1	1.00	0.79	13.31	1.35	1.96	2.42	2.08	1.15
time (sec)	N/A	0.023	0.015	0.208	0.597	0.447	0.928	0.291	3.478

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	63	57	85	109	88	49
normalized size	1	1.00	0.77	1.47	1.33	1.98	2.53	2.05	1.14
time (sec)	N/A	0.013	0.009	0.048	0.607	0.426	0.521	0.360	3.576

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	56	20	51	60	56	37
normalized size	1	1.00	1.00	2.55	0.91	2.32	2.73	2.55	1.68
time (sec)	N/A	0.024	0.003	0.026	0.456	0.431	22.815	0.287	3.416

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	704	70	77	110	86	56
normalized size	1	1.00	0.76	15.30	1.52	1.67	2.39	1.87	1.22
time (sec)	N/A	0.035	0.011	0.168	0.725	0.445	0.572	0.310	3.608

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	703	71	83	128	90	62
normalized size	1	1.00	0.79	13.52	1.37	1.60	2.46	1.73	1.19
time (sec)	N/A	0.035	0.013	0.161	0.603	0.474	1.123	0.337	3.429

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	2649	135	222	338	262	110
normalized size	1	1.00	0.86	34.40	1.75	2.88	4.39	3.40	1.43
time (sec)	N/A	0.063	0.032	0.315	0.621	0.418	4.396	0.289	3.664

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	2650	134	224	311	256	108
normalized size	1	1.00	0.87	34.42	1.74	2.91	4.04	3.32	1.40
time (sec)	N/A	0.060	0.013	0.310	0.679	0.448	2.890	0.296	3.374

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	60	2650	135	222	337	262	110
normalized size	1	1.00	0.78	34.42	1.75	2.88	4.38	3.40	1.43
time (sec)	N/A	0.039	0.030	0.318	0.688	0.443	1.777	0.375	3.440

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	2641	113	198	270	219	94
normalized size	1	1.00	0.76	40.02	1.71	3.00	4.09	3.32	1.42
time (sec)	N/A	0.024	0.010	0.286	0.590	0.456	1.035	0.249	3.663

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	75	20	100	92	114	56
normalized size	1	1.00	1.00	3.41	0.91	4.55	4.18	5.18	2.55
time (sec)	N/A	0.022	0.004	0.025	0.559	0.440	28.942	0.271	3.367

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	52	2674	133	180	272	197	104
normalized size	1	1.00	0.75	38.75	1.93	2.61	3.94	2.86	1.51
time (sec)	N/A	0.060	0.019	0.285	0.613	0.412	1.034	0.280	3.375

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	60	2673	135	189	338	203	111
normalized size	1	1.00	0.78	34.71	1.75	2.45	4.39	2.64	1.44
time (sec)	N/A	0.059	0.021	0.286	0.600	0.418	1.157	0.348	3.681

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	60	2674	136	191	313	204	110
normalized size	1	1.00	0.78	34.73	1.77	2.48	4.06	2.65	1.43
time (sec)	N/A	0.061	0.026	0.289	0.591	0.423	1.811	0.301	3.590

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	42	0	48	-1
normalized size	1	1.00	1.00	0.00	0.00	0.82	0.00	0.94	-0.02
time (sec)	N/A	0.056	0.062	0.297	0.000	0.437	0.000	0.337	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	42	0	48	-1
normalized size	1	1.00	1.00	0.00	0.00	0.82	0.00	0.94	-0.02
time (sec)	N/A	0.055	0.060	0.276	0.000	0.410	0.000	0.256	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	42	0	48	-1
normalized size	1	1.00	1.00	0.00	0.00	0.82	0.00	0.94	-0.02
time (sec)	N/A	0.045	0.052	0.280	0.000	0.430	0.000	0.306	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	240	0	39	0	42	-1
normalized size	1	1.00	1.00	5.00	0.00	0.81	0.00	0.88	-0.02
time (sec)	N/A	0.036	0.044	0.499	0.000	0.416	0.000	0.286	0.000



Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	19	34	45	18
normalized size	1	1.00	1.00	1.06	1.00	1.06	1.89	2.50	1.00
time (sec)	N/A	0.025	0.017	0.026	0.799	0.436	1.090	0.306	3.560

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	41	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.85	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.051	0.284	0.000	0.421	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	42	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.053	0.294	0.000	0.596	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	42	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.063	0.312	0.000	0.528	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	101	0	261	-1
normalized size	1	1.00	0.92	0.00	0.00	1.33	0.00	3.43	-0.01
time (sec)	N/A	0.079	0.148	1.000	0.000	0.616	0.000	0.498	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	101	0	261	-1
normalized size	1	1.00	0.92	0.00	0.00	1.33	0.00	3.43	-0.01
time (sec)	N/A	0.079	0.143	0.935	0.000	0.638	0.000	0.442	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	101	0	261	-1
normalized size	1	1.00	0.92	0.00	0.00	1.33	0.00	3.43	-0.01
time (sec)	N/A	0.059	0.124	0.894	0.000	0.422	0.000	0.424	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	350	0	95	0	238	-1
normalized size	1	1.00	0.94	5.00	0.00	1.36	0.00	3.40	-0.01
time (sec)	N/A	0.040	0.124	0.493	0.000	0.404	0.000	0.315	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	25	70	21	20
normalized size	1	1.00	1.00	1.05	1.00	1.25	3.50	1.05	1.00
time (sec)	N/A	0.024	0.004	0.025	0.497	0.401	22.744	0.252	3.269

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	76	0	0	88	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.120	0.917	0.000	0.421	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	80	0	0	102	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.092	0.978	0.000	0.457	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	80	0	0	102	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.099	1.025	0.000	0.420	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	89	0	0	211	0	1029	-1
normalized size	1	1.00	0.88	0.00	0.00	2.09	0.00	10.19	-0.01
time (sec)	N/A	0.108	0.136	0.982	0.000	0.436	0.000	0.550	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	0	0	211	0	1029	-1
normalized size	1	1.00	0.85	0.00	0.00	2.01	0.00	9.80	-0.01
time (sec)	N/A	0.108	0.131	0.927	0.000	0.445	0.000	0.556	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	89	0	0	211	0	1029	-1
normalized size	1	1.00	0.88	0.00	0.00	2.09	0.00	10.19	-0.01
time (sec)	N/A	0.080	0.131	0.897	0.000	0.459	0.000	0.565	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	82	459	0	198	0	982	-1
normalized size	1	1.00	0.84	4.68	0.00	2.02	0.00	10.02	-0.01
time (sec)	N/A	0.052	0.118	0.497	0.000	0.445	0.000	0.361	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	62	121	21	39
normalized size	1	1.00	1.00	0.95	0.91	2.82	5.50	0.95	1.77
time (sec)	N/A	0.024	0.004	0.025	0.576	0.427	93.787	0.326	3.534

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	94	0	0	192	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.096	0.949	0.000	0.441	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	89	0	0	221	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	2.21	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.119	0.981	0.000	0.464	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	0	0	221	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.117	1.024	0.000	0.466	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	41	50	0	117	-1
normalized size	1	1.00	0.71	3.12	1.00	1.22	0.00	2.85	-0.02
time (sec)	N/A	0.016	0.015	0.148	0.533	0.485	0.000	0.517	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	41	42	70	108	-1
normalized size	1	1.00	0.71	3.12	1.00	1.02	1.71	2.63	-0.02
time (sec)	N/A	0.016	0.010	0.123	0.556	0.447	25.449	0.533	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	124	41	32	70	105	-1
normalized size	1	1.00	0.71	3.02	1.00	0.78	1.71	2.56	-0.02
time (sec)	N/A	0.014	0.007	0.125	0.534	0.470	1.065	0.538	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	42	41	25	63	41	-1
normalized size	1	1.00	0.65	1.14	1.11	0.68	1.70	1.11	-0.03
time (sec)	N/A	0.015	0.006	0.040	0.560	0.467	0.769	0.289	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	122	41	28	65	43	-1
normalized size	1	1.00	0.65	3.30	1.11	0.76	1.76	1.16	-0.03
time (sec)	N/A	0.016	0.007	0.130	0.552	0.467	2.861	0.376	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	41	32	71	67	-1
normalized size	1	1.00	0.71	3.12	1.00	0.78	1.73	1.63	-0.02
time (sec)	N/A	0.015	0.009	0.134	0.630	0.478	28.214	0.353	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	102	141	0	425	-1
normalized size	1	1.00	0.84	9.81	1.40	1.93	0.00	5.82	-0.01
time (sec)	N/A	0.045	0.021	0.174	0.594	0.460	0.000	1.309	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	102	121	216	386	-1
normalized size	1	1.00	0.84	9.81	1.40	1.66	2.96	5.29	-0.01
time (sec)	N/A	0.046	0.018	0.173	0.622	0.472	50.356	1.425	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	710	102	99	216	383	-1
normalized size	1	1.00	0.84	9.73	1.40	1.36	2.96	5.25	-0.01
time (sec)	N/A	0.041	0.017	0.181	0.523	0.477	2.462	1.283	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	107	102	87	199	118	-1
normalized size	1	1.00	0.81	1.60	1.52	1.30	2.97	1.76	-0.01
time (sec)	N/A	0.041	0.014	0.056	0.534	0.438	1.457	0.427	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	707	101	87	201	149	-1
normalized size	1	1.00	0.81	10.55	1.51	1.30	3.00	2.22	-0.01
time (sec)	N/A	0.047	0.013	0.174	0.634	0.438	2.899	0.409	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	102	94	218	213	-1
normalized size	1	1.00	0.84	9.81	1.40	1.29	2.99	2.92	-0.01
time (sec)	N/A	0.046	0.016	0.176	0.657	0.462	26.711	0.407	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.084	0.141	0.000	0.433	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.075	0.139	0.000	0.460	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.075	0.141	0.000	0.458	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.066	0.143	0.000	0.485	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	62	0	0	0	0	49	-1
normalized size	1	0.96	0.93	0.00	0.00	0.00	0.00	0.73	-0.01
time (sec)	N/A	0.061	0.079	0.142	0.000	0.478	0.000	0.389	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.078	0.144	0.000	0.436	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.164	5.100	0.000	0.437	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.175	5.217	0.000	0.430	0.000	0.000	0.000



Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.168	4.997	0.000	0.414	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	427	0	0	0	0	-1
normalized size	1	1.00	0.85	4.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.198	2.000	0.000	0.406	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	98	93	0	0	0	0	293	-1
normalized size	1	0.97	0.92	0.00	0.00	0.00	0.00	2.90	-0.01
time (sec)	N/A	0.093	0.163	4.995	0.000	0.417	0.000	0.428	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.149	5.159	0.000	0.418	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.045	0.731	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.023	0.647	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.035	0.296	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.034	0.294	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.018	0.288	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	14	29	14	13
normalized size	1	1.00	1.00	0.82	0.76	0.82	1.71	0.82	0.76
time (sec)	N/A	0.014	0.002	0.028	0.638	0.454	1.174	0.363	3.541

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	65	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.051	0.289	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.048	0.289	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.044	0.293	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.054	0.305	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	79	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.058	0.303	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.040	0.276	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	34	75	72	13
normalized size	1	1.00	1.00	0.82	0.76	2.00	4.41	4.24	0.76
time (sec)	N/A	0.014	0.002	0.028	0.612	0.459	21.925	0.405	3.527

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.058	0.297	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.066	0.283	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.006	0.293	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.009	0.296	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.009	0.291	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.004	0.276	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	14	24	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.93	1.60	0.93	0.87
time (sec)	N/A	0.013	0.002	0.029	0.550	0.409	2.078	0.209	3.576

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.036	0.282	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.036	0.292	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	73	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.047	0.298	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.053	0.301	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.050	0.286	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	69	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.036	0.296	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	24	41	14	13
normalized size	1	1.00	1.00	0.93	0.87	1.60	2.73	0.93	0.87
time (sec)	N/A	0.014	0.002	0.031	0.697	0.461	93.977	0.271	3.450

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.050	0.283	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.049	0.290	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.066	0.281	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.069	0.277	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	92	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.066	0.294	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	83	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.060	0.295	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	37	0	14	13
normalized size	1	1.00	1.00	0.82	0.76	2.18	0.00	0.82	0.76
time (sec)	N/A	0.014	0.002	0.030	0.806	0.452	0.000	0.254	3.434

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.072	0.299	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	78	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.064	0.286	0.000	0.000	0.000	0.000	0.000



Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	260	102	26	27	214	-1
normalized size	1	1.00	0.81	12.38	4.86	1.24	1.29	10.19	-0.05
time (sec)	N/A	0.020	0.012	0.173	0.610	0.467	0.984	0.460	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	9684	247	574	2778	1133	-1
normalized size	1	1.00	0.66	83.48	2.13	4.95	23.95	9.77	-0.01
time (sec)	N/A	0.091	0.048	0.777	0.696	0.450	43.508	0.847	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	2126	132	208	891	402	-1
normalized size	1	1.00	0.94	26.25	1.63	2.57	11.00	4.96	-0.01
time (sec)	N/A	0.046	0.037	0.254	0.713	0.448	27.731	0.408	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	371	57	52	192	95	-1
normalized size	1	1.00	0.70	8.07	1.24	1.13	4.17	2.07	-0.02
time (sec)	N/A	0.015	0.013	0.168	0.567	0.510	10.453	0.324	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	0	0	68	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	1.03	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.120	0.515	0.000	0.440	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	89	0	0	131	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.242	1.962	0.000	0.451	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	113	0	0	322	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.369	1.842	0.000	0.468	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	40	2008	75	73	292	170	-1
normalized size	1	1.00	0.54	27.14	1.01	0.99	3.95	2.30	-0.01
time (sec)	N/A	0.053	0.008	0.302	0.586	0.439	123.181	0.533	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	750	53	42	163	99	-1
normalized size	1	1.00	0.57	14.15	1.00	0.79	3.08	1.87	-0.02
time (sec)	N/A	0.032	0.006	0.178	0.741	0.448	43.863	0.350	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	20	263	32	20	68	50	-1
normalized size	1	1.00	0.62	8.22	1.00	0.62	2.12	1.56	-0.03
time (sec)	N/A	0.011	0.004	0.141	0.666	0.484	13.721	0.356	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	20	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.74	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.008	0.486	0.000	0.458	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	50	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.02	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.017	1.836	0.000	0.441	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	84	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.024	1.751	0.000	0.436	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	101	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.189	0.291	0.000	0.502	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.052	0.316	0.000	0.475	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.011	0.297	0.000	0.445	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.198	0.291	0.000	0.485	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	103	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.422	0.285	0.000	0.509	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	107	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.142	1.685	0.000	0.466	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.097	1.367	0.000	0.418	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.084	1.266	0.000	0.421	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	52	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.073	0.933	0.000	0.453	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	35	56	27	26
normalized size	1	1.00	1.00	1.04	1.00	1.35	2.15	1.04	1.00
time (sec)	N/A	0.032	0.008	0.030	0.708	0.424	1.865	0.253	3.671

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.083	1.115	0.000	0.469	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.086	0.714	0.000	0.469	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.089	0.733	0.000	0.475	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.115	0.625	0.000	0.454	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.066	0.057	0.826	0.454	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.033	0.051	0.814	0.468	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	44	38	0	0	-1
normalized size	1	1.00	1.00	0.00	0.79	0.68	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.028	0.048	0.835	0.422	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	39	21	21
normalized size	1	1.00	1.00	1.05	1.00	1.24	1.86	1.00	1.00
time (sec)	N/A	0.028	0.006	0.027	0.529	0.418	1.307	0.394	3.704

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	40	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.77	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.035	0.057	0.632	0.455	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.037	0.043	0.696	0.432	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.036	0.043	0.691	0.456	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	103	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.195	0.071	0.000	0.475	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	48	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.106	0.039	0.644	0.446	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	48	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.066	0.039	0.676	0.482	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	48	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.66	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.031	0.037	0.802	0.470	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	24	31	48	25	24
normalized size	1	1.00	1.00	0.96	0.92	1.19	1.85	0.96	0.92
time (sec)	N/A	0.028	0.010	0.028	0.557	0.433	4.758	0.310	3.716

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	48	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.66	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.056	0.037	0.970	0.467	0.000	0.000	0.000



Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	48	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.64	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.051	0.041	0.785	0.463	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	48	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.050	0.040	0.769	0.454	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.052	4.261	0.000	0.494	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	118	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.234	2.127	0.000	0.434	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.244	24.092	0.000	0.445	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [117] had the largest ratio of [.4000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	8	0.125
2	A	1	1	1.00	8	0.125
3	A	1	1	1.00	6	0.167
4	A	1	1	1.00	4	0.250
5	A	1	1	1.00	8	0.125
6	A	1	1	1.00	8	0.125
7	A	1	1	1.00	8	0.125
8	A	2	2	1.00	10	0.200
9	A	2	2	1.00	10	0.200
10	A	2	2	1.00	8	0.250
11	A	2	2	1.00	6	0.333
12	A	2	2	1.00	10	0.200
13	A	2	2	1.00	10	0.200
14	A	2	2	1.00	10	0.200
15	A	3	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.00	10	0.200
17	A	3	2	1.00	8	0.250
18	A	3	2	1.00	6	0.333
19	A	2	2	1.00	10	0.200
20	A	3	2	1.00	10	0.200
21	A	3	2	1.00	10	0.200
22	A	2	2	1.00	10	0.200
23	A	2	2	1.00	10	0.200
24	A	2	2	1.00	8	0.250
25	A	1	1	1.00	6	0.167
26	A	2	2	1.00	10	0.200
27	A	2	2	1.00	10	0.200
28	A	2	2	1.00	10	0.200
29	A	3	3	1.00	10	0.300
30	A	3	3	1.00	10	0.300
31	A	3	3	1.00	8	0.375
32	A	2	2	1.00	6	0.333
33	A	2	2	1.00	10	0.200
34	A	3	3	1.00	10	0.300
35	A	3	3	1.00	10	0.300
36	A	4	3	1.00	10	0.300
37	A	4	3	1.00	10	0.300
38	A	4	3	1.00	8	0.375
39	A	3	2	1.00	6	0.333
40	A	2	2	1.00	10	0.200
41	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	4	3	1.00	10	0.300
43	A	1	1	1.00	14	0.071
44	A	1	1	1.00	14	0.071
45	A	1	1	1.00	12	0.083
46	A	2	1	1.00	10	0.100
47	A	1	1	1.00	14	0.071
48	A	1	1	1.00	14	0.071
49	A	1	1	1.00	14	0.071
50	A	2	2	1.00	16	0.125
51	A	2	2	1.00	16	0.125
52	A	2	2	1.00	14	0.143
53	A	3	2	1.00	12	0.167
54	A	2	2	1.00	16	0.125
55	A	2	2	1.00	16	0.125
56	A	2	2	1.00	16	0.125
57	A	3	2	1.00	16	0.125
58	A	3	2	1.00	16	0.125
59	A	3	2	1.00	14	0.143
60	A	4	2	1.00	12	0.167
61	A	2	2	1.00	16	0.125
62	A	3	2	1.00	16	0.125
63	A	3	2	1.00	16	0.125
64	A	3	2	1.00	16	0.125
65	A	2	2	1.00	16	0.125
66	A	2	2	1.00	16	0.125
67	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	2	2	1.00	12	0.167
69	A	2	2	1.00	16	0.125
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	16	0.125
72	A	2	2	1.00	16	0.125
73	A	3	3	1.00	16	0.188
74	A	3	3	1.00	16	0.188
75	A	3	3	1.00	14	0.214
76	A	3	3	1.00	12	0.250
77	A	2	2	1.00	16	0.125
78	A	3	3	1.00	16	0.188
79	A	3	3	1.00	16	0.188
80	A	3	3	1.00	16	0.188
81	A	4	3	1.00	16	0.188
82	A	4	3	1.00	16	0.188
83	A	4	3	1.00	14	0.214
84	A	4	3	1.00	12	0.250
85	A	2	2	1.00	16	0.125
86	A	4	3	1.00	16	0.188
87	A	4	3	1.00	16	0.188
88	A	4	3	1.00	16	0.188
89	A	1	1	1.00	18	0.056
90	A	1	1	1.00	18	0.056
91	A	1	1	1.00	18	0.056
92	A	1	1	1.00	18	0.056
93	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	1	1	1.00	18	0.056
95	A	2	2	1.00	20	0.100
96	A	2	2	1.00	20	0.100
97	A	2	2	1.00	20	0.100
98	A	2	2	1.00	20	0.100
99	A	2	2	1.00	20	0.100
100	A	2	2	1.00	20	0.100
101	A	2	2	1.00	20	0.100
102	A	2	2	1.00	20	0.100
103	A	2	2	1.00	20	0.100
104	A	2	2	1.00	20	0.100
105	A	2	2	0.96	20	0.100
106	A	2	2	1.00	20	0.100
107	A	3	3	1.00	20	0.150
108	A	3	3	1.00	20	0.150
109	A	3	3	1.00	20	0.150
110	A	3	3	1.00	20	0.150
111	A	3	3	0.97	20	0.150
112	A	3	3	1.00	20	0.150
113	A	4	4	1.00	14	0.286
114	A	4	4	1.00	14	0.286
115	A	4	4	1.00	14	0.286
116	A	4	4	1.00	12	0.333
117	A	4	4	1.00	10	0.400
118	A	2	2	1.00	14	0.143
119	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	4	4	1.00	14	0.286
121	A	5	4	1.00	14	0.286
122	A	5	4	1.00	14	0.286
123	A	5	4	1.00	12	0.333
124	A	5	4	1.00	10	0.400
125	A	2	2	1.00	14	0.143
126	A	5	4	1.00	14	0.286
127	A	5	4	1.00	14	0.286
128	A	3	3	1.00	14	0.214
129	A	3	3	1.00	14	0.214
130	A	3	3	1.00	12	0.250
131	A	3	3	1.00	10	0.300
132	A	2	2	1.00	14	0.143
133	A	3	3	1.00	14	0.214
134	A	3	3	1.00	14	0.214
135	A	4	4	1.00	14	0.286
136	A	4	4	1.00	14	0.286
137	A	4	4	1.00	12	0.333
138	A	4	4	1.00	10	0.400
139	A	2	2	1.00	14	0.143
140	A	4	4	1.00	14	0.286
141	A	4	4	1.00	14	0.286
142	A	5	4	1.00	14	0.286
143	A	5	4	1.00	14	0.286
144	A	5	4	1.00	12	0.333
145	A	5	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	2	2	1.00	14	0.143
147	A	5	4	1.00	14	0.286
148	A	5	4	1.00	14	0.286
149	A	1	1	1.00	22	0.045
150	A	3	2	1.00	18	0.111
151	A	2	2	1.00	18	0.111
152	A	1	1	1.00	16	0.062
153	A	2	2	1.00	18	0.111
154	A	3	3	1.00	18	0.167
155	A	4	3	1.00	18	0.167
156	A	3	2	1.00	16	0.125
157	A	2	2	1.00	16	0.125
158	A	1	1	1.00	14	0.071
159	A	3	3	1.00	16	0.188
160	A	4	4	1.00	16	0.250
161	A	5	4	1.00	16	0.250
162	A	5	4	1.00	14	0.286
163	A	4	4	1.00	14	0.286
164	A	3	3	1.00	14	0.214
165	A	4	4	1.00	14	0.286
166	A	5	4	1.00	14	0.286
167	A	2	2	1.00	18	0.111
168	A	2	2	1.00	16	0.125
169	A	2	2	1.00	14	0.143
170	A	2	2	1.00	12	0.167
171	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	2	2	1.00	16	0.125
173	A	2	2	1.00	16	0.125
174	A	2	2	1.00	16	0.125
175	A	2	2	1.00	16	0.125
176	A	2	2	1.00	14	0.143
177	A	2	2	1.00	12	0.167
178	A	2	2	1.00	10	0.200
179	A	2	2	1.00	14	0.143
180	A	2	2	1.00	14	0.143
181	A	2	2	1.00	14	0.143
182	A	2	2	1.00	14	0.143
183	A	2	2	1.00	20	0.100
184	A	2	2	1.00	18	0.111
185	A	2	2	1.00	16	0.125
186	A	2	2	1.00	14	0.143
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	18	0.111
189	A	2	2	1.00	18	0.111
190	A	2	2	1.00	18	0.111
191	A	2	2	1.00	18	0.111
192	A	3	3	1.00	20	0.150
193	A	4	3	1.00	27	0.111



# Chapter 3

## Listing of integrals

### 3.1 $\int x^3 \log(cx) dx$

Optimal. Leaf size=19

$$\frac{1}{4}x^4 \log(cx) - \frac{x^4}{16}$$

[Out]  $-1/16*x^4+1/4*x^4*\ln(c*x)$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$\frac{1}{4}x^4 \log(cx) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{Log}[c*x], x]$

[Out]  $-x^4/16 + (x^4*\text{Log}[c*x])/4$

Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))*((d_.)*(x_))^{(m_.)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x^3 \log(cx) dx = -\frac{x^4}{16} + \frac{1}{4}x^4 \log(cx)$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{4}x^4 \log(cx) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[c\*x],x]

[Out] -1/16\*x^4 + (x^4\*Log[c\*x])/4

**fricas** [A] time = 0.41, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 \log(cx) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x),x, algorithm="fricas")

[Out] 1/4\*x^4\*log(c\*x) - 1/16\*x^4

**giac** [A] time = 0.30, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 \log(cx) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x),x, algorithm="giac")

[Out] 1/4\*x^4\*log(c\*x) - 1/16\*x^4

**maple** [A] time = 0.03, size = 16, normalized size = 0.84

$$\frac{x^4 \ln(cx)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(c\*x),x)

[Out]  $-1/16*x^4+1/4*x^4*\ln(c*x)$

**maxima** [A] time = 0.49, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4\log(cx) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(c*x),x, algorithm="maxima")`

[Out]  $1/4*x^4*\log(c*x) - 1/16*x^4$

**mupad** [B] time = 0.04, size = 11, normalized size = 0.58

$$\frac{x^4 \left( \ln(cx) - \frac{1}{4} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*log(c*x),x)`

[Out]  $(x^4*(\log(c*x) - 1/4))/4$

**sympy** [A] time = 0.17, size = 14, normalized size = 0.74

$$\frac{x^4\log(cx)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*ln(c*x),x)`

[Out]  $x**4*\log(c*x)/4 - x**4/16$

## 3.2 $\int x^2 \log(cx) dx$

Optimal. Leaf size=19

$$\frac{1}{3}x^3 \log(cx) - \frac{x^3}{9}$$

[Out]  $-1/9*x^3+1/3*x^3*\ln(c*x)$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$\frac{1}{3}x^3 \log(cx) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Log}[c*x], x]$

[Out]  $-x^3/9 + (x^3*\text{Log}[c*x])/3$

Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]*((d_.)*(x_)^(m_.), x\_Symbol] :>$   
 $\text{Simp}[(d*x)^(m + 1)*(a + b*\text{Log}[c*x^n])/(d*(m + 1)), x] - \text{Simp}[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\int x^2 \log(cx) dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(cx)$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{3}x^3 \log(cx) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2*\text{Log}[c*x], x]$

[Out]  $-1/9*x^3 + (x^3*\text{Log}[c*x])/3$

fricas [A] time = 0.43, size = 15, normalized size = 0.79

$$\frac{1}{3}x^3 \log(cx) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x),x, algorithm="fricas")

[Out] 1/3\*x^3\*log(c\*x) - 1/9\*x^3

**giac** [A] time = 0.26, size = 15, normalized size = 0.79

$$\frac{1}{3}x^3 \log(cx) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x),x, algorithm="giac")

[Out] 1/3\*x^3\*log(c\*x) - 1/9\*x^3

**maple** [A] time = 0.03, size = 16, normalized size = 0.84

$$\frac{x^3 \ln(cx)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(c\*x),x)

[Out] -1/9\*x^3+1/3\*x^3\*ln(c\*x)

**maxima** [A] time = 0.61, size = 15, normalized size = 0.79

$$\frac{1}{3}x^3 \log(cx) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x),x, algorithm="maxima")

[Out] 1/3\*x^3\*log(c\*x) - 1/9\*x^3

**mupad** [B] time = 0.03, size = 11, normalized size = 0.58

$$\frac{x^3 \left( \ln(cx) - \frac{1}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*log(c\*x),x)

[Out]  $(x^3(\log(cx) - 1/3))/3$

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{x^3 \log(cx)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*ln(c*x),x)`

[Out] `x**3*log(c*x)/3 - x**3/9`



### 3.3 $\int x \log(cx) dx$

Optimal. Leaf size=19

$$\frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

[Out]  $-1/4*x^2+1/2*x^2*\ln(c*x)$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2304}

$$\frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*Log[c\*x],x]

[Out]  $-x^2/4 + (x^2*\text{Log}[c*x])/2$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x \log(cx) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(cx)$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[c\*x],x]

[Out]  $-1/4*x^2 + (x^2*\text{Log}[c*x])/2$

fricas [A] time = 0.42, size = 15, normalized size = 0.79

$$\frac{1}{2}x^2 \log(cx) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x),x, algorithm="fricas")

[Out] 1/2\*x^2\*log(c\*x) - 1/4\*x^2

**giac** [A] time = 0.22, size = 15, normalized size = 0.79

$$\frac{1}{2}x^2 \log(cx) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x),x, algorithm="giac")

[Out] 1/2\*x^2\*log(c\*x) - 1/4\*x^2

**maple** [A] time = 0.03, size = 16, normalized size = 0.84

$$\frac{x^2 \ln(cx)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(c\*x),x)

[Out] -1/4\*x^2+1/2\*x^2\*ln(c\*x)

**maxima** [A] time = 0.56, size = 15, normalized size = 0.79

$$\frac{1}{2}x^2 \log(cx) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x),x, algorithm="maxima")

[Out] 1/2\*x^2\*log(c\*x) - 1/4\*x^2

**mupad** [B] time = 0.03, size = 11, normalized size = 0.58

$$\frac{x^2 \left( \ln(cx) - \frac{1}{2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*log(c\*x),x)

[Out] (x^2\*(log(c\*x) - 1/2))/2

sympy [A] time = 0.10, size = 14, normalized size = 0.74

$$\frac{x^2 \log(cx)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*ln(c\*x),x)

[Out] x\*\*2\*log(c\*x)/2 - x\*\*2/4

### 3.4 $\int \log(cx) dx$

Optimal. Leaf size=10

$$x \log(cx) - x$$

[Out]  $-x+x*\ln(c*x)$

**Rubi** [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2295}

$$x \log(cx) - x$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x],x]

[Out]  $-x + x*\text{Log}[c*x]$

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rubi steps

$$\int \log(cx) dx = -x + x \log(cx)$$

**Mathematica** [A] time = 0.00, size = 10, normalized size = 1.00

$$x \log(cx) - x$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x],x]

[Out]  $-x + x*\text{Log}[c*x]$

**fricas** [A] time = 0.40, size = 10, normalized size = 1.00

$$x \log (cx) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x),x, algorithm="fricas")

[Out]  $x \cdot \log(cx) - x$

**giac** [A] time = 0.23, size = 16, normalized size = 1.60

$$\frac{cx \log(cx) - cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x),x, algorithm="giac")`

[Out]  $(c \cdot x \cdot \log(cx) - c \cdot x) / c$

**maple** [A] time = 0.02, size = 11, normalized size = 1.10

$$x \ln(cx) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x),x)`

[Out]  $-x + x \cdot \ln(cx)$

**maxima** [A] time = 0.56, size = 16, normalized size = 1.60

$$\frac{cx \log(cx) - cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x),x, algorithm="maxima")`

[Out]  $(c \cdot x \cdot \log(cx) - c \cdot x) / c$

**mupad** [B] time = 0.02, size = 8, normalized size = 0.80

$$x (\ln(cx) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(c*x),x)`

[Out]  $x \cdot (\log(cx) - 1)$

**sympy** [A] time = 0.09, size = 7, normalized size = 0.70

$$x \log(cx) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x),x)`

[Out]  $x \cdot \log(cx) - x$

$$3.5 \quad \int \frac{\log(cx)}{x} dx$$

Optimal. Leaf size=10

$$\frac{1}{2} \log^2(cx)$$

[Out] 1/2\*ln(c\*x)^2

**Rubi [A]** time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2301}

$$\frac{1}{2} \log^2(cx)$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]/x,x]

[Out] Log[c\*x]^2/2

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\int \frac{\log(cx)}{x} dx = \frac{1}{2} \log^2(cx)$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2} \log^2(cx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]/x,x]

[Out] Log[c\*x]^2/2

**fricas [A]** time = 0.41, size = 8, normalized size = 0.80

$$\frac{1}{2} \log(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x,x, algorithm="fricas")

[Out] 1/2\*log(c\*x)^2

**giac** [A] time = 0.20, size = 8, normalized size = 0.80

$$\frac{1}{2} \log(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x,x, algorithm="giac")

[Out] 1/2\*log(c\*x)^2

**maple** [A] time = 0.02, size = 9, normalized size = 0.90

$$\frac{\ln(cx)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)/x,x)

[Out] 1/2\*ln(c\*x)^2

**maxima** [A] time = 0.68, size = 8, normalized size = 0.80

$$\frac{1}{2} \log(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x,x, algorithm="maxima")

[Out] 1/2\*log(c\*x)^2

**mupad** [B] time = 3.60, size = 8, normalized size = 0.80

$$\frac{\ln(cx)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x)/x,x)

[Out] log(c\*x)^2/2

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\frac{\log(cx)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)/x,x)

[Out] log(c\*x)\*\*2/2



$$3.6 \quad \int \frac{\log(cx)}{x^2} dx$$

Optimal. Leaf size=15

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

[Out] -1/x-ln(c\*x)/x

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]/x^2,x]

[Out] -x^(-1) - Log[c\*x]/x

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{\log(cx)}{x^2} dx = -\frac{1}{x} - \frac{\log(cx)}{x}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]/x^2,x]

[Out] -x^(-1) - Log[c\*x]/x

fricas [A] time = 0.42, size = 11, normalized size = 0.73

$$-\frac{\log(cx) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x^2,x, algorithm="fricas")

[Out] -(log(c\*x) + 1)/x

**giac** [A] time = 0.21, size = 15, normalized size = 1.00

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x^2,x, algorithm="giac")

[Out] -log(c\*x)/x - 1/x

**maple** [A] time = 0.03, size = 16, normalized size = 1.07

$$-\frac{\ln(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)/x^2,x)

[Out] -1/x-ln(c\*x)/x

**maxima** [A] time = 0.53, size = 15, normalized size = 1.00

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x^2,x, algorithm="maxima")

[Out] -log(c\*x)/x - 1/x

**mupad** [B] time = 3.48, size = 11, normalized size = 0.73

$$-\frac{\ln(cx) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x)/x^2,x)

[Out] -(log(c\*x) + 1)/x

sympy [A] time = 0.10, size = 10, normalized size = 0.67

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)/x\*\*2,x)

[Out] -log(c\*x)/x - 1/x

$$3.7 \quad \int \frac{\log(cx)}{x^3} dx$$

**Optimal.** Leaf size=19

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

[Out]  $-1/4/x^2 - 1/2*\ln(c*x)/x^2$

**Rubi [A]** time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]/x^3,x]

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2)$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int \frac{\log(cx)}{x^3} dx = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2}$$

**Mathematica [A]** time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]/x^3,x]

[Out]  $-1/4*1/x^2 - \text{Log}[c*x]/(2*x^2)$

**fricas [A]** time = 0.40, size = 13, normalized size = 0.68

$$-\frac{2 \log(cx) + 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*log(c\*x) + 1)/x^2

**giac** [A] time = 0.20, size = 15, normalized size = 0.79

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x^3,x, algorithm="giac")

[Out] -1/2\*log(c\*x)/x^2 - 1/4/x^2

**maple** [A] time = 0.03, size = 16, normalized size = 0.84

$$-\frac{\ln(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)/x^3,x)

[Out] -1/4/x^2-1/2\*ln(c\*x)/x^2

**maxima** [A] time = 0.54, size = 15, normalized size = 0.79

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x^3,x, algorithm="maxima")

[Out] -1/2\*log(c\*x)/x^2 - 1/4/x^2

**mupad** [B] time = 0.02, size = 11, normalized size = 0.58

$$-\frac{\ln(cx) + \frac{1}{2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x)/x^3,x)

[Out] -(log(c\*x) + 1/2)/(2\*x^2)

sympy [A] time = 0.10, size = 17, normalized size = 0.89

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)/x\*\*3,x)

[Out] -log(c\*x)/(2\*x\*\*2) - 1/(4\*x\*\*2)

### 3.8 $\int x^3 \log^2(cx) dx$

Optimal. Leaf size=32

$$\frac{1}{4}x^4 \log^2(cx) - \frac{1}{8}x^4 \log(cx) + \frac{x^4}{32}$$

[Out]  $1/32*x^4-1/8*x^4*\ln(c*x)+1/4*x^4*\ln(c*x)^2$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$\frac{1}{4}x^4 \log^2(cx) - \frac{1}{8}x^4 \log(cx) + \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Log[c\*x]^2,x]

[Out]  $x^4/32 - (x^4*\text{Log}[c*x])/8 + (x^4*\text{Log}[c*x]^2)/4$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int x^3 \log^2(cx) dx &= \frac{1}{4}x^4 \log^2(cx) - \frac{1}{2} \int x^3 \log(cx) dx \\ &= \frac{x^4}{32} - \frac{1}{8}x^4 \log(cx) + \frac{1}{4}x^4 \log^2(cx) \end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$\frac{1}{4}x^4 \log^2(cx) - \frac{1}{8}x^4 \log(cx) + \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[c\*x]^2,x]

[Out] x^4/32 - (x^4\*Log[c\*x])/8 + (x^4\*Log[c\*x]^2)/4

**fricas** [A] time = 0.39, size = 26, normalized size = 0.81

$$\frac{1}{4}x^4 \log(cx)^2 - \frac{1}{8}x^4 \log(cx) + \frac{1}{32}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^2,x, algorithm="fricas")

[Out] 1/4\*x^4\*log(c\*x)^2 - 1/8\*x^4\*log(c\*x) + 1/32\*x^4

**giac** [A] time = 0.21, size = 26, normalized size = 0.81

$$\frac{1}{4}x^4 \log(cx)^2 - \frac{1}{8}x^4 \log(cx) + \frac{1}{32}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^2,x, algorithm="giac")

[Out] 1/4\*x^4\*log(c\*x)^2 - 1/8\*x^4\*log(c\*x) + 1/32\*x^4

**maple** [A] time = 0.03, size = 27, normalized size = 0.84

$$\frac{x^4 \ln(cx)^2}{4} - \frac{x^4 \ln(cx)}{8} + \frac{x^4}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(c\*x)^2,x)

[Out] 1/32\*x^4-1/8\*x^4\*ln(c\*x)+1/4\*x^4\*ln(c\*x)^2

**maxima** [A] time = 0.60, size = 21, normalized size = 0.66

$$\frac{1}{32} (8 \log(cx)^2 - 4 \log(cx) + 1)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^2,x, algorithm="maxima")

[Out] 1/32\*(8\*log(c\*x)^2 - 4\*log(c\*x) + 1)\*x^4



**mupad** [B] time = 3.60, size = 21, normalized size = 0.66

$$\frac{x^4 (8 \ln(cx)^2 - 4 \ln(cx) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*log(c*x)^2,x)`

[Out] `(x^4*(8*log(c*x)^2 - 4*log(c*x) + 1))/32`

**sympy** [A] time = 0.12, size = 26, normalized size = 0.81

$$\frac{x^4 \log(cx)^2}{4} - \frac{x^4 \log(cx)}{8} + \frac{x^4}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*ln(c*x)**2,x)`

[Out] `x**4*log(c*x)**2/4 - x**4*log(c*x)/8 + x**4/32`

### 3.9 $\int x^2 \log^2(cx) dx$

Optimal. Leaf size=32

$$\frac{1}{3}x^3 \log^2(cx) - \frac{2}{9}x^3 \log(cx) + \frac{2x^3}{27}$$

[Out]  $2/27*x^3-2/9*x^3*\ln(c*x)+1/3*x^3*\ln(c*x)^2$

**Rubi [A]** time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$\frac{1}{3}x^3 \log^2(cx) - \frac{2}{9}x^3 \log(cx) + \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Log[c\*x]^2,x]

[Out]  $(2*x^3)/27 - (2*x^3*\text{Log}[c*x])/9 + (x^3*\text{Log}[c*x]^2)/3$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x^2 \log^2(cx) dx &= \frac{1}{3}x^3 \log^2(cx) - \frac{2}{3} \int x^2 \log(cx) dx \\ &= \frac{2x^3}{27} - \frac{2}{9}x^3 \log(cx) + \frac{1}{3}x^3 \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 1.00

$$\frac{1}{3}x^3 \log^2(cx) - \frac{2}{9}x^3 \log(cx) + \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[c\*x]^2,x]

[Out] (2\*x^3)/27 - (2\*x^3\*Log[c\*x])/9 + (x^3\*Log[c\*x]^2)/3

**fricas** [A] time = 0.41, size = 26, normalized size = 0.81

$$\frac{1}{3}x^3 \log(cx)^2 - \frac{2}{9}x^3 \log(cx) + \frac{2}{27}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^2,x, algorithm="fricas")

[Out] 1/3\*x^3\*log(c\*x)^2 - 2/9\*x^3\*log(c\*x) + 2/27\*x^3

**giac** [A] time = 0.20, size = 26, normalized size = 0.81

$$\frac{1}{3}x^3 \log(cx)^2 - \frac{2}{9}x^3 \log(cx) + \frac{2}{27}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^2,x, algorithm="giac")

[Out] 1/3\*x^3\*log(c\*x)^2 - 2/9\*x^3\*log(c\*x) + 2/27\*x^3

**maple** [A] time = 0.03, size = 27, normalized size = 0.84

$$\frac{x^3 \ln(cx)^2}{3} - \frac{2x^3 \ln(cx)}{9} + \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(c\*x)^2,x)

[Out] 2/27\*x^3-2/9\*x^3\*ln(c\*x)+1/3\*x^3\*ln(c\*x)^2

**maxima** [A] time = 0.59, size = 21, normalized size = 0.66

$$\frac{1}{27} (9 \log(cx)^2 - 6 \log(cx) + 2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^2,x, algorithm="maxima")

[Out] 1/27\*(9\*log(c\*x)^2 - 6\*log(c\*x) + 2)\*x^3

**mupad** [B] time = 3.55, size = 21, normalized size = 0.66

$$\frac{x^3 (9 \ln(cx)^2 - 6 \ln(cx) + 2)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*log(c*x)^2,x)`

[Out] `(x^3*(9*log(c*x)^2 - 6*log(c*x) + 2))/27`

**sympy** [A] time = 0.11, size = 29, normalized size = 0.91

$$\frac{x^3 \log(cx)^2}{3} - \frac{2x^3 \log(cx)}{9} + \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*ln(c*x)**2,x)`

[Out] `x**3*log(c*x)**2/3 - 2*x**3*log(c*x)/9 + 2*x**3/27`

### 3.10 $\int x \log^2(cx) dx$

Optimal. Leaf size=32

$$\frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

[Out]  $1/4*x^2-1/2*x^2*\ln(c*x)+1/2*x^2*\ln(c*x)^2$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2305, 2304}

$$\frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*Log[c\*x]^2,x]

[Out]  $x^2/4 - (x^2*\text{Log}[c*x])/2 + (x^2*\text{Log}[c*x]^2)/2$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int x \log^2(cx) dx &= \frac{1}{2}x^2 \log^2(cx) - \int x \log(cx) dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2 \log(cx) + \frac{1}{2}x^2 \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 1.00

$$\frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[c\*x]^2,x]

[Out] x^2/4 - (x^2\*Log[c\*x])/2 + (x^2\*Log[c\*x]^2)/2

**fricas** [A] time = 0.43, size = 26, normalized size = 0.81

$$\frac{1}{2} x^2 \log(cx)^2 - \frac{1}{2} x^2 \log(cx) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^2,x, algorithm="fricas")

[Out] 1/2\*x^2\*log(c\*x)^2 - 1/2\*x^2\*log(c\*x) + 1/4\*x^2

**giac** [A] time = 0.21, size = 26, normalized size = 0.81

$$\frac{1}{2} x^2 \log(cx)^2 - \frac{1}{2} x^2 \log(cx) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^2,x, algorithm="giac")

[Out] 1/2\*x^2\*log(c\*x)^2 - 1/2\*x^2\*log(c\*x) + 1/4\*x^2

**maple** [A] time = 0.03, size = 27, normalized size = 0.84

$$\frac{x^2 \ln(cx)^2}{2} - \frac{x^2 \ln(cx)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(c\*x)^2,x)

[Out] 1/4\*x^2-1/2\*x^2\*ln(c\*x)+1/2\*x^2\*ln(c\*x)^2

**maxima** [A] time = 0.51, size = 21, normalized size = 0.66

$$\frac{1}{4} (2 \log(cx)^2 - 2 \log(cx) + 1) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^2,x, algorithm="maxima")

[Out] 1/4\*(2\*log(c\*x)^2 - 2\*log(c\*x) + 1)\*x^2

**mupad** [B] time = 3.49, size = 21, normalized size = 0.66

$$\frac{x^2 (2 \ln(cx)^2 - 2 \ln(cx) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*log(c*x)^2,x)`

[Out] `(x^2*(2*log(c*x)^2 - 2*log(c*x) + 1))/4`

**sympy** [A] time = 0.11, size = 26, normalized size = 0.81

$$\frac{x^2 \log(cx)^2}{2} - \frac{x^2 \log(cx)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(c*x)**2,x)`

[Out] `x**2*log(c*x)**2/2 - x**2*log(c*x)/2 + x**2/4`

### 3.11 $\int \log^2(cx) dx$

Optimal. Leaf size=19

$$x \log^2(cx) - 2x \log(cx) + 2x$$

[Out]  $2*x - 2*x*\ln(c*x) + x*\ln(c*x)^2$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2296, 2295}

$$x \log^2(cx) - 2x \log(cx) + 2x$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^2,x]

[Out]  $2*x - 2*x*\text{Log}[c*x] + x*\text{Log}[c*x]^2$

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

Rubi steps

$$\begin{aligned} \int \log^2(cx) dx &= x \log^2(cx) - 2 \int \log(cx) dx \\ &= 2x - 2x \log(cx) + x \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 19, normalized size = 1.00

$$x \log^2(cx) - 2x \log(cx) + 2x$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^2,x]



[Out]  $2*x - 2*x*\text{Log}[c*x] + x*\text{Log}[c*x]^2$

**fricas** [A] time = 0.40, size = 19, normalized size = 1.00

$$x \log(cx)^2 - 2x \log(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^2,x, algorithm="fricas")`

[Out]  $x*\log(c*x)^2 - 2*x*\log(c*x) + 2*x$

**giac** [A] time = 0.18, size = 19, normalized size = 1.00

$$x \log(cx)^2 - 2x \log(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^2,x, algorithm="giac")`

[Out]  $x*\log(c*x)^2 - 2*x*\log(c*x) + 2*x$

**maple** [A] time = 0.03, size = 20, normalized size = 1.05

$$x \ln(cx)^2 - 2x \ln(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x)^2,x)`

[Out]  $2*x-2*x*\ln(c*x)+x*\ln(c*x)^2$

**maxima** [A] time = 0.48, size = 16, normalized size = 0.84

$$(\log(cx)^2 - 2 \log(cx) + 2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^2,x, algorithm="maxima")`

[Out]  $(\log(c*x)^2 - 2*\log(c*x) + 2)*x$

**mupad** [B] time = 3.59, size = 16, normalized size = 0.84

$$x (\ln(cx)^2 - 2 \ln(cx) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x)^2,x)
```

```
[Out] x*(log(c*x)^2 - 2*log(c*x) + 2)
```

```
sympy [A] time = 0.10, size = 19, normalized size = 1.00
```

$$x \log(cx)^2 - 2x \log(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x)**2,x)
```

```
[Out] x*log(c*x)**2 - 2*x*log(c*x) + 2*x
```

$$3.12 \quad \int \frac{\log^2(cx)}{x} dx$$

Optimal. Leaf size=10

$$\frac{1}{3} \log^3(cx)$$

[Out] 1/3\*ln(c\*x)^3

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2302, 30}

$$\frac{1}{3} \log^3(cx)$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^2/x, x]

[Out] Log[c\*x]^3/3

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\log^2(cx)}{x} dx &= \text{Subst} \left( \int x^2 dx, x, \log(cx) \right) \\ &= \frac{1}{3} \log^3(cx) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3} \log^3(cx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^2/x,x]

[Out] Log[c\*x]^3/3

**fricas** [A] time = 0.42, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(cx)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x,x, algorithm="fricas")

[Out] 1/3\*log(c\*x)^3

**giac** [A] time = 0.20, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(cx)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x,x, algorithm="giac")

[Out] 1/3\*log(c\*x)^3

**maple** [A] time = 0.02, size = 9, normalized size = 0.90

$$\frac{\ln(cx)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^2/x,x)

[Out] 1/3\*ln(c\*x)^3

**maxima** [A] time = 0.55, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(cx)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x,x, algorithm="maxima")

[Out] 1/3\*log(c\*x)^3

mupad [B] time = 3.40, size = 8, normalized size = 0.80

$$\frac{\ln(cx)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(c*x)^2/x,x)`

[Out] `log(c*x)^3/3`

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\frac{\log(cx)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**2/x,x)`

[Out] `log(c*x)**3/3`

### 3.13

$$\int \frac{\log^2(cx)}{x^2} dx$$

Optimal. Leaf size=26

$$-\frac{\log^2(cx)}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

[Out]  $-2/x - 2*\ln(c*x)/x - \ln(c*x)^2/x$

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$-\frac{\log^2(cx)}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[c*x]^2/x^2, x]$

[Out]  $-2/x - (2*\text{Log}[c*x])/x - \text{Log}[c*x]^2/x$

Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]*((d_.)*(x_)^(m_.), x\_Symbol] :>$   
 $\text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^(m+1))/(d*(m+1)^2), x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1]$

Rule 2305

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]^(p_.)*((d_.)*(x_)^(m_.), x\_Symbol] :>$   
 $\text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^p]/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p-1), x], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\log^2(cx)}{x^2} dx &= -\frac{\log^2(cx)}{x} + 2 \int \frac{\log(cx)}{x^2} dx \\ &= -\frac{2}{x} - \frac{2 \log(cx)}{x} - \frac{\log^2(cx)}{x} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 26, normalized size = 1.00

$$-\frac{\log^2(cx)}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^2/x^2,x]

[Out] -2/x - (2\*Log[c\*x])/x - Log[c\*x]^2/x

**fricas** [A] time = 0.45, size = 19, normalized size = 0.73

$$-\frac{\log(cx)^2 + 2 \log(cx) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^2,x, algorithm="fricas")

[Out] -(log(c\*x)^2 + 2\*log(c\*x) + 2)/x

**giac** [A] time = 0.19, size = 26, normalized size = 1.00

$$-\frac{\log(cx)^2}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^2,x, algorithm="giac")

[Out] -log(c\*x)^2/x - 2\*log(c\*x)/x - 2/x

**maple** [A] time = 0.03, size = 27, normalized size = 1.04

$$-\frac{\ln(cx)^2}{x} - \frac{2 \ln(cx)}{x} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^2/x^2,x)

[Out] -2/x-2/x\*ln(c\*x)-ln(c\*x)^2/x

**maxima** [A] time = 0.53, size = 19, normalized size = 0.73

$$-\frac{\log(cx)^2 + 2 \log(cx) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^2,x, algorithm="maxima")

[Out] -(log(c\*x)^2 + 2\*log(c\*x) + 2)/x

mupad [B] time = 3.59, size = 19, normalized size = 0.73

$$\frac{\ln(cx)^2 + 2 \ln(cx) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x)^2/x^2,x)

[Out] -(2\*log(c\*x) + log(c\*x)^2 + 2)/x

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{\log(cx)^2}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*2/x\*\*2,x)

[Out] -log(c\*x)\*\*2/x - 2\*log(c\*x)/x - 2/x



$$3.14 \quad \int \frac{\log^2(cx)}{x^3} dx$$

Optimal. Leaf size=32

$$-\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

[Out]  $-1/4/x^2 - 1/2*\ln(c*x)/x^2 - 1/2*\ln(c*x)^2/x^2$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$-\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^2/x^3, x]

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2) - \text{Log}[c*x]^2/(2*x^2)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\log^2(cx)}{x^3} dx &= -\frac{\log^2(cx)}{2x^2} + \int \frac{\log(cx)}{x^3} dx \\ &= -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2} - \frac{\log^2(cx)}{2x^2} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 32, normalized size = 1.00

$$-\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^2/x^3,x]

[Out] -1/4\*1/x^2 - Log[c\*x]/(2\*x^2) - Log[c\*x]^2/(2\*x^2)

**fricas** [A] time = 0.43, size = 21, normalized size = 0.66

$$-\frac{2 \log (cx)^2 + 2 \log (cx) + 1}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*log(c\*x)^2 + 2\*log(c\*x) + 1)/x^2

**giac** [A] time = 0.21, size = 26, normalized size = 0.81

$$-\frac{\log (cx)^2}{2 x^2} - \frac{\log (cx)}{2 x^2} - \frac{1}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^3,x, algorithm="giac")

[Out] -1/2\*log(c\*x)^2/x^2 - 1/2\*log(c\*x)/x^2 - 1/4/x^2

**maple** [A] time = 0.03, size = 27, normalized size = 0.84

$$-\frac{\ln (cx)^2}{2 x^2} - \frac{\ln (cx)}{2 x^2} - \frac{1}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^2/x^3,x)

[Out] -1/4/x^2-1/2/x^2\*ln(c\*x)-1/2\*ln(c\*x)^2/x^2

**maxima** [A] time = 0.50, size = 21, normalized size = 0.66

$$-\frac{2 \log (cx)^2 + 2 \log (cx) + 1}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^3,x, algorithm="maxima")

[Out] -1/4\*(2\*log(c\*x)^2 + 2\*log(c\*x) + 1)/x^2

mupad [B] time = 3.38, size = 21, normalized size = 0.66

$$-\frac{\frac{\ln(cx)^2}{2} + \frac{\ln(cx)}{2} + \frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x)^2/x^3,x)

[Out] -(log(c\*x)/2 + log(c\*x)^2/2 + 1/4)/x^2

sympy [A] time = 0.13, size = 29, normalized size = 0.91

$$-\frac{\log(cx)^2}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*2/x\*\*3,x)

[Out] -log(c\*x)\*\*2/(2\*x\*\*2) - log(c\*x)/(2\*x\*\*2) - 1/(4\*x\*\*2)

### 3.15 $\int x^3 \log^3(cx) dx$

Optimal. Leaf size=45

$$\frac{1}{4}x^4 \log^3(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{3}{32}x^4 \log(cx) - \frac{3x^4}{128}$$

[Out]  $-3/128*x^4+3/32*x^4*\ln(c*x)-3/16*x^4*\ln(c*x)^2+1/4*x^4*\ln(c*x)^3$

**Rubi [A]** time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$\frac{1}{4}x^4 \log^3(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{3}{32}x^4 \log(cx) - \frac{3x^4}{128}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{Log}[c*x]^3,x]$

[Out]  $(-3*x^4)/128 + (3*x^4*\text{Log}[c*x])/32 - (3*x^4*\text{Log}[c*x]^2)/16 + (x^4*\text{Log}[c*x]^3)/4$

#### Rule 2304

$\text{Int}[(a_. + \text{Log}[c_.]*(x_.)^{n_.})*(b_.)]*((d_.)*(x_.))^{m_.}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1]$

#### Rule 2305

$\text{Int}[(a_. + \text{Log}[c_.]*(x_.)^{n_.})*(b_.)]^{(p_.)}*((d_.)*(x_.))^{m_.}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^p]/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned} \int x^3 \log^3(cx) dx &= \frac{1}{4}x^4 \log^3(cx) - \frac{3}{4} \int x^3 \log^2(cx) dx \\ &= -\frac{3}{16}x^4 \log^2(cx) + \frac{1}{4}x^4 \log^3(cx) + \frac{3}{8} \int x^3 \log(cx) dx \\ &= -\frac{3x^4}{128} + \frac{3}{32}x^4 \log(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{1}{4}x^4 \log^3(cx) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{1}{4}x^4 \log^3(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{3}{32}x^4 \log(cx) - \frac{3x^4}{128}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[c\*x]^3,x]

[Out] (-3\*x^4)/128 + (3\*x^4\*Log[c\*x])/32 - (3\*x^4\*Log[c\*x]^2)/16 + (x^4\*Log[c\*x]^3)/4

**fricas** [A] time = 0.42, size = 37, normalized size = 0.82

$$\frac{1}{4}x^4 \log(cx)^3 - \frac{3}{16}x^4 \log(cx)^2 + \frac{3}{32}x^4 \log(cx) - \frac{3}{128}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^3,x, algorithm="fricas")

[Out] 1/4\*x^4\*log(c\*x)^3 - 3/16\*x^4\*log(c\*x)^2 + 3/32\*x^4\*log(c\*x) - 3/128\*x^4

**giac** [A] time = 0.21, size = 37, normalized size = 0.82

$$\frac{1}{4}x^4 \log(cx)^3 - \frac{3}{16}x^4 \log(cx)^2 + \frac{3}{32}x^4 \log(cx) - \frac{3}{128}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^3,x, algorithm="giac")

[Out] 1/4\*x^4\*log(c\*x)^3 - 3/16\*x^4\*log(c\*x)^2 + 3/32\*x^4\*log(c\*x) - 3/128\*x^4

**maple** [A] time = 0.03, size = 38, normalized size = 0.84

$$\frac{x^4 \ln(cx)^3}{4} - \frac{3x^4 \ln(cx)^2}{16} + \frac{3x^4 \ln(cx)}{32} - \frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(c\*x)^3,x)

[Out] -3/128\*x^4+3/32\*x^4\*ln(c\*x)-3/16\*x^4\*ln(c\*x)^2+1/4\*x^4\*ln(c\*x)^3

**maxima** [A] time = 0.54, size = 29, normalized size = 0.64

$$\frac{1}{128} (32 \log(cx)^3 - 24 \log(cx)^2 + 12 \log(cx) - 3)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^3,x, algorithm="maxima")

[Out] 1/128\*(32\*log(c\*x)^3 - 24\*log(c\*x)^2 + 12\*log(c\*x) - 3)\*x^4

mupad [B] time = 3.29, size = 29, normalized size = 0.64

$$\frac{x^4 (32 \ln(cx)^3 - 24 \ln(cx)^2 + 12 \ln(cx) - 3)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*log(c\*x)^3,x)

[Out] (x^4\*(12\*log(c\*x) - 24\*log(c\*x)^2 + 32\*log(c\*x)^3 - 3))/128

sympy [A] time = 0.13, size = 42, normalized size = 0.93

$$\frac{x^4 \log(cx)^3}{4} - \frac{3x^4 \log(cx)^2}{16} + \frac{3x^4 \log(cx)}{32} - \frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*ln(c\*x)\*\*3,x)

[Out] x\*\*4\*log(c\*x)\*\*3/4 - 3\*x\*\*4\*log(c\*x)\*\*2/16 + 3\*x\*\*4\*log(c\*x)/32 - 3\*x\*\*4/128

### 3.16 $\int x^2 \log^3(cx) dx$

Optimal. Leaf size=45

$$\frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

[Out]  $-2/27*x^3+2/9*x^3*\ln(c*x)-1/3*x^3*\ln(c*x)^2+1/3*x^3*\ln(c*x)^3$

**Rubi** [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$\frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Log}[c*x]^3, x]$

[Out]  $(-2*x^3)/27 + (2*x^3*\text{Log}[c*x])/9 - (x^3*\text{Log}[c*x]^2)/3 + (x^3*\text{Log}[c*x]^3)/3$

#### Rule 2304

$\text{Int}[(a + \text{Log}[c*(x_)^(n_)]*(b_))*((d_)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^(m+1))/(d*(m+1)^2), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1]$

#### Rule 2305

$\text{Int}[(a + \text{Log}[c*(x_)^(n_)]*(b_))^(p_)*((d_)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^p/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p-1), x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned} \int x^2 \log^3(cx) dx &= \frac{1}{3}x^3 \log^3(cx) - \int x^2 \log^2(cx) dx \\ &= -\frac{1}{3}x^3 \log^2(cx) + \frac{1}{3}x^3 \log^3(cx) + \frac{2}{3} \int x^2 \log(cx) dx \\ &= -\frac{2x^3}{27} + \frac{2}{9}x^3 \log(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{1}{3}x^3 \log^3(cx) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[c\*x]^3,x]

[Out] (-2\*x^3)/27 + (2\*x^3\*Log[c\*x])/9 - (x^3\*Log[c\*x]^2)/3 + (x^3\*Log[c\*x]^3)/3

**fricas** [A] time = 0.39, size = 37, normalized size = 0.82

$$\frac{1}{3}x^3 \log(cx)^3 - \frac{1}{3}x^3 \log(cx)^2 + \frac{2}{9}x^3 \log(cx) - \frac{2}{27}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^3,x, algorithm="fricas")

[Out] 1/3\*x^3\*log(c\*x)^3 - 1/3\*x^3\*log(c\*x)^2 + 2/9\*x^3\*log(c\*x) - 2/27\*x^3

**giac** [A] time = 0.17, size = 37, normalized size = 0.82

$$\frac{1}{3}x^3 \log(cx)^3 - \frac{1}{3}x^3 \log(cx)^2 + \frac{2}{9}x^3 \log(cx) - \frac{2}{27}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^3,x, algorithm="giac")

[Out] 1/3\*x^3\*log(c\*x)^3 - 1/3\*x^3\*log(c\*x)^2 + 2/9\*x^3\*log(c\*x) - 2/27\*x^3

**maple** [A] time = 0.03, size = 38, normalized size = 0.84

$$\frac{x^3 \ln(cx)^3}{3} - \frac{x^3 \ln(cx)^2}{3} + \frac{2x^3 \ln(cx)}{9} - \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(c\*x)^3,x)

[Out] -2/27\*x^3+2/9\*x^3\*ln(c\*x)-1/3\*x^3\*ln(c\*x)^2+1/3\*x^3\*ln(c\*x)^3

**maxima** [A] time = 0.51, size = 29, normalized size = 0.64

$$\frac{1}{27} \left( 9 \log(cx)^3 - 9 \log(cx)^2 + 6 \log(cx) - 2 \right) x^3$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^3,x, algorithm="maxima")

[Out] 1/27\*(9\*log(c\*x)^3 - 9\*log(c\*x)^2 + 6\*log(c\*x) - 2)\*x^3

mupad [B] time = 3.56, size = 29, normalized size = 0.64

$$\frac{x^3 (9 \ln(cx)^3 - 9 \ln(cx)^2 + 6 \ln(cx) - 2)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*log(c\*x)^3,x)

[Out] (x^3\*(6\*log(c\*x) - 9\*log(c\*x)^2 + 9\*log(c\*x)^3 - 2))/27

sympy [A] time = 0.13, size = 41, normalized size = 0.91

$$\frac{x^3 \log(cx)^3}{3} - \frac{x^3 \log(cx)^2}{3} + \frac{2x^3 \log(cx)}{9} - \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*ln(c\*x)\*\*3,x)

[Out] x\*\*3\*log(c\*x)\*\*3/3 - x\*\*3\*log(c\*x)\*\*2/3 + 2\*x\*\*3\*log(c\*x)/9 - 2\*x\*\*3/27

### 3.17 $\int x \log^3(cx) dx$

Optimal. Leaf size=45

$$\frac{1}{2}x^2 \log^3(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{3}{4}x^2 \log(cx) - \frac{3x^2}{8}$$

[Out]  $-3/8*x^2+3/4*x^2*\ln(c*x)-3/4*x^2*\ln(c*x)^2+1/2*x^2*\ln(c*x)^3$

**Rubi [A]** time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2305, 2304}

$$\frac{1}{2}x^2 \log^3(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{3}{4}x^2 \log(cx) - \frac{3x^2}{8}$$

Antiderivative was successfully verified.

[In] Int[x\*Log[c\*x]^3,x]

[Out]  $(-3*x^2)/8 + (3*x^2*\text{Log}[c*x])/4 - (3*x^2*\text{Log}[c*x]^2)/4 + (x^2*\text{Log}[c*x]^3)/2$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*(d\*x)^(m + 1))/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int x \log^3(cx) dx &= \frac{1}{2}x^2 \log^3(cx) - \frac{3}{2} \int x \log^2(cx) dx \\ &= -\frac{3}{4}x^2 \log^2(cx) + \frac{1}{2}x^2 \log^3(cx) + \frac{3}{2} \int x \log(cx) dx \\ &= -\frac{3x^2}{8} + \frac{3}{4}x^2 \log(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{1}{2}x^2 \log^3(cx) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{1}{2}x^2 \log^3(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{3}{4}x^2 \log(cx) - \frac{3x^2}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[c\*x]^3,x]

[Out] (-3\*x^2)/8 + (3\*x^2\*Log[c\*x])/4 - (3\*x^2\*Log[c\*x]^2)/4 + (x^2\*Log[c\*x]^3)/2

**fricas** [A] time = 0.41, size = 37, normalized size = 0.82

$$\frac{1}{2}x^2 \log(cx)^3 - \frac{3}{4}x^2 \log(cx)^2 + \frac{3}{4}x^2 \log(cx) - \frac{3}{8}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^3,x, algorithm="fricas")

[Out] 1/2\*x^2\*log(c\*x)^3 - 3/4\*x^2\*log(c\*x)^2 + 3/4\*x^2\*log(c\*x) - 3/8\*x^2

**giac** [A] time = 0.22, size = 37, normalized size = 0.82

$$\frac{1}{2}x^2 \log(cx)^3 - \frac{3}{4}x^2 \log(cx)^2 + \frac{3}{4}x^2 \log(cx) - \frac{3}{8}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^3,x, algorithm="giac")

[Out] 1/2\*x^2\*log(c\*x)^3 - 3/4\*x^2\*log(c\*x)^2 + 3/4\*x^2\*log(c\*x) - 3/8\*x^2

**maple** [A] time = 0.03, size = 38, normalized size = 0.84

$$\frac{x^2 \ln(cx)^3}{2} - \frac{3x^2 \ln(cx)^2}{4} + \frac{3x^2 \ln(cx)}{4} - \frac{3x^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(c\*x)^3,x)

[Out] -3/8\*x^2+3/4\*x^2\*ln(c\*x)-3/4\*x^2\*ln(c\*x)^2+1/2\*x^2\*ln(c\*x)^3

**maxima** [A] time = 0.50, size = 29, normalized size = 0.64

$$\frac{1}{8} \left( 4 \log(cx)^3 - 6 \log(cx)^2 + 6 \log(cx) - 3 \right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^3,x, algorithm="maxima")

[Out] 1/8\*(4\*log(c\*x)^3 - 6\*log(c\*x)^2 + 6\*log(c\*x) - 3)\*x^2

mupad [B] time = 3.48, size = 29, normalized size = 0.64

$$\frac{x^2 (4 \ln(cx)^3 - 6 \ln(cx)^2 + 6 \ln(cx) - 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*log(c\*x)^3,x)

[Out] (x^2\*(6\*log(c\*x) - 6\*log(c\*x)^2 + 4\*log(c\*x)^3 - 3))/8

sympy [A] time = 0.13, size = 42, normalized size = 0.93

$$\frac{x^2 \log(cx)^3}{2} - \frac{3x^2 \log(cx)^2}{4} + \frac{3x^2 \log(cx)}{4} - \frac{3x^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*ln(c\*x)\*\*3,x)

[Out] x\*\*2\*log(c\*x)\*\*3/2 - 3\*x\*\*2\*log(c\*x)\*\*2/4 + 3\*x\*\*2\*log(c\*x)/4 - 3\*x\*\*2/8

### 3.18 $\int \log^3(cx) dx$

Optimal. Leaf size=28

$$x \log^3(cx) - 3x \log^2(cx) + 6x \log(cx) - 6x$$

[Out]  $-6*x+6*x*\ln(c*x)-3*x*\ln(c*x)^2+x*\ln(c*x)^3$

**Rubi** [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2296, 2295}

$$x \log^3(cx) - 3x \log^2(cx) + 6x \log(cx) - 6x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[c*x]^3, x]$

[Out]  $-6*x + 6*x*\text{Log}[c*x] - 3*x*\text{Log}[c*x]^2 + x*\text{Log}[c*x]^3$

Rule 2295

$\text{Int}[\text{Log}[(c_.)*(x_)^(n_.)], x\_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x]$  /;  $\text{FreeQ}\{c, n\}, x]$

Rule 2296

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^(p-1), x], x]$  /;  $\text{FreeQ}\{a, b, c, n\}, x \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[2*p]$

Rubi steps

$$\begin{aligned} \int \log^3(cx) dx &= x \log^3(cx) - 3 \int \log^2(cx) dx \\ &= -3x \log^2(cx) + x \log^3(cx) + 6 \int \log(cx) dx \\ &= -6x + 6x \log(cx) - 3x \log^2(cx) + x \log^3(cx) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 28, normalized size = 1.00

$$x \log^3(cx) - 3x \log^2(cx) + 6x \log(cx) - 6x$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3,x]

[Out] -6\*x + 6\*x\*Log[c\*x] - 3\*x\*Log[c\*x]^2 + x\*Log[c\*x]^3

**fricas** [A] time = 0.40, size = 28, normalized size = 1.00

$$x \log(cx)^3 - 3x \log(cx)^2 + 6x \log(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3,x, algorithm="fricas")

[Out] x\*log(c\*x)^3 - 3\*x\*log(c\*x)^2 + 6\*x\*log(c\*x) - 6\*x

**giac** [A] time = 0.20, size = 28, normalized size = 1.00

$$x \log(cx)^3 - 3x \log(cx)^2 + 6x \log(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3,x, algorithm="giac")

[Out] x\*log(c\*x)^3 - 3\*x\*log(c\*x)^2 + 6\*x\*log(c\*x) - 6\*x

**maple** [A] time = 0.03, size = 29, normalized size = 1.04

$$x \ln(cx)^3 - 3x \ln(cx)^2 + 6x \ln(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^3,x)

[Out] -6\*x+6\*x\*ln(c\*x)-3\*x\*ln(c\*x)^2+x\*ln(c\*x)^3

**maxima** [A] time = 0.48, size = 24, normalized size = 0.86

$$(\log(cx)^3 - 3 \log(cx)^2 + 6 \log(cx) - 6)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3,x, algorithm="maxima")

[Out] (log(c\*x)^3 - 3\*log(c\*x)^2 + 6\*log(c\*x) - 6)\*x

**mupad** [B] time = 3.56, size = 24, normalized size = 0.86

$$x (\ln(cx)^3 - 3 \ln(cx)^2 + 6 \ln(cx) - 6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(c*x)^3,x)`

[Out] `x*(6*log(c*x) - 3*log(c*x)^2 + log(c*x)^3 - 6)`

sympy [A] time = 0.11, size = 29, normalized size = 1.04

$$x \log(cx)^3 - 3x \log(cx)^2 + 6x \log(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**3,x)`

[Out] `x*log(c*x)**3 - 3*x*log(c*x)**2 + 6*x*log(c*x) - 6*x`

$$3.19 \quad \int \frac{\log^3(cx)}{x} dx$$

Optimal. Leaf size=10

$$\frac{1}{4} \log^4(cx)$$

[Out] 1/4\*ln(c\*x)^4

**Rubi [A]** time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2302, 30}

$$\frac{1}{4} \log^4(cx)$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^3/x,x]

[Out] Log[c\*x]^4/4

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\log^3(cx)}{x} dx &= \text{Subst} \left( \int x^3 dx, x, \log(cx) \right) \\ &= \frac{1}{4} \log^4(cx) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{4} \log^4(cx)$$



Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3/x,x]

[Out] Log[c\*x]^4/4

**fricas** [A] time = 0.44, size = 8, normalized size = 0.80

$$\frac{1}{4} \log(cx)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x,x, algorithm="fricas")

[Out] 1/4\*log(c\*x)^4

**giac** [A] time = 0.20, size = 8, normalized size = 0.80

$$\frac{1}{4} \log(cx)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x,x, algorithm="giac")

[Out] 1/4\*log(c\*x)^4

**maple** [A] time = 0.02, size = 9, normalized size = 0.90

$$\frac{\ln(cx)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^3/x,x)

[Out] 1/4\*ln(c\*x)^4

**maxima** [A] time = 0.49, size = 8, normalized size = 0.80

$$\frac{1}{4} \log(cx)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x,x, algorithm="maxima")

[Out] 1/4\*log(c\*x)^4

mupad [B] time = 3.53, size = 8, normalized size = 0.80

$$\frac{\ln(cx)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(c*x)^3/x,x)`

[Out] `log(c*x)^4/4`

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\frac{\log(cx)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**3/x,x)`

[Out] `log(c*x)**4/4`

### 3.20 $\int \frac{\log^3(cx)}{x^2} dx$

Optimal. Leaf size=37

$$-\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

[Out]  $-6/x - 6*\ln(c*x)/x - 3*\ln(c*x)^2/x - \ln(c*x)^3/x$

**Rubi [A]** time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$-\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^3/x^2, x]

[Out]  $-6/x - (6*\text{Log}[c*x])/x - (3*\text{Log}[c*x]^2)/x - \text{Log}[c*x]^3/x$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\log^3(cx)}{x^2} dx &= -\frac{\log^3(cx)}{x} + 3 \int \frac{\log^2(cx)}{x^2} dx \\ &= -\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} + 6 \int \frac{\log(cx)}{x^2} dx \\ &= -\frac{6}{x} - \frac{6\log(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{\log^3(cx)}{x} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 37, normalized size = 1.00

$$\frac{\log^3(cx)}{x} - \frac{3 \log^2(cx)}{x} - \frac{6 \log(cx)}{x} - \frac{6}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3/x^2,x]

[Out] -6/x - (6\*Log[c\*x])/x - (3\*Log[c\*x]^2)/x - Log[c\*x]^3/x

**fricas** [A] time = 0.41, size = 27, normalized size = 0.73

$$\frac{\log(cx)^3 + 3 \log(cx)^2 + 6 \log(cx) + 6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^2,x, algorithm="fricas")

[Out] -(log(c\*x)^3 + 3\*log(c\*x)^2 + 6\*log(c\*x) + 6)/x

**giac** [A] time = 0.20, size = 37, normalized size = 1.00

$$\frac{\log(cx)^3}{x} - \frac{3 \log(cx)^2}{x} - \frac{6 \log(cx)}{x} - \frac{6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^2,x, algorithm="giac")

[Out] -log(c\*x)^3/x - 3\*log(c\*x)^2/x - 6\*log(c\*x)/x - 6/x

**maple** [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{\ln(cx)^3}{x} - \frac{3 \ln(cx)^2}{x} - \frac{6 \ln(cx)}{x} - \frac{6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^3/x^2,x)

[Out] -6/x-6/x\*ln(c\*x)-3/x\*ln(c\*x)^2-ln(c\*x)^3/x

**maxima** [A] time = 0.52, size = 27, normalized size = 0.73

$$\frac{\log(cx)^3 + 3 \log(cx)^2 + 6 \log(cx) + 6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3/x^2,x, algorithm="maxima")`

[Out]  $-(\log(c*x)^3 + 3*\log(c*x)^2 + 6*\log(c*x) + 6)/x$

mupad [B] time = 3.46, size = 27, normalized size = 0.73

$$\frac{\ln(cx)^3 + 3\ln(cx)^2 + 6\ln(cx) + 6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(c*x)^3/x^2,x)`

[Out]  $-(6*\log(c*x) + 3*\log(c*x)^2 + \log(c*x)^3 + 6)/x$

sympy [A] time = 0.13, size = 31, normalized size = 0.84

$$\frac{\log(cx)^3}{x} - \frac{3\log(cx)^2}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**3/x**2,x)`

[Out]  $-\log(c*x)**3/x - 3*\log(c*x)**2/x - 6*\log(c*x)/x - 6/x$

$$3.21 \quad \int \frac{\log^3(cx)}{x^3} dx$$

Optimal. Leaf size=45

$$-\frac{\log^3(cx)}{2x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

[Out]  $-3/8/x^2-3/4*\ln(c*x)/x^2-3/4*\ln(c*x)^2/x^2-1/2*\ln(c*x)^3/x^2$

**Rubi [A]** time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2305, 2304}

$$-\frac{\log^3(cx)}{2x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^3/x^3,x]

[Out]  $-3/(8*x^2) - (3*\text{Log}[c*x])/(4*x^2) - (3*\text{Log}[c*x]^2)/(4*x^2) - \text{Log}[c*x]^3/(2*x^2)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\log^3(cx)}{x^3} dx &= -\frac{\log^3(cx)}{2x^2} + \frac{3}{2} \int \frac{\log^2(cx)}{x^3} dx \\ &= -\frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2} + \frac{3}{2} \int \frac{\log(cx)}{x^3} dx \\ &= -\frac{3}{8x^2} - \frac{3\log(cx)}{4x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 45, normalized size = 1.00

$$-\frac{\log^3(cx)}{2x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3/x^3,x]

[Out] -3/(8\*x^2) - (3\*Log[c\*x])/(4\*x^2) - (3\*Log[c\*x]^2)/(4\*x^2) - Log[c\*x]^3/(2\*x^2)

**fricas** [A] time = 0.43, size = 29, normalized size = 0.64

$$-\frac{4\log(cx)^3 + 6\log(cx)^2 + 6\log(cx) + 3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^3,x, algorithm="fricas")

[Out] -1/8\*(4\*log(c\*x)^3 + 6\*log(c\*x)^2 + 6\*log(c\*x) + 3)/x^2

**giac** [A] time = 0.22, size = 37, normalized size = 0.82

$$-\frac{\log(cx)^3}{2x^2} - \frac{3\log(cx)^2}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^3,x, algorithm="giac")

[Out] -1/2\*log(c\*x)^3/x^2 - 3/4\*log(c\*x)^2/x^2 - 3/4\*log(c\*x)/x^2 - 3/8/x^2

**maple** [A] time = 0.03, size = 38, normalized size = 0.84

$$-\frac{\ln(cx)^3}{2x^2} - \frac{3\ln(cx)^2}{4x^2} - \frac{3\ln(cx)}{4x^2} - \frac{3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x)^3/x^3,x)`

[Out]  $-3/8/x^2-3/4/x^2*\ln(c*x)-3/4/x^2*\ln(c*x)^2-1/2*\ln(c*x)^3/x^2$

**maxima** [A] time = 0.57, size = 29, normalized size = 0.64

$$-\frac{4 \log(cx)^3 + 6 \log(cx)^2 + 6 \log(cx) + 3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3/x^3,x, algorithm="maxima")`

[Out]  $-1/8*(4*\log(c*x)^3 + 6*\log(c*x)^2 + 6*\log(c*x) + 3)/x^2$

**mupad** [B] time = 3.60, size = 29, normalized size = 0.64

$$-\frac{\frac{\ln(cx)^3}{2} + \frac{3\ln(cx)^2}{4} + \frac{3\ln(cx)}{4} + \frac{3}{8}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(c*x)^3/x^3,x)`

[Out]  $-((3*\log(c*x))/4 + (3*\log(c*x)^2)/4 + \log(c*x)^3/2 + 3/8)/x^2$

**sympy** [A] time = 0.14, size = 44, normalized size = 0.98

$$-\frac{\log(cx)^3}{2x^2} - \frac{3\log(cx)^2}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**3/x**3,x)`

[Out]  $-\log(c*x)**3/(2*x**2) - 3*\log(c*x)**2/(4*x**2) - 3*\log(c*x)/(4*x**2) - 3/(8*x**2)$



$$3.22 \quad \int \frac{x^3}{\log(cx)} dx$$

Optimal. Leaf size=11

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

[Out] Ei(4\*ln(c\*x))/c^4

**Rubi [A]** time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2309, 2178}

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[c\*x], x]

[Out] ExpIntegralEi[4\*Log[c\*x]]/c^4

Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2309

Int[((a\_) + Log[(c\_)\*(x\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\int \frac{x^3}{\log(cx)} dx = \frac{\text{Subst}\left(\int \frac{e^{4x}}{x} dx, x, \log(cx)\right)}{c^4} = \frac{\text{Ei}(4 \log(cx))}{c^4}$$

**Mathematica [A]** time = 0.02, size = 11, normalized size = 1.00

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[c\*x],x]

[Out] ExpIntegralEi[4\*Log[c\*x]]/c^4

**fricas** [A] time = 0.45, size = 12, normalized size = 1.09

$$\frac{\log\_integral(c^4 x^4)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x),x, algorithm="fricas")

[Out] log\_integral(c^4\*x^4)/c^4

**giac** [A] time = 0.20, size = 11, normalized size = 1.00

$$\frac{Ei(4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x),x, algorithm="giac")

[Out] Ei(4\*log(c\*x))/c^4

**maple** [A] time = 0.04, size = 14, normalized size = 1.27

$$-\frac{Ei(1, -4 \ln(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(c\*x),x)

[Out] -1/c^4\*Ei(1,-4\*ln(c\*x))

**maxima** [A] time = 0.96, size = 11, normalized size = 1.00

$$\frac{Ei(4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x),x, algorithm="maxima")

[Out]  $Ei(4*\log(c*x))/c^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x^3}{\ln(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/log(c*x), x)`

[Out] `int(x^3/log(c*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/ln(c*x), x)`

[Out] `Integral(x**3/log(c*x), x)`

$$3.23 \quad \int \frac{x^2}{\log(cx)} dx$$

Optimal. Leaf size=11

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

[Out] Ei(3\*ln(c\*x))/c^3

**Rubi [A]** time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2309, 2178}

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[c\*x],x]

[Out] ExpIntegralEi[3\*Log[c\*x]]/c^3

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^((p\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\log(cx)} dx &= \frac{\text{Subst}\left(\int \frac{e^{3x}}{x} dx, x, \log(cx)\right)}{c^3} \\ &= \frac{\text{Ei}(3 \log(cx))}{c^3} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 11, normalized size = 1.00

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[c\*x],x]

[Out] ExpIntegralEi[3\*Log[c\*x]]/c^3

**fricas** [A] time = 0.42, size = 12, normalized size = 1.09

$$\frac{\log\_integral(c^3x^3)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x),x, algorithm="fricas")

[Out] log\_integral(c^3\*x^3)/c^3

**giac** [A] time = 0.24, size = 11, normalized size = 1.00

$$\frac{Ei(3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x),x, algorithm="giac")

[Out] Ei(3\*log(c\*x))/c^3

**maple** [A] time = 0.04, size = 14, normalized size = 1.27

$$\frac{Ei(1, -3 \ln(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(c\*x),x)

[Out] -1/c^3\*Ei(1,-3\*ln(c\*x))

**maxima** [A] time = 0.87, size = 11, normalized size = 1.00

$$\frac{Ei(3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x),x, algorithm="maxima")

[Out]  $Ei(3*\log(c*x))/c^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x^2}{\ln(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/log(c*x), x)`

[Out] `int(x^2/log(c*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/ln(c*x), x)`

[Out] `Integral(x**2/log(c*x), x)`

$$3.24 \quad \int \frac{x}{\log(cx)} dx$$

Optimal. Leaf size=11

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

[Out] Ei(2\*ln(c\*x))/c^2

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2309, 2178}

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c\*x],x]

[Out] ExpIntegralEi[2\*Log[c\*x]]/c^2

Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2309

Int[((a\_) + Log[(c\_)\*(x\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{\log(cx)} dx &= \frac{\text{Subst}\left(\int \frac{e^{2x}}{x} dx, x, \log(cx)\right)}{c^2} \\ &= \frac{\text{Ei}(2 \log(cx))}{c^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[c\*x],x]

[Out] ExpIntegralEi[2\*Log[c\*x]]/c^2

**fricas** [A] time = 0.40, size = 12, normalized size = 1.09

$$\frac{\log\_integral(c^2x^2)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x),x, algorithm="fricas")

[Out] log\_integral(c^2\*x^2)/c^2

**giac** [A] time = 0.16, size = 11, normalized size = 1.00

$$\frac{Ei(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x),x, algorithm="giac")

[Out] Ei(2\*log(c\*x))/c^2

**maple** [A] time = 0.04, size = 14, normalized size = 1.27

$$-\frac{Ei(1, -2 \ln(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(c\*x),x)

[Out] -1/c^2\*Ei(1,-2\*ln(c\*x))

**maxima** [A] time = 0.93, size = 11, normalized size = 1.00

$$\frac{Ei(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x),x, algorithm="maxima")



[Out]  $Ei(2*\log(c*x))/c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{\ln(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/log(c*x), x)`

[Out] `int(x/log(c*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/ln(c*x), x)`

[Out] `Integral(x/log(c*x), x)`

$$3.25 \quad \int \frac{1}{\log(cx)} dx$$

**Optimal.** Leaf size=8

$$\frac{\text{li}(cx)}{c}$$

[Out] Li(c\*x)/c

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2298}

$$\frac{\text{li}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^(-1), x]

[Out] LogIntegral[c\*x]/c

**Rule 2298**

Int[Log[(c\_.)\*(x\_)]^(-1), x\_Symbol] :> Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

**Rubi steps**

$$\int \frac{1}{\log(cx)} dx = \frac{\text{li}(cx)}{c}$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$\frac{\text{li}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^(-1), x]

[Out] LogIntegral[c\*x]/c

**fricas [A]** time = 0.40, size = 8, normalized size = 1.00

$$\frac{\text{log\_integral}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x),x, algorithm="fricas")

[Out] log\_integral(c\*x)/c

**giac** [A] time = 0.18, size = 9, normalized size = 1.12

$$\frac{\text{Ei}(\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x),x, algorithm="giac")

[Out] Ei(log(c\*x))/c

**maple** [A] time = 0.03, size = 14, normalized size = 1.75

$$-\frac{\text{Ei}(1, -\ln(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(c\*x),x)

[Out] -1/c\*Ei(1,-ln(c\*x))

**maxima** [A] time = 0.89, size = 9, normalized size = 1.12

$$\frac{\text{Ei}(\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x),x, algorithm="maxima")

[Out] Ei(log(c\*x))/c

**mupad** [B] time = 3.43, size = 8, normalized size = 1.00

$$\frac{\text{logint}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/log(c\*x),x)

[Out] logint(c\*x)/c

sympy [A] time = 0.48, size = 5, normalized size = 0.62

$$\frac{\operatorname{li}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(c\*x),x)

[Out] li(c\*x)/c

$$3.26 \quad \int \frac{1}{x \log(cx)} dx$$

Optimal. Leaf size=5

$$\log(\log(cx))$$

[Out] ln(ln(c\*x))

**Rubi** [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2302, 29}

$$\log(\log(cx))$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[c\*x]), x]

[Out] Log[Log[c\*x]]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\int \frac{1}{x \log(cx)} dx = \text{Subst} \left( \int \frac{1}{x} dx, x, \log(cx) \right) \\ = \log(\log(cx))$$

**Mathematica** [A] time = 0.00, size = 5, normalized size = 1.00

$$\log(\log(cx))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[c\*x]), x]

[Out] Log[Log[c\*x]]

**fricas** [A] time = 0.42, size = 5, normalized size = 1.00

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x),x, algorithm="fricas")

[Out] log(log(c\*x))

**giac** [A] time = 0.21, size = 5, normalized size = 1.00

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x),x, algorithm="giac")

[Out] log(log(c\*x))

**maple** [A] time = 0.02, size = 6, normalized size = 1.20

$$\ln(\ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(c\*x),x)

[Out] ln(ln(c\*x))

**maxima** [A] time = 0.68, size = 5, normalized size = 1.00

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x),x, algorithm="maxima")

[Out] log(log(c\*x))

**mupad** [B] time = 3.54, size = 5, normalized size = 1.00

$$\ln(\ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*log(c\*x)),x)

[Out]  $\log(\log(c*x))$

sympy [A] time = 0.10, size = 5, normalized size = 1.00

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(c*x),x)`

[Out]  $\log(\log(c*x))$

$$3.27 \quad \int \frac{1}{x^2 \log(cx)} dx$$

**Optimal.** Leaf size=9

$$c\text{Ei}(-\log(cx))$$

[Out] c\*Ei(-ln(c\*x))

**Rubi [A]** time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2309, 2178}

$$c\text{Ei}(-\log(cx))$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Log[c\*x]),x]

[Out] c\*ExpIntegralEi[-Log[c\*x]]

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^((p\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \log(cx)} dx &= c \text{Subst} \left( \int \frac{e^{-x}}{x} dx, x, \log(cx) \right) \\ &= c\text{Ei}(-\log(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 9, normalized size = 1.00

$$c\text{Ei}(-\log(cx))$$

Antiderivative was successfully verified.



[In] Integrate[1/(x^2\*Log[c\*x]),x]

[Out] c\*ExpIntegralEi[-Log[c\*x]]

**fricas** [A] time = 0.42, size = 10, normalized size = 1.11

$$c \log\_integral\left(\frac{1}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x),x, algorithm="fricas")

[Out] c\*log\_integral(1/(c\*x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x),x, algorithm="giac")

[Out] integrate(1/(x^2\*log(c\*x)), x)

**maple** [A] time = 0.04, size = 10, normalized size = 1.11

$$-c \operatorname{Ei}(1, \ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(c\*x),x)

[Out] -c\*Ei(1,ln(c\*x))

**maxima** [A] time = 0.71, size = 9, normalized size = 1.00

$$c \operatorname{Ei}(-\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x),x, algorithm="maxima")

[Out] c\*Ei(-log(c\*x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{x^2 \ln(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(c*x)),x)
```

```
[Out] int(1/(x^2*log(c*x)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{x^2 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(c*x),x)
```

```
[Out] Integral(1/(x**2*log(c*x)), x)
```

$$3.28 \quad \int \frac{1}{x^3 \log(cx)} dx$$

Optimal. Leaf size=11

$$c^2 \text{Ei}(-2 \log(cx))$$

[Out]  $c^2 \text{Ei}(-2 \ln(c*x))$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2309, 2178}

$$c^2 \text{Ei}(-2 \log(cx))$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3 \text{Log}[c*x]), x]$

[Out]  $c^2 \text{ExpIntegralEi}[-2 \text{Log}[c*x]]$

Rule 2178

$\text{Int}[(F_)^((g_.) * ((e_.) + (f_.) * (x_))) / ((c_.) + (d_.) * (x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)}) * \text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d]) / d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_)] * (b_.)^{(p_)} * (x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x} * (a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$  FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\int \frac{1}{x^3 \log(cx)} dx = c^2 \text{Subst} \left( \int \frac{e^{-2x}}{x} dx, x, \log(cx) \right) = c^2 \text{Ei}(-2 \log(cx))$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$c^2 \text{Ei}(-2 \log(cx))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[c\*x]),x]

[Out] c^2\*ExpIntegralEi[-2\*Log[c\*x]]

**fricas** [A] time = 0.42, size = 12, normalized size = 1.09

$$c^2 \log\_integral\left(\frac{1}{c^2 x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x),x, algorithm="fricas")

[Out] c^2\*log\_integral(1/(c^2\*x^2))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x),x, algorithm="giac")

[Out] integrate(1/(x^3\*log(c\*x)), x)

**maple** [A] time = 0.04, size = 14, normalized size = 1.27

$$-c^2 \operatorname{Ei}(1, 2 \ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(c\*x),x)

[Out] -c^2\*Ei(1,2\*ln(c\*x))

**maxima** [A] time = 0.90, size = 11, normalized size = 1.00

$$c^2 \operatorname{Ei}(-2 \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x),x, algorithm="maxima")

[Out] c^2\*Ei(-2\*log(c\*x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{x^3 \ln(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(c*x)),x)
```

```
[Out] int(1/(x^3*log(c*x)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{x^3 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(c*x),x)
```

```
[Out] Integral(1/(x**3*log(c*x)), x)
```

$$3.29 \quad \int \frac{x^3}{\log^2(cx)} dx$$

Optimal. Leaf size=24

$$\frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

[Out]  $4*\text{Ei}(4*\ln(c*x))/c^4 - x^4/\ln(c*x)$

**Rubi [A]** time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$\frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/\text{Log}[c*x]^2, x]$

[Out]  $(4*\text{ExpIntegralEi}[4*\text{Log}[c*x]])/c^4 - x^4/\text{Log}[c*x]$

Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d))*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d]})/d, x] /;$   
 $\text{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ !\$UseGamma == True$

Rule 2306

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_)}]*(b_.)^{(p_)}*((d_.)*(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p+1)}, x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n, x\} \ \&\amp; \ \text{NeQ}[m, -1] \ \&\amp; \ \text{LtQ}[p, -1]$

Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)]*(b_.)^{(p_)}*(x_)^{(m_)}], x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$   
 $\text{FreeQ}\{a, b, c, p, x\} \ \&\amp; \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\log^2(cx)} dx &= -\frac{x^4}{\log(cx)} + 4 \int \frac{x^3}{\log(cx)} dx \\ &= -\frac{x^4}{\log(cx)} + \frac{4 \operatorname{Subst}\left(\int \frac{e^{4x}}{x} dx, x, \log(cx)\right)}{c^4} \\ &= \frac{4\operatorname{Ei}(4 \log(cx))}{c^4} - \frac{x^4}{\log(cx)} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{4\operatorname{Ei}(4 \log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[c\*x]^2,x]

[Out] (4\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/Log[c\*x]

**fricas** [A] time = 0.42, size = 33, normalized size = 1.38

$$-\frac{c^4 x^4 - 4 \log(cx) \log\_integral(c^4 x^4)}{c^4 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c^4\*x^4 - 4\*log(c\*x)\*log\_integral(c^4\*x^4))/(c^4\*log(c\*x))

**giac** [A] time = 0.19, size = 24, normalized size = 1.00

$$-\frac{x^4}{\log(cx)} + \frac{4 \operatorname{Ei}(4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^2,x, algorithm="giac")

[Out] -x^4/log(c\*x) + 4\*Ei(4\*log(c\*x))/c^4

**maple** [A] time = 0.03, size = 26, normalized size = 1.08

$$-\frac{x^4}{\ln(cx)} - \frac{4 \operatorname{Ei}(1, -4 \ln(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/ln(c*x)^2,x)`

[Out] `-x^4/ln(c*x)-4/c^4*Ei(1,-4*ln(c*x))`

**maxima** [A] time = 0.77, size = 13, normalized size = 0.54

$$\frac{4 \Gamma(-1, -4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/log(c*x)^2,x, algorithm="maxima")`

[Out] `4*gamma(-1, -4*log(c*x))/c^4`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\ln(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/log(c*x)^2,x)`

[Out] `int(x^3/log(c*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^4}{\log(cx)} + 4 \int \frac{x^3}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/ln(c*x)**2,x)`

[Out] `-x**4/log(c*x) + 4*Integral(x**3/log(c*x), x)`



$$3.30 \quad \int \frac{x^2}{\log^2(cx)} dx$$

Optimal. Leaf size=24

$$\frac{3\text{Ei}(3 \log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

[Out] 3\*Ei(3\*ln(c\*x))/c^3-x^3/ln(c\*x)

**Rubi [A]** time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$\frac{3\text{Ei}(3 \log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[c\*x]^2,x]

[Out] (3\*ExpIntegralEi[3\*Log[c\*x]])/c^3 - x^3/Log[c\*x]

Rule 2178

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\_)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^p\_\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\log^2(cx)} dx &= -\frac{x^3}{\log(cx)} + 3 \int \frac{x^2}{\log(cx)} dx \\ &= -\frac{x^3}{\log(cx)} + \frac{3 \operatorname{Subst}\left(\int \frac{e^{3x}}{x} dx, x, \log(cx)\right)}{c^3} \\ &= \frac{3\operatorname{Ei}(3 \log(cx))}{c^3} - \frac{x^3}{\log(cx)} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{3\operatorname{Ei}(3 \log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[c\*x]^2,x]

[Out] (3\*ExpIntegralEi[3\*Log[c\*x]])/c^3 - x^3/Log[c\*x]

**fricas** [A] time = 0.40, size = 33, normalized size = 1.38

$$-\frac{c^3 x^3 - 3 \log(cx) \log\_integral(c^3 x^3)}{c^3 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c^3\*x^3 - 3\*log(c\*x)\*log\_integral(c^3\*x^3))/(c^3\*log(c\*x))

**giac** [A] time = 0.19, size = 24, normalized size = 1.00

$$-\frac{x^3}{\log(cx)} + \frac{3 \operatorname{Ei}(3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^2,x, algorithm="giac")

[Out] -x^3/log(c\*x) + 3\*Ei(3\*log(c\*x))/c^3

**maple** [A] time = 0.03, size = 26, normalized size = 1.08

$$-\frac{x^3}{\ln(cx)} - \frac{3 \operatorname{Ei}(1, -3 \ln(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/ln(c*x)^2,x)`

[Out] `-x^3/ln(c*x)-3/c^3*Ei(1,-3*ln(c*x))`

**maxima** [A] time = 0.96, size = 13, normalized size = 0.54

$$\frac{3 \Gamma(-1, -3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(c*x)^2,x, algorithm="maxima")`

[Out] `3*gamma(-1, -3*log(c*x))/c^3`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2}{\ln(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/log(c*x)^2,x)`

[Out] `int(x^2/log(c*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^3}{\log(cx)} + 3 \int \frac{x^2}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/ln(c*x)**2,x)`

[Out] `-x**3/log(c*x) + 3*Integral(x**2/log(c*x), x)`

$$3.31 \quad \int \frac{x}{\log^2(cx)} dx$$

**Optimal.** Leaf size=24

$$\frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

[Out] 2\*Ei(2\*ln(c\*x))/c^2-x^2/ln(c\*x)

**Rubi [A]** time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2306, 2309, 2178}

$$\frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c\*x]^2,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/Log[c\*x]

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{\log^2(cx)} dx &= -\frac{x^2}{\log(cx)} + 2 \int \frac{x}{\log(cx)} dx \\ &= -\frac{x^2}{\log(cx)} + \frac{2 \operatorname{Subst}\left(\int \frac{e^{2x}}{x} dx, x, \log(cx)\right)}{c^2} \\ &= \frac{2\operatorname{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{2\operatorname{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[c\*x]^2,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/Log[c\*x]

**fricas** [A] time = 0.43, size = 33, normalized size = 1.38

$$-\frac{c^2 x^2 - 2 \log(cx) \log\_integral(c^2 x^2)}{c^2 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c^2\*x^2 - 2\*log(c\*x)\*log\_integral(c^2\*x^2))/(c^2\*log(c\*x))

**giac** [A] time = 0.20, size = 24, normalized size = 1.00

$$-\frac{x^2}{\log(cx)} + \frac{2 \operatorname{Ei}(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^2,x, algorithm="giac")

[Out] -x^2/log(c\*x) + 2\*Ei(2\*log(c\*x))/c^2

**maple** [A] time = 0.03, size = 26, normalized size = 1.08

$$-\frac{x^2}{\ln(cx)} - \frac{2 \operatorname{Ei}(1, -2 \ln(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/ln(c*x)^2,x)`

[Out] `-x^2/ln(c*x)-2/c^2*Ei(1,-2*ln(c*x))`

**maxima** [A] time = 0.82, size = 13, normalized size = 0.54

$$\frac{2\Gamma(-1, -2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/log(c*x)^2,x, algorithm="maxima")`

[Out] `2*gamma(-1, -2*log(c*x))/c^2`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\ln(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/log(c*x)^2,x)`

[Out] `int(x/log(c*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^2}{\log(cx)} + 2 \int \frac{x}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/ln(c*x)**2,x)`

[Out] `-x**2/log(c*x) + 2*Integral(x/log(c*x), x)`

$$3.32 \quad \int \frac{1}{\log^2(cx)} dx$$

Optimal. Leaf size=18

$$\frac{\text{li}(cx)}{c} - \frac{x}{\log(cx)}$$

[Out] Li(c\*x)/c-x/ln(c\*x)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2297, 2298}

$$\frac{\text{li}(cx)}{c} - \frac{x}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^(-2), x]

[Out] -(x/Log[c\*x]) + LogIntegral[c\*x]/c

Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

Rule 2298

Int[Log[(c\_.)\*(x\_)^(-1)], x\_Symbol] :> Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\log^2(cx)} dx &= -\frac{x}{\log(cx)} + \int \frac{1}{\log(cx)} dx \\ &= -\frac{x}{\log(cx)} + \frac{\text{li}(cx)}{c} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\text{li}(cx)}{c} - \frac{x}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^(-2),x]

[Out] -(x/Log[c\*x]) + LogIntegral[c\*x]/c

**fricas** [A] time = 0.41, size = 25, normalized size = 1.39

$$\frac{cx - \log(cx) \log\_integral(cx)}{c \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c\*x - log(c\*x)\*log\_integral(c\*x))/(c\*log(c\*x))

**giac** [A] time = 0.20, size = 19, normalized size = 1.06

$$\frac{Ei(\log(cx))}{c} - \frac{x}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^2,x, algorithm="giac")

[Out] Ei(log(c\*x))/c - x/log(c\*x)

**maple** [A] time = 0.03, size = 24, normalized size = 1.33

$$-\frac{Ei(1, -\ln(cx))}{c} - \frac{x}{\ln(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(c\*x)^2,x)

[Out] -x/ln(c\*x)-1/c\*Ei(1,-ln(c\*x))

**maxima** [A] time = 0.91, size = 12, normalized size = 0.67

$$\frac{\Gamma(-1, -\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^2,x, algorithm="maxima")



[Out]  $\text{gamma}(-1, -\log(cx))/c$

**mupad [B]** time = 3.34, size = 18, normalized size = 1.00

$$\frac{\text{logint}(cx)}{c} - \frac{x}{\ln(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/\log(cx)^2, x)$

[Out]  $\text{logint}(cx)/c - x/\log(cx)$

**sympy [A]** time = 0.50, size = 12, normalized size = 0.67

$$-\frac{x}{\log(cx)} + \frac{\text{li}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/\ln(cx)**2, x)$

[Out]  $-x/\log(cx) + \text{li}(cx)/c$

$$3.33 \quad \int \frac{1}{x \log^2(cx)} dx$$

Optimal. Leaf size=8

$$-\frac{1}{\log(cx)}$$

[Out] -1/ln(c\*x)

**Rubi [A]** time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2302, 30}

$$-\frac{1}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[c\*x]^2), x]

[Out] -Log[c\*x]^(-1)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b^n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^2(cx)} dx &= \text{Subst} \left( \int \frac{1}{x^2} dx, x, \log(cx) \right) \\ &= -\frac{1}{\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[c\*x]^2),x]

[Out] -Log[c\*x]^(-1)

**fricas** [A] time = 0.42, size = 8, normalized size = 1.00

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^2,x, algorithm="fricas")

[Out] -1/log(c\*x)

**giac** [A] time = 0.20, size = 8, normalized size = 1.00

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^2,x, algorithm="giac")

[Out] -1/log(c\*x)

**maple** [A] time = 0.02, size = 9, normalized size = 1.12

$$-\frac{1}{\ln(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(c\*x)^2,x)

[Out] -1/ln(c\*x)

**maxima** [A] time = 0.64, size = 8, normalized size = 1.00

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^2,x, algorithm="maxima")

[Out] -1/log(c\*x)

mupad [B] time = 3.56, size = 8, normalized size = 1.00

$$-\frac{1}{\ln(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*log(c*x)^2),x)`

[Out] `-1/log(c*x)`

sympy [A] time = 0.09, size = 7, normalized size = 0.88

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(c*x)**2,x)`

[Out] `-1/log(c*x)`

$$3.34 \quad \int \frac{1}{x^2 \log^2(cx)} dx$$

Optimal. Leaf size=22

$$-c\text{Ei}(-\log(cx)) - \frac{1}{x \log(cx)}$$

[Out]  $-c\text{Ei}(-\ln(c*x))-1/x/\ln(c*x)$

**Rubi [A]** time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$-c\text{Ei}(-\log(cx)) - \frac{1}{x \log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*\text{Log}[c*x]^2), x]$

[Out]  $-(c*\text{ExpIntegralEi}[-\text{Log}[c*x]]) - 1/(x*\text{Log}[c*x])$

#### Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)})*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$   $\text{FreeQ}\{F, c, d, e, f, g\}, x \ \&\amp; \ !\$UseGamma == True$

#### Rule 2306

$\text{Int}[((a_.) + \text{Log}[(c_.)*(x_)^(n_)]*(b_))^(p_)*((d_.)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^(p+1)/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p+1), x], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\amp; \ \text{NeQ}[m, -1] \ \&\amp; \ \text{LtQ}[p, -1]$

#### Rule 2309

$\text{Int}[((a_.) + \text{Log}[(c_.)*(x_)]*(b_))^(p_)*(x_)^(m_), x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$   $\text{FreeQ}\{a, b, c, p\}, x \ \&\amp; \ \text{IntegerQ}[m]$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \log^2(cx)} dx &= -\frac{1}{x \log(cx)} - \int \frac{1}{x^2 \log(cx)} dx \\ &= -\frac{1}{x \log(cx)} - c \operatorname{Subst} \left( \int \frac{e^{-x}}{x} dx, x, \log(cx) \right) \\ &= -c \operatorname{Ei}(-\log(cx)) - \frac{1}{x \log(cx)} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 22, normalized size = 1.00

$$-c \operatorname{Ei}(-\log(cx)) - \frac{1}{x \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[c\*x]^2),x]

[Out] -(c\*ExpIntegralEi[-Log[c\*x]]) - 1/(x\*Log[c\*x])

**fricas** [A] time = 0.40, size = 28, normalized size = 1.27

$$-\frac{cx \log(cx) \log\_integral\left(\frac{1}{cx}\right) + 1}{x \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c\*x\*log(c\*x)\*log\_integral(1/(c\*x)) + 1)/(x\*log(c\*x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^2(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^2,x, algorithm="giac")

[Out] integrate(1/(x^2\*log(c\*x)^2), x)

**maple** [A] time = 0.03, size = 21, normalized size = 0.95

$$c \operatorname{Ei}(1, \ln(cx)) - \frac{1}{x \ln(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/ln(c*x)^2,x)`

[Out] `-1/x/ln(c*x)+c*Ei(1,ln(c*x))`

**maxima** [A] time = 0.84, size = 9, normalized size = 0.41

$$-c\Gamma(-1, \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/log(c*x)^2,x, algorithm="maxima")`

[Out] `-c*gamma(-1, log(c*x))`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \ln(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*log(c*x)^2),x)`

[Out] `int(1/(x^2*log(c*x)^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2 \log(cx)} dx - \frac{1}{x \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/ln(c*x)**2,x)`

[Out] `-Integral(1/(x**2*log(c*x)), x) - 1/(x*log(c*x))`

$$3.35 \quad \int \frac{1}{x^3 \log^2(cx)} dx$$

Optimal. Leaf size=24

$$-2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)}$$

[Out]  $-2*c^2*Ei(-2*\ln(c*x))-1/x^2/\ln(c*x)$

**Rubi [A]** time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$-2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*\text{Log}[c*x]^2), x]$

[Out]  $-2*c^2*\text{ExpIntegralEi}[-2*\text{Log}[c*x]] - 1/(x^2*\text{Log}[c*x])$

Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^(g*(e - (c*f)/d))*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$   
 $\text{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \text{!}\$UseGamma == \text{True}$

Rule 2306

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)]*(b_.)]^(p_)*((d_.)*(x_))^(m_.), x\_Symbol]$   
 $\rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^(p+1)/(b*d*n*(p+1)), x] -$   
 $\text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p+1), x], x]$   
 $/;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[m, -1] \&\& \text{LtQ}[p, -1]$

Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)]*(b_.)]^(p_)*x^(m_.), x\_Symbol] \rightarrow \text{Dist}[1/c^(m+1), \text{Subst}[\text{Int}[E^((m+1)*x)*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$   
 $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IntegerQ}[m]$

Rubi steps



$$\begin{aligned} \int \frac{1}{x^3 \log^2(cx)} dx &= -\frac{1}{x^2 \log(cx)} - 2 \int \frac{1}{x^3 \log(cx)} dx \\ &= -\frac{1}{x^2 \log(cx)} - (2c^2) \text{Subst} \left( \int \frac{e^{-2x}}{x} dx, x, \log(cx) \right) \\ &= -2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 24, normalized size = 1.00

$$-2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[c\*x]^2),x]

[Out] -2\*c^2\*ExpIntegralEi[-2\*Log[c\*x]] - 1/(x^2\*Log[c\*x])

**fricas** [A] time = 0.40, size = 33, normalized size = 1.38

$$\frac{2c^2x^2 \log(cx) \log\_integral\left(\frac{1}{c^2x^2}\right) + 1}{x^2 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^2,x, algorithm="fricas")

[Out] -(2\*c^2\*x^2\*log(c\*x)\*log\_integral(1/(c^2\*x^2)) + 1)/(x^2\*log(c\*x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^2(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^2,x, algorithm="giac")

[Out] integrate(1/(x^3\*log(c\*x)^2), x)

**maple** [A] time = 0.03, size = 26, normalized size = 1.08

$$2c^2 \text{Ei}(1, 2 \ln(cx)) - \frac{1}{x^2 \ln(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/ln(c*x)^2,x)`

[Out] `-1/x^2/ln(c*x)+2*c^2*Ei(1,2*ln(c*x))`

**maxima** [A] time = 0.83, size = 13, normalized size = 0.54

$$-2c^2\Gamma(-1, 2\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(c*x)^2,x, algorithm="maxima")`

[Out] `-2*c^2*gamma(-1, 2*log(c*x))`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^3 \ln(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*log(c*x)^2),x)`

[Out] `int(1/(x^3*log(c*x)^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \int \frac{1}{x^3 \log(cx)} dx - \frac{1}{x^2 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/ln(c*x)**2,x)`

[Out] `-2*Integral(1/(x**3*log(c*x)), x) - 1/(x**2*log(c*x))`

$$3.36 \quad \int \frac{x^3}{\log^3(cx)} dx$$

Optimal. Leaf size=37

$$\frac{8\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

[Out]  $8\text{Ei}(4\ln(c*x))/c^4 - 1/2*x^4/\ln(c*x)^2 - 2*x^4/\ln(c*x)$

**Rubi [A]** time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$\frac{8\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/\text{Log}[c*x]^3, x]$

[Out]  $(8*\text{ExpIntegralEi}[4*\text{Log}[c*x]])/c^4 - x^4/(2*\text{Log}[c*x]^2) - (2*x^4)/\text{Log}[c*x]$

Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d))*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d]})/d, x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \text{!}\$UseGamma === \text{True}$

Rule 2306

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)^(p_)*((d_.)*(x_))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^(p+1)/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p+1), x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{LtQ}[p, -1]$

Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)]*(b_.)^(p_)*(x_)^(m_.), x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\log^3(cx)} dx &= -\frac{x^4}{2\log^2(cx)} + 2 \int \frac{x^3}{\log^2(cx)} dx \\
&= -\frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)} + 8 \int \frac{x^3}{\log(cx)} dx \\
&= -\frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)} + \frac{8 \operatorname{Subst}\left(\int \frac{e^{4x}}{x} dx, x, \log(cx)\right)}{c^4} \\
&= \frac{8\operatorname{Ei}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{8\operatorname{Ei}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[c\*x]^3,x]

[Out] (8\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/(2\*Log[c\*x]^2) - (2\*x^4)/Log[c\*x]

**fricas** [A] time = 0.42, size = 47, normalized size = 1.27

$$-\frac{4c^4x^4\log(cx) + c^4x^4 - 16\log(cx)^2\log\_integral(c^4x^4)}{2c^4\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(4\*c^4\*x^4\*log(c\*x) + c^4\*x^4 - 16\*log(c\*x)^2\*log\_integral(c^4\*x^4))/(c^4\*log(c\*x)^2)

**giac** [A] time = 0.23, size = 35, normalized size = 0.95

$$-\frac{2x^4}{\log(cx)} - \frac{x^4}{2\log(cx)^2} + \frac{8\operatorname{Ei}(4\log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^3,x, algorithm="giac")

[Out]  $-2x^4/\log(cx) - 1/2x^4/\log(cx)^2 + 8\text{Ei}(4\log(cx))/c^4$

maple [A] time = 0.03, size = 37, normalized size = 1.00

$$-\frac{2x^4}{\ln(cx)} - \frac{x^4}{2\ln(cx)^2} - \frac{8\text{Ei}(1, -4\ln(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/ln(c*x)^3,x)`

[Out]  $-1/2x^4/\ln(c*x)^2 - 2x^4/\ln(c*x) - 8/c^4\text{Ei}(1, -4*\ln(c*x))$

maxima [A] time = 0.84, size = 13, normalized size = 0.35

$$-\frac{16\Gamma(-2, -4\log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/log(c*x)^3,x, algorithm="maxima")`

[Out]  $-16*\text{gamma}(-2, -4*\log(c*x))/c^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\ln(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/log(c*x)^3,x)`

[Out] `int(x^3/log(c*x)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-4x^4\log(cx) - x^4}{2\log(cx)^2} + 8 \int \frac{x^3}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/ln(c*x)**3,x)`

[Out]  $(-4*x**4*\log(c*x) - x**4)/(2*\log(c*x)**2) + 8*\text{Integral}(x**3/\log(c*x), x)$

$$3.37 \quad \int \frac{x^2}{\log^3(cx)} dx$$

Optimal. Leaf size=41

$$\frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

[Out]  $9/2*\text{Ei}(3*\ln(c*x))/c^3-1/2*x^3/\ln(c*x)^2-3/2*x^3/\ln(c*x)$

**Rubi [A]** time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$\frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[c\*x]^3,x]

[Out]  $(9*\text{ExpIntegralEi}[3*\text{Log}[c*x]])/(2*c^3) - x^3/(2*\text{Log}[c*x]^2) - (3*x^3)/(2*\text{Log}[c*x])$

#### Rule 2178

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_) \* ((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^ (p\_) \* (x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\log^3(cx)} dx &= -\frac{x^3}{2\log^2(cx)} + \frac{3}{2} \int \frac{x^2}{\log^2(cx)} dx \\
&= -\frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} + \frac{9}{2} \int \frac{x^2}{\log(cx)} dx \\
&= -\frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} + \frac{9 \operatorname{Subst}\left(\int \frac{e^{3x}}{x} dx, x, \log(cx)\right)}{2c^3} \\
&= \frac{9\operatorname{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 41, normalized size = 1.00

$$\frac{9\operatorname{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[c\*x]^3,x]

[Out] (9\*ExpIntegralEi[3\*Log[c\*x]])/(2\*c^3) - x^3/(2\*Log[c\*x]^2) - (3\*x^3)/(2\*Log[c\*x])

**fricas [A]** time = 0.60, size = 47, normalized size = 1.15

$$-\frac{3c^3x^3\log(cx) + c^3x^3 - 9\log(cx)^2\log\_integral(c^3x^3)}{2c^3\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(3\*c^3\*x^3\*log(c\*x) + c^3\*x^3 - 9\*log(c\*x)^2\*log\_integral(c^3\*x^3))/(c^3\*log(c\*x)^2)

**giac [A]** time = 0.22, size = 35, normalized size = 0.85

$$-\frac{3x^3}{2\log(cx)} - \frac{x^3}{2\log(cx)^2} + \frac{9\operatorname{Ei}(3\log(cx))}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^3,x, algorithm="giac")

[Out]  $-3/2*x^3/\log(c*x) - 1/2*x^3/\log(c*x)^2 + 9/2*Ei(3*\log(c*x))/c^3$

**maple** [A] time = 0.03, size = 37, normalized size = 0.90

$$-\frac{3x^3}{2\ln(cx)} - \frac{x^3}{2\ln(cx)^2} - \frac{9Ei(1, -3\ln(cx))}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(c\*x)^3,x)

[Out]  $-1/2*x^3/\ln(c*x)^2 - 3/2*x^3/\ln(c*x) - 9/2/c^3*Ei(1, -3*\ln(c*x))$

**maxima** [A] time = 0.87, size = 13, normalized size = 0.32

$$-\frac{9\Gamma(-2, -3\log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^3,x, algorithm="maxima")

[Out]  $-9*\gamma(-2, -3*\log(c*x))/c^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\ln(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/log(c\*x)^3,x)

[Out] int(x^2/log(c\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-3x^3 \log(cx) - x^3}{2\log(cx)^2} + \frac{9 \int \frac{x^2}{\log(cx)} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(c\*x)\*\*3,x)

[Out]  $(-3*x**3*\log(c*x) - x**3)/(2*\log(c*x)**2) + 9*Integral(x**2/\log(c*x), x)/2$



$$3.38 \quad \int \frac{x}{\log^3(cx)} dx$$

Optimal. Leaf size=37

$$\frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

[Out]  $2*\text{Ei}(2*\ln(c*x))/c^2-1/2*x^2/\ln(c*x)^2-x^2/\ln(c*x)$

**Rubi [A]** time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2306, 2309, 2178}

$$\frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c\*x]^3,x]

[Out]  $(2*\text{ExpIntegralEi}[2*\text{Log}[c*x]])/c^2 - x^2/(2*\text{Log}[c*x]^2) - x^2/\text{Log}[c*x]$

Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2309

Int[((a\_) + Log[(c\_)\*(x\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\log^3(cx)} dx &= -\frac{x^2}{2\log^2(cx)} + \int \frac{x}{\log^2(cx)} dx \\
&= -\frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)} + 2 \int \frac{x}{\log(cx)} dx \\
&= -\frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)} + \frac{2 \operatorname{Subst}\left(\int \frac{e^{2x}}{x} dx, x, \log(cx)\right)}{c^2} \\
&= \frac{2\operatorname{Ei}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{2\operatorname{Ei}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[c\*x]^3,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/(2\*Log[c\*x]^2) - x^2/Log[c\*x]

**fricas** [A] time = 0.42, size = 47, normalized size = 1.27

$$-\frac{2c^2x^2\log(cx) + c^2x^2 - 4\log(cx)^2\log\_integral(c^2x^2)}{2c^2\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(2\*c^2\*x^2\*log(c\*x) + c^2\*x^2 - 4\*log(c\*x)^2\*log\_integral(c^2\*x^2))/(c^2\*log(c\*x)^2)

**giac** [A] time = 0.22, size = 35, normalized size = 0.95

$$-\frac{x^2}{\log(cx)} - \frac{x^2}{2\log(cx)^2} + \frac{2\operatorname{Ei}(2\log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^3,x, algorithm="giac")

[Out]  $-x^2/\log(cx) - 1/2*x^2/\log(cx)^2 + 2*Ei(2*\log(cx))/c^2$

**maple** [A] time = 0.03, size = 37, normalized size = 1.00

$$-\frac{x^2}{\ln(cx)} - \frac{x^2}{2\ln(cx)^2} - \frac{2\operatorname{Ei}(1, -2\ln(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/ln(c*x)^3,x)`

[Out]  $-1/2*x^2/\ln(c*x)^2 - x^2/\ln(c*x) - 2/c^2*Ei(1, -2*\ln(c*x))$

**maxima** [A] time = 0.85, size = 13, normalized size = 0.35

$$-\frac{4\Gamma(-2, -2\log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/log(c*x)^3,x, algorithm="maxima")`

[Out]  $-4*\gamma(-2, -2*\log(c*x))/c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\ln(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/log(c*x)^3,x)`

[Out] `int(x/log(c*x)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2x^2 \log(cx) - x^2}{2\log(cx)^2} + 2 \int \frac{x}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/ln(c*x)**3,x)`

[Out]  $(-2*x**2*\log(c*x) - x**2)/(2*\log(c*x)**2) + 2*Integral(x/\log(c*x), x)$

$$3.39 \quad \int \frac{1}{\log^3(cx)} dx$$

Optimal. Leaf size=34

$$\frac{\text{li}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

[Out]  $1/2*\text{Li}(c*x)/c-1/2*x/\ln(c*x)^2-1/2*x/\ln(c*x)$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2297, 2298}

$$\frac{\text{li}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^(-3), x]

[Out]  $-x/(2*\text{Log}[c*x]^2) - x/(2*\text{Log}[c*x]) + \text{LogIntegral}[c*x]/(2*c)$

Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

Rule 2298

Int[Log[(c\_.)\*(x\_)]^(-1), x\_Symbol] := Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\log^3(cx)} dx &= -\frac{x}{2\log^2(cx)} + \frac{1}{2} \int \frac{1}{\log^2(cx)} dx \\ &= -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{1}{2} \int \frac{1}{\log(cx)} dx \\ &= -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{\text{li}(cx)}{2c} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{\text{li}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^(-3),x]

[Out] -1/2\*x/Log[c\*x]^2 - x/(2\*Log[c\*x]) + LogIntegral[c\*x]/(2\*c)

**fricas** [A] time = 0.40, size = 34, normalized size = 1.00

$$-\frac{cx \log(cx) - \log(cx)^2 \log\_integral(cx) + cx}{2c \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(c\*x\*log(c\*x) - log(c\*x)^2\*log\_integral(c\*x) + c\*x)/(c\*log(c\*x)^2)

**giac** [A] time = 0.22, size = 29, normalized size = 0.85

$$\frac{\text{Ei}(\log(cx))}{2c} - \frac{x}{2\log(cx)} - \frac{x}{2\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^3,x, algorithm="giac")

[Out] 1/2\*Ei(log(c\*x))/c - 1/2\*x/log(c\*x) - 1/2\*x/log(c\*x)^2

**maple** [A] time = 0.03, size = 33, normalized size = 0.97

$$-\frac{\text{Ei}(1, -\ln(cx))}{2c} - \frac{x}{2\ln(cx)} - \frac{x}{2\ln(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(c\*x)^3,x)

[Out] -1/2\*x/ln(c\*x)^2-1/2\*x/ln(c\*x)-1/2/c\*Ei(1,-ln(c\*x))

**maxima** [A] time = 0.79, size = 13, normalized size = 0.38

$$-\frac{\Gamma(-2, -\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^3,x, algorithm="maxima")

[Out] -gamma(-2, -log(c\*x))/c

mupad [B] time = 3.53, size = 29, normalized size = 0.85

$$\frac{\operatorname{logint}(cx)}{2c} - \frac{\frac{x}{2} + \frac{x \ln(cx)}{2}}{\ln(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/log(c\*x)^3,x)

[Out] logint(c\*x)/(2\*c) - (x/2 + (x\*log(c\*x))/2)/log(c\*x)^2

sympy [A] time = 0.50, size = 26, normalized size = 0.76

$$\frac{-x \log(cx) - x}{2 \log(cx)^2} + \frac{\operatorname{li}(cx)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(c\*x)\*\*3,x)

[Out] (-x\*log(c\*x) - x)/(2\*log(c\*x)\*\*2) + li(c\*x)/(2\*c)

$$3.40 \quad \int \frac{1}{x \log^3(cx)} dx$$

Optimal. Leaf size=10

$$-\frac{1}{2 \log^2(cx)}$$

[Out] -1/2/ln(c\*x)^2

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2302, 30}

$$-\frac{1}{2 \log^2(cx)}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[c\*x]^3), x]

[Out] -1/(2\*Log[c\*x]^2)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^3(cx)} dx &= \text{Subst} \left( \int \frac{1}{x^3} dx, x, \log(cx) \right) \\ &= -\frac{1}{2 \log^2(cx)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2 \log^2(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[c\*x]^3),x]

[Out] -1/2\*1/Log[c\*x]^2

**fricas** [A] time = 0.41, size = 8, normalized size = 0.80

$$-\frac{1}{2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2/log(c\*x)^2

**giac** [A] time = 0.20, size = 8, normalized size = 0.80

$$-\frac{1}{2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^3,x, algorithm="giac")

[Out] -1/2/log(c\*x)^2

**maple** [A] time = 0.02, size = 9, normalized size = 0.90

$$-\frac{1}{2 \ln(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(c\*x)^3,x)

[Out] -1/2/ln(c\*x)^2

**maxima** [A] time = 0.51, size = 8, normalized size = 0.80

$$-\frac{1}{2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^3,x, algorithm="maxima")



[Out]  $-1/2/\log(c*x)^2$

**mupad** [B] time = 3.45, size = 8, normalized size = 0.80

$$-\frac{1}{2 \ln(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*log(c*x)^3),x)`

[Out]  $-1/(2*\log(c*x)^2)$

**sympy** [A] time = 0.09, size = 10, normalized size = 1.00

$$-\frac{1}{2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(c*x)**3,x)`

[Out]  $-1/(2*\log(c*x)**2)$

$$3.41 \quad \int \frac{1}{x^2 \log^3(cx)} dx$$

Optimal. Leaf size=39

$$\frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

[Out]  $1/2*c*\text{Ei}(-\ln(c*x))-1/2/x/\ln(c*x)^2+1/2/x/\ln(c*x)$

**Rubi [A]** time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$\frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*Log[c*x]^3),x]`

[Out] `(c*ExpIntegralEi[-Log[c*x]])/2 - 1/(2*x*Log[c*x]^2) + 1/(2*x*Log[c*x])`

Rule 2178

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True`

Rule 2306

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

Rule 2309

`Int[((a_.) + Log[(c_.)*(x_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]`

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \log^3(cx)} dx &= -\frac{1}{2x \log^2(cx)} - \frac{1}{2} \int \frac{1}{x^2 \log^2(cx)} dx \\
&= -\frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)} + \frac{1}{2} \int \frac{1}{x^2 \log(cx)} dx \\
&= -\frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)} + \frac{1}{2} c \operatorname{Subst} \left( \int \frac{e^{-x}}{x} dx, x, \log(cx) \right) \\
&= \frac{1}{2} c \operatorname{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{1}{2} c \operatorname{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[c\*x]^3), x]

[Out] (c\*ExpIntegralEi[-Log[c\*x]])/2 - 1/(2\*x\*Log[c\*x]^2) + 1/(2\*x\*Log[c\*x])

**fricas** [A] time = 0.40, size = 34, normalized size = 0.87

$$\frac{cx \log(cx)^2 \log\_integral\left(\frac{1}{cx}\right) + \log(cx) - 1}{2x \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^3,x, algorithm="fricas")

[Out] 1/2\*(c\*x\*log(c\*x)^2\*log\_integral(1/(c\*x)) + log(c\*x) - 1)/(x\*log(c\*x)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log^3(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^3,x, algorithm="giac")

[Out] integrate(1/(x^2\*log(c\*x)^3), x)

**maple** [A] time = 0.03, size = 33, normalized size = 0.85

$$-\frac{c \operatorname{Ei}(1, \ln(cx))}{2} + \frac{1}{2x \ln(cx)} - \frac{1}{2x \ln(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/ln(c*x)^3,x)`

[Out] `-1/2/x/ln(c*x)^2+1/2/x/ln(c*x)-1/2*c*Ei(1,ln(c*x))`

**maxima** [A] time = 0.95, size = 9, normalized size = 0.23

$$-c\Gamma(-2, \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/log(c*x)^3,x, algorithm="maxima")`

[Out] `-c*gamma(-2, log(c*x))`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2 \ln(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*log(c*x)^3),x)`

[Out] `int(1/(x^2*log(c*x)^3), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2 \log(cx)} dx}{2} + \frac{\log(cx) - 1}{2x \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/ln(c*x)**3,x)`

[Out] `Integral(1/(x**2*log(c*x)), x)/2 + (log(c*x) - 1)/(2*x*log(c*x)**2)`

$$3.42 \quad \int \frac{1}{x^3 \log^3(cx)} dx$$

Optimal. Leaf size=36

$$2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

[Out]  $2*c^2*Ei(-2*\ln(c*x))-1/2/x^2/\ln(c*x)^2+1/x^2/\ln(c*x)$

**Rubi [A]** time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2306, 2309, 2178}

$$2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Log[c\*x]^3),x]

[Out]  $2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(2*x^2*Log[c*x]^2) + 1/(x^2*Log[c*x])$

Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2309

Int[((a\_) + Log[(c\_)\*(x\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \log^3(cx)} dx &= -\frac{1}{2x^2 \log^2(cx)} - \int \frac{1}{x^3 \log^2(cx)} dx \\
&= -\frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)} + 2 \int \frac{1}{x^3 \log(cx)} dx \\
&= -\frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)} + (2c^2) \text{Subst} \left( \int \frac{e^{-2x}}{x} dx, x, \log(cx) \right) \\
&= 2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 36, normalized size = 1.00

$$2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[c\*x]^3),x]

[Out] 2\*c^2\*ExpIntegralEi[-2\*Log[c\*x]] - 1/(2\*x^2\*Log[c\*x]^2) + 1/(x^2\*Log[c\*x])

**fricas** [A] time = 0.41, size = 41, normalized size = 1.14

$$\frac{4c^2x^2 \log(cx)^2 \log\_integral\left(\frac{1}{c^2x^2}\right) + 2 \log(cx) - 1}{2x^2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^3,x, algorithm="fricas")

[Out] 1/2\*(4\*c^2\*x^2\*log(c\*x)^2\*log\_integral(1/(c^2\*x^2)) + 2\*log(c\*x) - 1)/(x^2\*log(c\*x)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log^3(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^3,x, algorithm="giac")

[Out] integrate(1/(x^3\*log(c\*x)^3), x)

**maple** [A] time = 0.03, size = 36, normalized size = 1.00

$$-2c^2 \operatorname{Ei}(1, 2 \ln(cx)) + \frac{1}{x^2 \ln(cx)} - \frac{1}{2x^2 \ln(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(c\*x)^3,x)

[Out] -1/2/x^2/ln(c\*x)^2+1/x^2/ln(c\*x)-2\*c^2\*Ei(1,2\*ln(c\*x))

**maxima** [A] time = 0.81, size = 13, normalized size = 0.36

$$-4c^2\Gamma(-2, 2 \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^3,x, algorithm="maxima")

[Out] -4\*c^2\*gamma(-2, 2\*log(c\*x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3 \ln(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*log(c\*x)^3),x)

[Out] int(1/(x^3\*log(c\*x)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$2 \int \frac{1}{x^3 \log(cx)} dx + \frac{2 \log(cx) - 1}{2x^2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(c\*x)\*\*3,x)

[Out] 2\*Integral(1/(x\*\*3\*log(c\*x)), x) + (2\*log(c\*x) - 1)/(2\*x\*\*2\*log(c\*x)\*\*2)

### 3.43 $\int x^3 (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=27

$$\frac{1}{4}x^4(a + b \log(cx^n)) - \frac{1}{16}bnx^4$$

[Out]  $-1/16*b*n*x^4+1/4*x^4*(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$\frac{1}{4}x^4(a + b \log(cx^n)) - \frac{1}{16}bnx^4$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*\text{Log}[c*x^n]), x]$

[Out]  $-(b*n*x^4)/16 + (x^4*(a + b*\text{Log}[c*x^n]))/4$

**Rule 2304**

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

**Rubi steps**

$$\int x^3 (a + b \log(cx^n)) dx = -\frac{1}{16}bnx^4 + \frac{1}{4}x^4 (a + b \log(cx^n))$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 1.19

$$\frac{ax^4}{4} + \frac{1}{4}bx^4 \log(cx^n) - \frac{1}{16}bnx^4$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3*(a + b*\text{Log}[c*x^n]), x]$

[Out]  $(a*x^4)/4 - (b*n*x^4)/16 + (b*x^4*\text{Log}[c*x^n])/4$



**fricas** [A] time = 0.41, size = 30, normalized size = 1.11

$$\frac{1}{4} b n x^4 \log(x) + \frac{1}{4} b x^4 \log(c) - \frac{1}{16} (b n - 4 a) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] 1/4\*b\*n\*x^4\*log(x) + 1/4\*b\*x^4\*log(c) - 1/16\*(b\*n - 4\*a)\*x^4

**giac** [A] time = 0.23, size = 31, normalized size = 1.15

$$\frac{1}{4} b n x^4 \log(x) - \frac{1}{16} b n x^4 + \frac{1}{4} b x^4 \log(c) + \frac{1}{4} a x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 1/4\*b\*n\*x^4\*log(x) - 1/16\*b\*n\*x^4 + 1/4\*b\*x^4\*log(c) + 1/4\*a\*x^4

**maple** [C] time = 0.22, size = 112, normalized size = 4.15

$$\frac{b x^4 \ln(x^n)}{4} + \frac{(-2i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 2i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 2i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*ln(c\*x^n)),x)

[Out] 1/4\*b\*x^4\*ln(x^n)+1/16\*x^4\*(2\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-2\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-2\*I\*b\*Pi\*csgn(I\*c\*x^n)^3+2\*I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+4\*b\*ln(c)-b\*n+4\*a)

**maxima** [A] time = 0.50, size = 26, normalized size = 0.96

$$-\frac{1}{16} b n x^4 + \frac{1}{4} b x^4 \log(c x^n) + \frac{1}{4} a x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -1/16\*b\*n\*x^4 + 1/4\*b\*x^4\*log(c\*x^n) + 1/4\*a\*x^4

**mupad** [B] time = 3.59, size = 25, normalized size = 0.93

$$x^4 \left( \frac{a}{4} - \frac{b n}{16} \right) + \frac{b x^4 \ln(c x^n)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b*log(c*x^n)),x)`

[Out]  $x^4*(a/4 - (b*n)/16) + (b*x^4*log(c*x^n))/4$

sympy [A] time = 1.36, size = 36, normalized size = 1.33

$$\frac{ax^4}{4} + \frac{bnx^4 \log(x)}{4} - \frac{bnx^4}{16} + \frac{bx^4 \log(c)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*ln(c*x**n)),x)`

[Out]  $a*x**4/4 + b*n*x**4*log(x)/4 - b*n*x**4/16 + b*x**4*log(c)/4$

### 3.44 $\int x^2 (a + b \log(cx^n)) dx$

Optimal. Leaf size=27

$$\frac{1}{3}x^3(a + b \log(cx^n)) - \frac{1}{9}bnx^3$$

[Out]  $-1/9*b*n*x^3+1/3*x^3*(a+b*\ln(c*x^n))$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$\frac{1}{3}x^3(a + b \log(cx^n)) - \frac{1}{9}bnx^3$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{Log}[c*x^n]), x]$

[Out]  $-(b*n*x^3)/9 + (x^3*(a + b*\text{Log}[c*x^n]))/3$

Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]*((d_.)*(x_))^(m_.), x\_Symbol] \text{ :>}$   
 $\text{Simp}[(d*x)^(m + 1)*(a + b*\text{Log}[c*x^n])]/(d*(m + 1)), x] - \text{Simp}[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x^2 (a + b \log(cx^n)) dx = -\frac{1}{9}bnx^3 + \frac{1}{3}x^3 (a + b \log(cx^n))$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.19

$$\frac{ax^3}{3} + \frac{1}{3}bx^3 \log(cx^n) - \frac{1}{9}bnx^3$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2*(a + b*\text{Log}[c*x^n]), x]$

[Out]  $(a*x^3)/3 - (b*n*x^3)/9 + (b*x^3*\text{Log}[c*x^n])/3$

**fricas** [A] time = 0.42, size = 30, normalized size = 1.11

$$\frac{1}{3} b n x^3 \log(x) + \frac{1}{3} b x^3 \log(c) - \frac{1}{9} (b n - 3 a) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] 1/3\*b\*n\*x^3\*log(x) + 1/3\*b\*x^3\*log(c) - 1/9\*(b\*n - 3\*a)\*x^3

**giac** [A] time = 0.29, size = 31, normalized size = 1.15

$$\frac{1}{3} b n x^3 \log(x) - \frac{1}{9} b n x^3 + \frac{1}{3} b x^3 \log(c) + \frac{1}{3} a x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 1/3\*b\*n\*x^3\*log(x) - 1/9\*b\*n\*x^3 + 1/3\*b\*x^3\*log(c) + 1/3\*a\*x^3

**maple** [C] time = 0.18, size = 112, normalized size = 4.15

$$\frac{b x^3 \ln(x^n)}{3} + \frac{(-3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 3i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*ln(c\*x^n)),x)

[Out] 1/3\*b\*x^3\*ln(x^n)+1/18\*x^3\*(3\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-3\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-3\*I\*b\*Pi\*csgn(I\*c\*x^n)^3+3\*I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+6\*b\*ln(c)-2\*b\*n+6\*a)

**maxima** [A] time = 0.55, size = 26, normalized size = 0.96

$$-\frac{1}{9} b n x^3 + \frac{1}{3} b x^3 \log(c x^n) + \frac{1}{3} a x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -1/9\*b\*n\*x^3 + 1/3\*b\*x^3\*log(c\*x^n) + 1/3\*a\*x^3

**mupad** [B] time = 3.36, size = 25, normalized size = 0.93

$$x^3 \left( \frac{a}{3} - \frac{b n}{9} \right) + \frac{b x^3 \ln(c x^n)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*log(c*x^n)),x)`

[Out]  $x^3*(a/3 - (b*n)/9) + (b*x^3*log(c*x^n))/3$

sympy [A] time = 0.78, size = 36, normalized size = 1.33

$$\frac{ax^3}{3} + \frac{bnx^3 \log(x)}{3} - \frac{bnx^3}{9} + \frac{bx^3 \log(c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*ln(c*x**n)),x)`

[Out]  $a*x**3/3 + b*n*x**3*log(x)/3 - b*n*x**3/9 + b*x**3*log(c)/3$

### 3.45 $\int x (a + b \log (cx^n)) dx$

**Optimal.** Leaf size=27

$$\frac{1}{2}x^2 (a + b \log (cx^n)) - \frac{1}{4}bnx^2$$

[Out]  $-1/4*b*n*x^2+1/2*x^2*(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {2304}

$$\frac{1}{2}x^2 (a + b \log (cx^n)) - \frac{1}{4}bnx^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{Log}[c*x^n]), x]$

[Out]  $-(b*n*x^2)/4 + (x^2*(a + b*\text{Log}[c*x^n]))/2$

**Rule 2304**

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^(n_.)]*(b_.)]*((d_.)*(x_)^(m_.), x\_Symbol] :>$   
 $\text{Simp}[(d*x)^(m + 1)*(a + b*\text{Log}[c*x^n])]/(d*(m + 1)), x] - \text{Simp}[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int x (a + b \log (cx^n)) dx = -\frac{1}{4}bnx^2 + \frac{1}{2}x^2 (a + b \log (cx^n))$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 1.19

$$\frac{ax^2}{2} + \frac{1}{2}bx^2 \log (cx^n) - \frac{1}{4}bnx^2$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x*(a + b*\text{Log}[c*x^n]), x]$

[Out]  $(a*x^2)/2 - (b*n*x^2)/4 + (b*x^2*\text{Log}[c*x^n])/2$

**fricas** [A] time = 0.45, size = 30, normalized size = 1.11

$$\frac{1}{2} b n x^2 \log(x) + \frac{1}{2} b x^2 \log(c) - \frac{1}{4} (b n - 2 a) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] 1/2\*b\*n\*x^2\*log(x) + 1/2\*b\*x^2\*log(c) - 1/4\*(b\*n - 2\*a)\*x^2

**giac** [A] time = 0.25, size = 31, normalized size = 1.15

$$\frac{1}{2} b n x^2 \log(x) - \frac{1}{4} b n x^2 + \frac{1}{2} b x^2 \log(c) + \frac{1}{2} a x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 1/2\*b\*n\*x^2\*log(x) - 1/4\*b\*n\*x^2 + 1/2\*b\*x^2\*log(c) + 1/2\*a\*x^2

**maple** [A] time = 0.05, size = 29, normalized size = 1.07

$$-\frac{b n x^2}{4} + \frac{b x^2 \ln(c e^{n \ln(x)})}{2} + \frac{a x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*ln(c\*x^n)),x)

[Out] 1/2\*a\*x^2+1/2\*b\*x^2\*ln(c\*exp(n\*ln(x)))-1/4\*b\*n\*x^2

**maxima** [A] time = 0.61, size = 26, normalized size = 0.96

$$-\frac{1}{4} b n x^2 + \frac{1}{2} b x^2 \log(c x^n) + \frac{1}{2} a x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -1/4\*b\*n\*x^2 + 1/2\*b\*x^2\*log(c\*x^n) + 1/2\*a\*x^2

**mupad** [B] time = 3.26, size = 25, normalized size = 0.93

$$x^2 \left( \frac{a}{2} - \frac{b n}{4} \right) + \frac{b x^2 \ln(c x^n)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*log(c*x^n)),x)`

[Out]  $x^2*(a/2 - (b*n)/4) + (b*x^2*log(c*x^n))/2$

sympy [A] time = 0.46, size = 36, normalized size = 1.33

$$\frac{ax^2}{2} + \frac{bnx^2 \log(x)}{2} - \frac{bnx^2}{4} + \frac{bx^2 \log(c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*ln(c*x**n)),x)`

[Out]  $a*x**2/2 + b*n*x**2*log(x)/2 - b*n*x**2/4 + b*x**2*log(c)/2$



### 3.46 $\int (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=18

$$ax + bx \log(cx^n) - bnx$$

[Out] a\*x-b\*n\*x+b\*x\*ln(c\*x^n)

**Rubi [A]** time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2295}

$$ax + bx \log(cx^n) - bnx$$

Antiderivative was successfully verified.

[In] Int[a + b\*Log[c\*x^n], x]

[Out] a\*x - b\*n\*x + b\*x\*Log[c\*x^n]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rubi steps

$$\begin{aligned} \int (a + b \log(cx^n)) dx &= ax + b \int \log(cx^n) dx \\ &= ax - bnx + bx \log(cx^n) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 18, normalized size = 1.00

$$ax + bx \log(cx^n) - bnx$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*Log[c\*x^n], x]

[Out] a\*x - b\*n\*x + b\*x\*Log[c\*x^n]

**fricas [A]** time = 0.44, size = 22, normalized size = 1.22

$$bnx \log(x) + bx \log(c) - (bn - a)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*log(c\*x^n),x, algorithm="fricas")

[Out] b\*n\*x\*log(x) + b\*x\*log(c) - (b\*n - a)\*x

**giac** [A] time = 0.27, size = 20, normalized size = 1.11

$$(nx \log(x) - nx + x \log(c))b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*log(c\*x^n),x, algorithm="giac")

[Out] (n\*x\*log(x) - n\*x + x\*log(c))\*b + a\*x

**maple** [A] time = 0.03, size = 19, normalized size = 1.06

$$-bnx + bx \ln(cx^n) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*ln(c\*x^n),x)

[Out] a\*x-b\*n\*x+b\*x\*ln(c\*x^n)

**maxima** [A] time = 0.59, size = 18, normalized size = 1.00

$$-bnx + bx \log(cx^n) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*log(c\*x^n),x, algorithm="maxima")

[Out] -b\*n\*x + b\*x\*log(c\*x^n) + a\*x

**mupad** [B] time = 3.50, size = 18, normalized size = 1.00

$$x(a - bn) + bx \ln(cx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b\*log(c\*x^n),x)

[Out] x\*(a - b\*n) + b\*x\*log(c\*x^n)

**sympy** [A] time = 0.25, size = 19, normalized size = 1.06

$$ax + b(nx \log(x) - nx + x \log(c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*ln(c\*x\*\*n),x)

[Out] a\*x + b\*(n\*x\*log(x) - n\*x + x\*log(c))

$$3.47 \quad \int \frac{a+b \log(cx^n)}{x} dx$$

Optimal. Leaf size=22

$$\frac{(a + b \log(cx^n))^2}{2bn}$$

[Out] 1/2\*(a+b\*ln(c\*x^n))^2/b/n

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2301}

$$\frac{(a + b \log(cx^n))^2}{2bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/x,x]

[Out] (a + b\*Log[c\*x^n])^2/(2\*b\*n)

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\int \frac{a + b \log(cx^n)}{x} dx = \frac{(a + b \log(cx^n))^2}{2bn}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.95

$$a \log(x) + \frac{b \log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/x,x]

[Out] a\*Log[x] + (b\*Log[c\*x^n]^2)/(2\*n)

**fricas** [A] time = 0.45, size = 18, normalized size = 0.82

$$\frac{1}{2}bn \log(x)^2 + (b \log(c) + a) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x,x, algorithm="fricas")

[Out] 1/2\*b\*n\*log(x)^2 + (b\*log(c) + a)\*log(x)

**giac** [A] time = 0.22, size = 19, normalized size = 0.86

$$\frac{1}{2}bn \log(x)^2 + b \log(c) \log(x) + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x,x, algorithm="giac")

[Out] 1/2\*b\*n\*log(x)^2 + b\*log(c)\*log(x) + a\*log(x)

**maple** [A] time = 0.03, size = 27, normalized size = 1.23

$$\frac{b \ln(cx^n)^2}{2n} + \frac{a \ln(cx^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/x,x)

[Out] 1/n\*a\*ln(c\*x^n)+1/2/n\*b\*ln(c\*x^n)^2

**maxima** [A] time = 0.44, size = 20, normalized size = 0.91

$$\frac{(b \log(cx^n) + a)^2}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x,x, algorithm="maxima")

[Out] 1/2\*(b\*log(c\*x^n) + a)^2/(b\*n)

**mupad** [B] time = 3.41, size = 19, normalized size = 0.86

$$a \ln(x) + \frac{b \ln(cx^n)^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*x^n))/x,x)
```

```
[Out] a*log(x) + (b*log(c*x^n)^2)/(2*n)
```

sympy [A] time = 8.60, size = 34, normalized size = 1.55

$$\begin{cases} a \log(x) & \text{for } b = 0 \\ -(-a - b \log(c)) \log(x) & \text{for } n = 0 \\ \frac{(-a - b \log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**n))**2/(2*b*n), True))
```

$$3.48 \quad \int \frac{a+b \log(cx^n)}{x^2} dx$$

Optimal. Leaf size=23

$$-\frac{a+b \log(cx^n)}{x} - \frac{bn}{x}$$

[Out]  $-b*n/x+(-a-b*\ln(c*x^n))/x$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$-\frac{a+b \log(cx^n)}{x} - \frac{bn}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/x^2, x]

[Out] -((b\*n)/x) - (a + b\*Log[c\*x^n])/x

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{a+b \log(cx^n)}{x^2} dx = -\frac{bn}{x} - \frac{a+b \log(cx^n)}{x}$$

**Mathematica [A]** time = 0.00, size = 26, normalized size = 1.13

$$-\frac{a}{x} - \frac{b \log(cx^n)}{x} - \frac{bn}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/x^2, x]

[Out] -(a/x) - (b\*n)/x - (b\*Log[c\*x^n])/x

**fricas [A]** time = 0.46, size = 19, normalized size = 0.83

$$\frac{bn \log(x) + bn + b \log(c) + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^2,x, algorithm="fricas")

[Out] -(b\*n\*log(x) + b\*n + b\*log(c) + a)/x

**giac** [A] time = 0.25, size = 24, normalized size = 1.04

$$-\frac{bn \log(x)}{x} - \frac{bn + b \log(c) + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^2,x, algorithm="giac")

[Out] -b\*n\*log(x)/x - (b\*n + b\*log(c) + a)/x

**maple** [C] time = 0.12, size = 112, normalized size = 4.87

$$\frac{b \ln(x^n)}{x} - \frac{-i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2 - i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/x^2,x)

[Out] -b/x\*ln(x^n)-1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*b\*ln(c)+2\*b\*n+2\*a)/x

**maxima** [A] time = 0.55, size = 26, normalized size = 1.13

$$-\frac{bn}{x} - \frac{b \log(cx^n)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^2,x, algorithm="maxima")

[Out] -b\*n/x - b\*log(c\*x^n)/x - a/x

**mupad** [B] time = 3.56, size = 23, normalized size = 1.00

$$-\frac{a + bn}{x} - \frac{b \ln(cx^n)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))/x^2,x)

[Out]  $-(a + b*n)/x - (b*\log(c*x^n))/x$

sympy [A] time = 0.46, size = 24, normalized size = 1.04

$$-\frac{a}{x} - \frac{bn \log(x)}{x} - \frac{bn}{x} - \frac{b \log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/x**2,x)`

[Out]  $-a/x - b*n*\log(x)/x - b*n/x - b*\log(c)/x$



$$3.49 \quad \int \frac{a+b \log(cx^n)}{x^3} dx$$

Optimal. Leaf size=27

$$-\frac{a+b \log(cx^n)}{2x^2} - \frac{bn}{4x^2}$$

[Out]  $-1/4*b*n/x^2+1/2*(-a-b*\ln(c*x^n))/x^2$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$-\frac{a+b \log(cx^n)}{2x^2} - \frac{bn}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/x^3, x]

[Out]  $-(b*n)/(4*x^2) - (a + b*Log[c*x^n])/(2*x^2)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{a+b \log(cx^n)}{x^3} dx = -\frac{bn}{4x^2} - \frac{a+b \log(cx^n)}{2x^2}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.19

$$-\frac{a}{2x^2} - \frac{b \log(cx^n)}{2x^2} - \frac{bn}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/x^3, x]

[Out]  $-1/2*a/x^2 - (b*n)/(4*x^2) - (b*Log[c*x^n])/(2*x^2)$

fricas [A] time = 0.43, size = 23, normalized size = 0.85

$$-\frac{2bn \log(x) + bn + 2b \log(c) + 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*b\*n\*log(x) + b\*n + 2\*b\*log(c) + 2\*a)/x^2

**giac** [A] time = 0.25, size = 27, normalized size = 1.00

$$-\frac{bn \log(x)}{2x^2} - \frac{bn + 2b \log(c) + 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^3,x, algorithm="giac")

[Out] -1/2\*b\*n\*log(x)/x^2 - 1/4\*(b\*n + 2\*b\*log(c) + 2\*a)/x^2

**maple** [C] time = 0.11, size = 111, normalized size = 4.11

$$-\frac{b \ln(x^n)}{2x^2} - \frac{-i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(icx^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)^2 - i\pi b}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/x^3,x)

[Out] -1/2\*b/x^2\*ln(x^n)-1/4\*(I\*Pi\*b\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*b\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)-I\*Pi\*b\*csgn(I\*c\*x^n)^3+I\*Pi\*b\*csgn(I\*c)\*csgn(I\*c\*x^n)^2+2\*b\*ln(c)+b\*n+2\*a)/x^2

**maxima** [A] time = 0.53, size = 26, normalized size = 0.96

$$-\frac{bn}{4x^2} - \frac{b \log(cx^n)}{2x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^3,x, algorithm="maxima")

[Out] -1/4\*b\*n/x^2 - 1/2\*b\*log(c\*x^n)/x^2 - 1/2\*a/x^2

**mupad** [B] time = 3.49, size = 26, normalized size = 0.96

$$-\frac{\frac{a}{2} + \frac{bn}{4}}{x^2} - \frac{b \ln(cx^n)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))/x^3,x)`

[Out]  $-(a/2 + (b*n)/4)/x^2 - (b*\log(c*x^n))/(2*x^2)$

**sympy [A]** time = 0.94, size = 37, normalized size = 1.37

$$-\frac{a}{2x^2} - \frac{bn \log(x)}{2x^2} - \frac{bn}{4x^2} - \frac{b \log(c)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/x**3,x)`

[Out]  $-a/(2*x**2) - b*n*\log(x)/(2*x**2) - b*n/(4*x**2) - b*\log(c)/(2*x**2)$

### 3.50 $\int x^3 (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=52

$$\frac{1}{4}x^4 (a + b \log(cx^n))^2 - \frac{1}{8}bnx^4 (a + b \log(cx^n)) + \frac{1}{32}b^2n^2x^4$$

[Out]  $1/32*b^2*n^2*x^4-1/8*b*n*x^4*(a+b*\ln(c*x^n))+1/4*x^4*(a+b*\ln(c*x^n))^2$

**Rubi [A]** time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{1}{4}x^4 (a + b \log(cx^n))^2 - \frac{1}{8}bnx^4 (a + b \log(cx^n)) + \frac{1}{32}b^2n^2x^4$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*\text{Log}[c*x^n])^2, x]$

[Out]  $(b^2*n^2*x^4)/32 - (b*n*x^4*(a + b*\text{Log}[c*x^n]))/8 + (x^4*(a + b*\text{Log}[c*x^n])^2)/4$

#### Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 2305

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^p]/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{GtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned} \int x^3 (a + b \log(cx^n))^2 dx &= \frac{1}{4}x^4 (a + b \log(cx^n))^2 - \frac{1}{2}(bn) \int x^3 (a + b \log(cx^n)) dx \\ &= \frac{1}{32}b^2n^2x^4 - \frac{1}{8}bnx^4 (a + b \log(cx^n)) + \frac{1}{4}x^4 (a + b \log(cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 43, normalized size = 0.83

$$\frac{1}{32}x^4 \left( -4bn(a + b \log(cx^n)) + 8(a + b \log(cx^n))^2 + b^2n^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Log[c\*x^n])^2,x]

[Out] (x^4\*(b^2\*n^2 - 4\*b\*n\*(a + b\*Log[c\*x^n]) + 8\*(a + b\*Log[c\*x^n])^2))/32

**fricas [B]** time = 0.42, size = 102, normalized size = 1.96

$$\frac{1}{4}b^2n^2x^4 \log(x)^2 + \frac{1}{4}b^2x^4 \log(c)^2 - \frac{1}{8}(b^2n - 4ab)x^4 \log(c) + \frac{1}{32}(b^2n^2 - 4abn + 8a^2)x^4 + \frac{1}{8}(4b^2nx^4 \log(c) - (b^2n^2 - 4abn + 8a^2)x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 1/4\*b^2\*n^2\*x^4\*log(x)^2 + 1/4\*b^2\*x^4\*log(c)^2 - 1/8\*(b^2\*n - 4\*a\*b)\*x^4\*log(c) + 1/32\*(b^2\*n^2 - 4\*a\*b\*n + 8\*a^2)\*x^4 + 1/8\*(4\*b^2\*n\*x^4\*log(c) - (b^2\*n^2 - 4\*a\*b\*n)\*x^4)\*log(x)

**giac [B]** time = 0.39, size = 111, normalized size = 2.13

$$\frac{1}{4}b^2n^2x^4 \log(x)^2 - \frac{1}{8}b^2n^2x^4 \log(x) + \frac{1}{2}b^2nx^4 \log(c) \log(x) + \frac{1}{32}b^2n^2x^4 - \frac{1}{8}b^2nx^4 \log(c) + \frac{1}{4}b^2x^4 \log(c)^2 + \frac{1}{2}abnx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 1/4\*b^2\*n^2\*x^4\*log(x)^2 - 1/8\*b^2\*n^2\*x^4\*log(x) + 1/2\*b^2\*n\*x^4\*log(c)\*log(x) + 1/32\*b^2\*n^2\*x^4 - 1/8\*b^2\*n\*x^4\*log(c) + 1/4\*b^2\*x^4\*log(c)^2 + 1/2\*a\*b\*n\*x^4\*log(x) - 1/8\*a\*b\*n\*x^4 + 1/2\*a\*b\*x^4\*log(c) + 1/4\*a^2\*x^4

**maple [C]** time = 0.21, size = 691, normalized size = 13.29

$$\frac{b^2x^4 \ln(x^n)^2}{4} + \frac{(-2i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) + 2i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(icx^n)^2 + 2i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*ln(c\*x^n))^2,x)

[Out] 1/4\*b^2\*x^4\*ln(x^n)^2+1/8\*b\*x^4\*(2\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-2\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-2\*I\*b\*Pi\*csgn(I\*c\*x^n)^3+2\*I\*b\*Pi\*c

$\text{sgn}(I*c*x^n)^2 * \text{csgn}(I*c) + 4*b*\ln(c) - b*n + 4*a) * \ln(x^n) + 1/32*x^4 * (-8*I*\ln(c)*\text{Pi} * b^2 * \text{csgn}(I*c*x^n)^3 - 8*I*\text{Pi}*a*b * \text{csgn}(I*c*x^n)^3 - 2*\text{Pi}^2*b^2 * \text{csgn}(I*x^n)^2 * \text{csgn}(I*c*x^n)^2 * \text{csgn}(I*c) - 2*8*\text{Pi}^2*b^2 * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n)^4 * \text{csgn}(I*c) + 4*\text{Pi}^2*b^2 * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n)^3 * \text{csgn}(I*c)^2 + 2*I*\text{Pi}*b^2*n * \text{csgn}(I*c*x^n)^3 + 4*\text{Pi}^2*b^2 * \text{csgn}(I*x^n)^2 * \text{csgn}(I*c*x^n)^3 * \text{csgn}(I*c) + 8*a^2 + b^2*n^2 + 16*\ln(c) * a*b - 4*\ln(c) * b^2*n + 8*\ln(c)^2 * b^2 - 2*\text{Pi}^2*b^2 * \text{csgn}(I*x^n)^2 * \text{csgn}(I*c*x^n)^4 + 4*\text{Pi}^2*b^2 * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n)^5 - 4*a*b*n - 2*\text{Pi}^2*b^2 * \text{csgn}(I*c*x^n)^6 + 4*\text{Pi}^2*b^2 * \text{csgn}(I*c*x^n)^5 * \text{csgn}(I*c) - 2*\text{Pi}^2*b^2 * \text{csgn}(I*c*x^n)^4 * \text{csgn}(I*c)^2 + 8*I*\text{Pi}*a*b * \text{csgn}(I*c*x^n)^2 * \text{csgn}(I*c) - 2*I*\text{Pi}*b^2*n * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n)^2 - 2*I*\text{Pi}*b^2*n * \text{csgn}(I*c*x^n)^2 * \text{csgn}(I*c) + 8*I*\ln(c) * \text{Pi}*b^2 * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n)^2 + 8*I*\ln(c) * \text{Pi}*b^2 * \text{csgn}(I*c*x^n)^2 * \text{csgn}(I*c) + 8*I*\text{Pi}*a*b * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n)^2 - 8*I*\ln(c) * \text{Pi}*b^2 * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n) * \text{csgn}(I*c) - 8 * I*\text{Pi}*a*b * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n) * \text{csgn}(I*c) + 2*I*\text{Pi}*b^2*n * \text{csgn}(I*x^n) * \text{csgn}(I*c*x^n) * \text{csgn}(I*c))$

**maxima** [A] time = 0.51, size = 71, normalized size = 1.37

$$\frac{1}{4} b^2 x^4 \log(cx^n)^2 - \frac{1}{8} abn x^4 + \frac{1}{2} abx^4 \log(cx^n) + \frac{1}{4} a^2 x^4 + \frac{1}{32} (n^2 x^4 - 4nx^4 \log(cx^n)) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 1/4\*b^2\*x^4\*log(c\*x^n)^2 - 1/8\*a\*b\*n\*x^4 + 1/2\*a\*b\*x^4\*log(c\*x^n) + 1/4\*a^2\*x^4 + 1/32\*(n^2\*x^4 - 4\*n\*x^4\*log(c\*x^n))\*b^2

**mupad** [B] time = 3.60, size = 61, normalized size = 1.17

$$x^4 \left( \frac{a^2}{4} - \frac{abn}{8} + \frac{b^2 n^2}{32} \right) + \frac{x^4 \ln(cx^n) \left( ab - \frac{b^2 n}{4} \right)}{2} + \frac{b^2 x^4 \ln(cx^n)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b\*log(c\*x^n))^2,x)

[Out] x^4\*(a^2/4 + (b^2\*n^2)/32 - (a\*b\*n)/8) + (x^4\*log(c\*x^n)\*(a\*b - (b^2\*n)/4))/2 + (b^2\*x^4\*log(c\*x^n)^2)/4

**sympy** [B] time = 2.40, size = 131, normalized size = 2.52

$$\frac{a^2 x^4}{4} + \frac{abn x^4 \log(x)}{2} - \frac{abn x^4}{8} + \frac{abx^4 \log(c)}{2} + \frac{b^2 n^2 x^4 \log(x)^2}{4} - \frac{b^2 n^2 x^4 \log(x)}{8} + \frac{b^2 n^2 x^4}{32} + \frac{b^2 n x^4 \log(c) \log(x)}{2} - \frac{b^2 n x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*x**4/4 + a*b*n*x**4*log(x)/2 - a*b*n*x**4/8 + a*b*x**4*log(c)/2 + b**2  
*n**2*x**4*log(x)**2/4 - b**2*n**2*x**4*log(x)/8 + b**2*n**2*x**4/32 + b**2  
*n*x**4*log(c)*log(x)/2 - b**2*n*x**4*log(c)/8 + b**2*x**4*log(c)**2/4
```

### 3.51 $\int x^2 (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=52

$$\frac{1}{3}x^3 (a + b \log(cx^n))^2 - \frac{2}{9}bnx^3 (a + b \log(cx^n)) + \frac{2}{27}b^2n^2x^3$$

[Out]  $2/27*b^2*n^2*x^3-2/9*b*n*x^3*(a+b*\ln(c*x^n))+1/3*x^3*(a+b*\ln(c*x^n))^2$

**Rubi [A]** time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{1}{3}x^3 (a + b \log(cx^n))^2 - \frac{2}{9}bnx^3 (a + b \log(cx^n)) + \frac{2}{27}b^2n^2x^3$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{Log}[c*x^n])^2, x]$

[Out]  $(2*b^2*n^2*x^3)/27 - (2*b*n*x^3*(a + b*\text{Log}[c*x^n]))/9 + (x^3*(a + b*\text{Log}[c*x^n])^2)/3$

#### Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^p]/(d*(m+1)), x] - \text{Dist}[(b*n^p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \log(cx^n))^2 dx &= \frac{1}{3}x^3 (a + b \log(cx^n))^2 - \frac{1}{3}(2bn) \int x^2 (a + b \log(cx^n)) dx \\ &= \frac{2}{27}b^2n^2x^3 - \frac{2}{9}bnx^3 (a + b \log(cx^n)) + \frac{1}{3}x^3 (a + b \log(cx^n))^2 \end{aligned}$$



**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.88

$$\frac{1}{3} \left( x^3 (a + b \log(cx^n))^2 + \frac{2}{9} b n x^3 (-3a - 3b \log(cx^n) + b n) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x^n])^2,x]

[Out] ((2\*b\*n\*x^3\*(-3\*a + b\*n - 3\*b\*Log[c\*x^n]))/9 + x^3\*(a + b\*Log[c\*x^n])^2)/3

**fricas [B]** time = 0.42, size = 103, normalized size = 1.98

$$\frac{1}{3} b^2 n^2 x^3 \log(x)^2 + \frac{1}{3} b^2 x^3 \log(c)^2 - \frac{2}{9} (b^2 n - 3 a b) x^3 \log(c) + \frac{1}{27} (2 b^2 n^2 - 6 a b n + 9 a^2) x^3 + \frac{2}{9} (3 b^2 n x^3 \log(c) - (b^2 n^2 - 3 a b n + 9 a^2) x^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 1/3\*b^2\*n^2\*x^3\*log(x)^2 + 1/3\*b^2\*x^3\*log(c)^2 - 2/9\*(b^2\*n - 3\*a\*b)\*x^3\*log(c) + 1/27\*(2\*b^2\*n^2 - 6\*a\*b\*n + 9\*a^2)\*x^3 + 2/9\*(3\*b^2\*n\*x^3\*log(c) - (b^2\*n^2 - 3\*a\*b\*n)\*x^3)\*log(x)

**giac [B]** time = 0.35, size = 111, normalized size = 2.13

$$\frac{1}{3} b^2 n^2 x^3 \log(x)^2 - \frac{2}{9} b^2 n^2 x^3 \log(x) + \frac{2}{3} b^2 n x^3 \log(c) \log(x) + \frac{2}{27} b^2 n^2 x^3 - \frac{2}{9} b^2 n x^3 \log(c) + \frac{1}{3} b^2 x^3 \log(c)^2 + \frac{2}{3} a b n x^3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 1/3\*b^2\*n^2\*x^3\*log(x)^2 - 2/9\*b^2\*n^2\*x^3\*log(x) + 2/3\*b^2\*n\*x^3\*log(c)\*log(x) + 2/27\*b^2\*n^2\*x^3 - 2/9\*b^2\*n\*x^3\*log(c) + 1/3\*b^2\*x^3\*log(c)^2 + 2/3\*a\*b\*n\*x^3\*log(x) - 2/9\*a\*b\*n\*x^3 + 2/3\*a\*b\*x^3\*log(c) + 1/3\*a^2\*x^3

**maple [C]** time = 0.21, size = 692, normalized size = 13.31

$$\frac{b^2 x^3 \ln(x^n)^2}{3} + \frac{(-3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 3i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)) x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*ln(c\*x^n))^2,x)

[Out] 1/3\*x^3\*b^2\*ln(x^n)^2+1/9\*b\*x^3\*(-3\*I\*Pi\*b\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)+3\*I\*Pi\*b\*csgn(I\*c)\*csgn(I\*c\*x^n)^2+3\*I\*Pi\*b\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2)

$$\begin{aligned}
& -3*I*Pi*b*csgn(I*c*x^n)^3-2*b*n+6*b*ln(c)+6*a)*ln(x^n)+1/108*x^3*(-36*I*ln(c) \\
& *Pi*b^2*csgn(I*c*x^n)^3-9*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n) \\
& )^2-36*I*Pi*a*b*csgn(I*c*x^n)^3-36*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n) \\
& ^4+18*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+12*I*Pi*b^2*n*csgn(I*c*x^n) \\
& ^3+18*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+36*a^2+8*b^2*n^2+72*a*b*ln(c) \\
& -24*b^2*n*ln(c)+36*b^2*ln(c)^2-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+18*Pi^2*b^2 \\
& *csgn(I*x^n)*csgn(I*c*x^n)^5-24*a*b*n-9*Pi^2*b^2*csgn(I*c*x^n)^6+18*Pi^2*b^2*csgn(I*c) \\
& *csgn(I*c*x^n)^5-9*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4+36*I*ln(c)*Pi*b^2*csgn(I*x^n) \\
& *csgn(I*c*x^n)^2+36*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+36*I*Pi*a*b*csgn(I*x^n) \\
& *csgn(I*c*x^n)^2+36*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-12*I*Pi*b^2*n*csgn(I*x^n) \\
& *csgn(I*c*x^n)^2-12*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-36*I*Pi*a*b*csgn(I*x^n) \\
& *csgn(I*c*x^n)*csgn(I*c)+12*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-36*I*ln(c) \\
& *Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)
\end{aligned}$$

**maxima** [A] time = 0.67, size = 71, normalized size = 1.37

$$\frac{1}{3}b^2x^3\log(cx^n)^2 - \frac{2}{9}abnx^3 + \frac{2}{3}abx^3\log(cx^n) + \frac{1}{3}a^2x^3 + \frac{2}{27}(n^2x^3 - 3nx^3\log(cx^n))b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 1/3\*b^2\*x^3\*log(c\*x^n)^2 - 2/9\*a\*b\*n\*x^3 + 2/3\*a\*b\*x^3\*log(c\*x^n) + 1/3\*a^2\*x^3 + 2/27\*(n^2\*x^3 - 3\*n\*x^3\*log(c\*x^n))\*b^2

**mupad** [B] time = 3.56, size = 62, normalized size = 1.19

$$x^3 \left( \frac{a^2}{3} - \frac{2abn}{9} + \frac{2b^2n^2}{27} \right) + \frac{x^3 \ln(cx^n) \left( 2ab - \frac{2b^2n}{3} \right)}{3} + \frac{b^2x^3 \ln(cx^n)^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*log(c\*x^n))^2,x)

[Out] x^3\*(a^2/3 + (2\*b^2\*n^2)/27 - (2\*a\*b\*n)/9) + (x^3\*log(c\*x^n)\*(2\*a\*b - (2\*b^2\*n)/3))/3 + (b^2\*x^3\*log(c\*x^n)^2)/3

**sympy** [B] time = 1.52, size = 143, normalized size = 2.75

$$\frac{a^2x^3}{3} + \frac{2abnx^3\log(x)}{3} - \frac{2abnx^3}{9} + \frac{2abx^3\log(c)}{3} + \frac{b^2n^2x^3\log(x)^2}{3} - \frac{2b^2n^2x^3\log(x)}{9} + \frac{2b^2n^2x^3}{27} + \frac{2b^2nx^3\log(c)\log(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*n*x**3*log(x)/3 - 2*a*b*n*x**3/9 + 2*a*b*x**3*log(c)/3  
+ b**2*n**2*x**3*log(x)**2/3 - 2*b**2*n**2*x**3*log(x)/9 + 2*b**2*n**2*x**3  
/27 + 2*b**2*n*x**3*log(c)*log(x)/3 - 2*b**2*n*x**3*log(c)/9 + b**2*x**3*lo  
g(c)**2/3
```

### 3.52 $\int x (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=52

$$\frac{1}{2}x^2 (a + b \log(cx^n))^2 - \frac{1}{2}bnx^2 (a + b \log(cx^n)) + \frac{1}{4}b^2n^2x^2$$

[Out]  $1/4*b^2*n^2*x^2-1/2*b*n*x^2*(a+b*\ln(c*x^n))+1/2*x^2*(a+b*\ln(c*x^n))^2$

**Rubi [A]** time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2305, 2304}

$$\frac{1}{2}x^2 (a + b \log(cx^n))^2 - \frac{1}{2}bnx^2 (a + b \log(cx^n)) + \frac{1}{4}b^2n^2x^2$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(b^2*n^2*x^2)/4 - (b*n*x^2*(a + b*Log[c*x^n]))/2 + (x^2*(a + b*Log[c*x^n])^2)/2$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int x (a + b \log(cx^n))^2 dx &= \frac{1}{2}x^2 (a + b \log(cx^n))^2 - (bn) \int x (a + b \log(cx^n)) dx \\ &= \frac{1}{4}b^2n^2x^2 - \frac{1}{2}bnx^2 (a + b \log(cx^n)) + \frac{1}{2}x^2 (a + b \log(cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 41, normalized size = 0.79

$$\frac{1}{4}x^2 \left( 2 \left( a + b \log(cx^n) \right)^2 + bn \left( -2a - 2b \log(cx^n) + bn \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n])^2,x]

[Out] (x^2\*(b\*n\*(-2\*a + b\*n - 2\*b\*Log[c\*x^n]) + 2\*(a + b\*Log[c\*x^n])^2))/4

**fricas [B]** time = 0.45, size = 102, normalized size = 1.96

$$\frac{1}{2} b^2 n^2 x^2 \log(x)^2 + \frac{1}{2} b^2 x^2 \log(c)^2 - \frac{1}{2} (b^2 n - 2ab) x^2 \log(c) + \frac{1}{4} (b^2 n^2 - 2abn + 2a^2) x^2 + \frac{1}{2} (2b^2 n x^2 \log(c) - (b^2 n^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 1/2\*b^2\*n^2\*x^2\*log(x)^2 + 1/2\*b^2\*x^2\*log(c)^2 - 1/2\*(b^2\*n - 2\*a\*b)\*x^2\*log(c) + 1/4\*(b^2\*n^2 - 2\*a\*b\*n + 2\*a^2)\*x^2 + 1/2\*(2\*b^2\*n\*x^2\*log(c) - (b^2\*n^2 - 2\*a\*b\*n)\*x^2)\*log(x)

**giac [B]** time = 0.29, size = 108, normalized size = 2.08

$$\frac{1}{2} b^2 n^2 x^2 \log(x)^2 - \frac{1}{2} b^2 n^2 x^2 \log(x) + b^2 n x^2 \log(c) \log(x) + \frac{1}{4} b^2 n^2 x^2 - \frac{1}{2} b^2 n x^2 \log(c) + \frac{1}{2} b^2 x^2 \log(c)^2 + abn x^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 1/2\*b^2\*n^2\*x^2\*log(x)^2 - 1/2\*b^2\*n^2\*x^2\*log(x) + b^2\*n\*x^2\*log(c)\*log(x) + 1/4\*b^2\*n^2\*x^2 - 1/2\*b^2\*n\*x^2\*log(c) + 1/2\*b^2\*x^2\*log(c)^2 + a\*b\*n\*x^2\*log(x) - 1/2\*a\*b\*n\*x^2 + a\*b\*x^2\*log(c) + 1/2\*a^2\*x^2

**maple [C]** time = 0.21, size = 692, normalized size = 13.31

$$\frac{b^2 x^2 \ln(x^n)^2}{2} + \frac{(-i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*ln(c\*x^n))^2,x)

[Out] 1/2\*b^2\*x^2\*ln(x^n)^2+1/2\*b\*x^2\*(I\*Pi\*b\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*b\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)-I\*Pi\*b\*csgn(I\*c\*x^n)^3+I\*Pi\*b\*csgn(I\*c)

```
*csgn(I*c*x^n)^2+2*b*ln(c)-b*n+2*a)*ln(x^n)+1/8*x^2*(-Pi^2*b^2*csgn(I*c)^2*
csgn(I*x^n)^2*csgn(I*c*x^n)^2-4*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)^4+2*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+2*I*Pi*b^2*n*csgn(I*
c*x^n)^3+2*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-4*I*Pi*a*b*csgn
(I*c*x^n)^3-4*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^3+4*a^2+2*b^2*n^2+8*a*b*ln(c)-4*
b^2*n*ln(c)+4*b^2*ln(c)^2-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^2
*csgn(I*x^n)*csgn(I*c*x^n)^5-4*a*b*n-Pi^2*b^2*csgn(I*c*x^n)^6+2*Pi^2*b^2*cs
gn(I*c)*csgn(I*c*x^n)^5-Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4+4*I*Pi*a*b*cs
gn(I*c*x^n)^2*csgn(I*c)-2*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-2*I*Pi*b^2*
n*csgn(I*c)*csgn(I*c*x^n)^2+4*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+4*
I*ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^
n)^2-4*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2*I*Pi*b^2*n*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I
*c))
```

**maxima** [A] time = 0.60, size = 70, normalized size = 1.35

$$\frac{1}{2} b^2 x^2 \log(cx^n)^2 - \frac{1}{2} abn x^2 + abx^2 \log(cx^n) + \frac{1}{2} a^2 x^2 + \frac{1}{4} (n^2 x^2 - 2nx^2 \log(cx^n)) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log(c*x^n)^2 - 1/2*a*b*n*x^2 + a*b*x^2*log(c*x^n) + 1/2*a^2*x^2
+ 1/4*(n^2*x^2 - 2*n*x^2*log(c*x^n))*b^2
```

**mupad** [B] time = 3.48, size = 60, normalized size = 1.15

$$x^2 \left( \frac{a^2}{2} - \frac{abn}{2} + \frac{b^2 n^2}{4} \right) + x^2 \ln(cx^n) \left( ab - \frac{b^2 n}{2} \right) + \frac{b^2 x^2 \ln(cx^n)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*x^n))^2,x)
```

```
[Out] x^2*(a^2/2 + (b^2*n^2)/4 - (a*b*n)/2) + x^2*log(c*x^n)*(a*b - (b^2*n)/2) +
(b^2*x^2*log(c*x^n)^2)/2
```

**sympy** [B] time = 0.93, size = 126, normalized size = 2.42

$$\frac{a^2 x^2}{2} + abn x^2 \log(x) - \frac{abn x^2}{2} + abx^2 \log(c) + \frac{b^2 n^2 x^2 \log(x)^2}{2} - \frac{b^2 n^2 x^2 \log(x)}{2} + \frac{b^2 n^2 x^2}{4} + b^2 n x^2 \log(c) \log(x) - \frac{b^2 n x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*x**2/2 + a*b*n*x**2*log(x) - a*b*n*x**2/2 + a*b*x**2*log(c) + b**2*n**  
2*x**2*log(x)**2/2 - b**2*n**2*x**2*log(x)/2 + b**2*n**2*x**2/4 + b**2*n*x*  
*2*log(c)*log(x) - b**2*n*x**2*log(c)/2 + b**2*x**2*log(c)**2/2
```

### 3.53 $\int (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=43

$$x(a + b \log(cx^n))^2 - 2abnx - 2b^2nx \log(cx^n) + 2b^2n^2x$$

[Out]  $-2*a*b*n*x+2*b^2*n^2*x-2*b^2*n*x*\ln(c*x^n)+x*(a+b*\ln(c*x^n))^2$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2296, 2295}

$$x(a + b \log(cx^n))^2 - 2abnx - 2b^2nx \log(cx^n) + 2b^2n^2x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[c*x^n])^2, x]$

[Out]  $-2*a*b*n*x + 2*b^2*n^2*x - 2*b^2*n*x*\text{Log}[c*x^n] + x*(a + b*\text{Log}[c*x^n])^2$

**Rule 2295**

$\text{Int}[\text{Log}[(c_*)*(x_)^(n_)], x\_Symbol] := \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}\{c, n\}, x]$

**Rule 2296**

$\text{Int}[(a_*) + \text{Log}[(c_*)*(x_)^(n_)]*(b_*)^(p_), x\_Symbol] := \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^(p - 1), x], x] /; \text{FreeQ}\{a, b, c, n\}, x \ \&\& \text{GtQ}[p, 0] \ \&\& \text{IntegerQ}[2*p]$

**Rubi steps**

$$\begin{aligned} \int (a + b \log(cx^n))^2 dx &= x(a + b \log(cx^n))^2 - (2bn) \int (a + b \log(cx^n)) dx \\ &= -2abnx + x(a + b \log(cx^n))^2 - (2b^2n) \int \log(cx^n) dx \\ &= -2abnx + 2b^2n^2x - 2b^2nx \log(cx^n) + x(a + b \log(cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 0.77

$$x \left( (a + b \log(cx^n))^2 - 2bn(a + b \log(cx^n) - bn) \right)$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2,x]

[Out] x\*((a + b\*Log[c\*x^n])^2 - 2\*b\*n\*(a - b\*n + b\*Log[c\*x^n]))

**fricas** [A] time = 0.43, size = 85, normalized size = 1.98

$$b^2n^2x \log(x)^2 + b^2x \log(c)^2 - 2(b^2n - ab)x \log(c) + (2b^2n^2 - 2abn + a^2)x + 2(b^2nx \log(c) - (b^2n^2 - abn)x) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] b^2\*n^2\*x\*log(x)^2 + b^2\*x\*log(c)^2 - 2\*(b^2\*n - a\*b)\*x\*log(c) + (2\*b^2\*n^2 - 2\*a\*b\*n + a^2)\*x + 2\*(b^2\*n\*x\*log(c) - (b^2\*n^2 - a\*b\*n)\*x)\*log(x)

**giac** [B] time = 0.36, size = 88, normalized size = 2.05

$$b^2n^2x \log(x)^2 - 2b^2n^2x \log(x) + 2b^2nx \log(c) \log(x) + 2b^2n^2x - 2b^2nx \log(c) + b^2x \log(c)^2 + 2abnx \log(x) - 2abnx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] b^2\*n^2\*x\*log(x)^2 - 2\*b^2\*n^2\*x\*log(x) + 2\*b^2\*n\*x\*log(c)\*log(x) + 2\*b^2\*n^2\*x - 2\*b^2\*n\*x\*log(c) + b^2\*x\*log(c)^2 + 2\*a\*b\*n\*x\*log(x) - 2\*a\*b\*n\*x + 2\*a\*b\*x\*log(c) + a^2\*x

**maple** [A] time = 0.05, size = 63, normalized size = 1.47

$$2b^2n^2x - 2b^2nx \ln(c e^{n \ln(x)}) + b^2x \ln(c e^{n \ln(x)})^2 - 2abnx + 2abx \ln(c x^n) + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2,x)

[Out] a^2\*x + b^2\*x\*ln(c\*exp(n\*ln(x)))^2 + 2\*b^2\*n^2\*x - 2\*b^2\*n\*x\*ln(c\*exp(n\*ln(x))) + 2\*x\*a\*b\*ln(c\*x^n) - 2\*a\*b\*n\*x

**maxima** [A] time = 0.61, size = 57, normalized size = 1.33

$$b^2x \log(cx^n)^2 - 2abnx + 2abx \log(cx^n) + 2(n^2x - nx \log(cx^n))b^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out]  $b^2*x*\log(c*x^n)^2 - 2*a*b*n*x + 2*a*b*x*\log(c*x^n) + 2*(n^2*x - n*x*\log(c*x^n))*b^2 + a^2*x$

mupad [B] time = 3.58, size = 49, normalized size = 1.14

$$x \left( a^2 - 2abn + 2b^2n^2 \right) + b^2x \ln(cx^n)^2 + 2bx \ln(cx^n) (a - bn)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))^2,x)`

[Out]  $x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + b^2*x*\log(c*x^n)^2 + 2*b*x*\log(c*x^n)*(a - b*n)$

sympy [B] time = 0.52, size = 109, normalized size = 2.53

$a^2x+2abnx \log(x)-2abnx+2abx \log(c)+b^2n^2x \log(x)^2-2b^2n^2x \log(x)+2b^2n^2x+2b^2nx \log(c) \log(x)-2b^2nx \log$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**2,x)`

[Out]  $a**2*x + 2*a*b*n*x*\log(x) - 2*a*b*n*x + 2*a*b*x*\log(c) + b**2*n**2*x*\log(x)**2 - 2*b**2*n**2*x*\log(x) + 2*b**2*n**2*x + 2*b**2*n*x*\log(c)*\log(x) - 2*b**2*n*x*\log(c) + b**2*x*\log(c)**2$

$$3.54 \quad \int \frac{(a+b \log(cx^n))^2}{x} dx$$

Optimal. Leaf size=22

$$\frac{(a + b \log(cx^n))^3}{3bn}$$

[Out] 1/3\*(a+b\*ln(c\*x^n))^3/b/n

**Rubi [A]** time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$\frac{(a + b \log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/x, x]

[Out] (a + b\*Log[c\*x^n])^3/(3\*b\*n)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(cx^n))^2}{x} dx &= \frac{\text{Subst}\left(\int x^2 dx, x, a + b \log(cx^n)\right)}{bn} \\ &= \frac{(a + b \log(cx^n))^3}{3bn} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{(a + b \log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/x,x]

[Out] (a + b\*Log[c\*x^n])^3/(3\*b\*n)

**fricas** [B] time = 0.43, size = 51, normalized size = 2.32

$$\frac{1}{3} b^2 n^2 \log(x)^3 + (b^2 n \log(c) + abn) \log(x)^2 + (b^2 \log(c)^2 + 2 ab \log(c) + a^2) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x,x, algorithm="fricas")

[Out] 1/3\*b^2\*n^2\*log(x)^3 + (b^2\*n\*log(c) + a\*b\*n)\*log(x)^2 + (b^2\*log(c)^2 + 2\*a\*b\*log(c) + a^2)\*log(x)

**giac** [B] time = 0.29, size = 56, normalized size = 2.55

$$\frac{1}{3} b^2 n^2 \log(x)^3 + b^2 n \log(c) \log(x)^2 + b^2 \log(c)^2 \log(x) + abn \log(x)^2 + 2 ab \log(c) \log(x) + a^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x,x, algorithm="giac")

[Out] 1/3\*b^2\*n^2\*log(x)^3 + b^2\*n\*log(c)\*log(x)^2 + b^2\*log(c)^2\*log(x) + a\*b\*n\*log(x)^2 + 2\*a\*b\*log(c)\*log(x) + a^2\*log(x)

**maple** [B] time = 0.03, size = 56, normalized size = 2.55

$$\frac{b^2 \ln(cx^n)^3}{3n} + \frac{ab \ln(cx^n)^2}{n} + \frac{a^2 \ln(cx^n)}{n} + \frac{a^3}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2/x,x)

[Out] 1/3/n\*b^2\*ln(c\*x^n)^3+1/n\*b\*ln(c\*x^n)^2\*a+1/n\*ln(c\*x^n)\*a^2+1/3/n/b\*a^3

**maxima** [A] time = 0.46, size = 20, normalized size = 0.91

$$\frac{(b \log(cx^n) + a)^3}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x,x, algorithm="maxima")

[Out] 1/3\*(b\*log(c\*x^n) + a)^3/(b\*n)

mupad [B] time = 3.42, size = 37, normalized size = 1.68

$$a^2 \ln(x) + \frac{b^2 \ln(c x^n)^3}{3n} + \frac{a b \ln(c x^n)^2}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^2/x,x)

[Out] a^2\*log(x) + (b^2\*log(c\*x^n)^3)/(3\*n) + (a\*b\*log(c\*x^n)^2)/n

sympy [A] time = 22.81, size = 60, normalized size = 2.73

$$\begin{cases} \frac{a^2 \log(cx^n) + ab \log(cx^n)^2 + \frac{b^2 \log(cx^n)^3}{3}}{n} & \text{for } n \neq 0 \\ (a^2 + 2ab \log(c) + b^2 \log(c)^2) \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/x,x)

[Out] Piecewise(((a\*\*2\*log(c\*x\*\*n) + a\*b\*log(c\*x\*\*n)\*\*2 + b\*\*2\*log(c\*x\*\*n)\*\*3/3)/n, Ne(n, 0)), ((a\*\*2 + 2\*a\*b\*log(c) + b\*\*2\*log(c)\*\*2)\*log(x), True))

$$3.55 \quad \int \frac{(a+b \log(cx^n))^2}{x^2} dx$$

Optimal. Leaf size=46

$$-\frac{2bn(a+b \log(cx^n))}{x} - \frac{(a+b \log(cx^n))^2}{x} - \frac{2b^2n^2}{x}$$

[Out]  $-2*b^2*n^2/x - 2*b*n*(a+b*\ln(c*x^n))/x - (a+b*\ln(c*x^n))^2/x$

**Rubi [A]** time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{2bn(a+b \log(cx^n))}{x} - \frac{(a+b \log(cx^n))^2}{x} - \frac{2b^2n^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/x^2, x]

[Out]  $(-2*b^2*n^2)/x - (2*b*n*(a + b*Log[c*x^n]))/x - (a + b*Log[c*x^n])^2/x$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{x^2} dx &= -\frac{(a+b \log(cx^n))^2}{x} + (2bn) \int \frac{a+b \log(cx^n)}{x^2} dx \\ &= -\frac{2b^2n^2}{x} - \frac{2bn(a+b \log(cx^n))}{x} - \frac{(a+b \log(cx^n))^2}{x} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 0.76

$$\frac{(a + b \log(cx^n))^2 + 2bn(a + b \log(cx^n) + bn)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/x^2,x]

[Out] -(((a + b\*Log[c\*x^n])^2 + 2\*b\*n\*(a + b\*n + b\*Log[c\*x^n]))/x)

**fricas [A]** time = 0.45, size = 77, normalized size = 1.67

$$\frac{b^2 n^2 \log(x)^2 + 2 b^2 n^2 + b^2 \log(c)^2 + 2 abn + a^2 + 2 (b^2 n + ab) \log(c) + 2 (b^2 n^2 + b^2 n \log(c) + abn) \log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^2,x, algorithm="fricas")

[Out] -(b^2\*n^2\*log(x)^2 + 2\*b^2\*n^2 + b^2\*log(c)^2 + 2\*a\*b\*n + a^2 + 2\*(b^2\*n + a\*b)\*log(c) + 2\*(b^2\*n^2 + b^2\*n\*log(c) + a\*b\*n)\*log(x))/x

**giac [A]** time = 0.31, size = 86, normalized size = 1.87

$$\frac{b^2 n^2 \log(x)^2}{x} - \frac{2 (b^2 n^2 + b^2 n \log(c) + abn) \log(x)}{x} - \frac{2 b^2 n^2 + 2 b^2 n \log(c) + b^2 \log(c)^2 + 2 abn + 2 ab \log(c) + a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^2,x, algorithm="giac")

[Out] -b^2\*n^2\*log(x)^2/x - 2\*(b^2\*n^2 + b^2\*n\*log(c) + a\*b\*n)\*log(x)/x - (2\*b^2\*n^2 + 2\*b^2\*n\*log(c) + b^2\*log(c)^2 + 2\*a\*b\*n + 2\*a\*b\*log(c) + a^2)/x

**maple [C]** time = 0.17, size = 704, normalized size = 15.30

$$\frac{b^2 \ln(x^n)^2}{x} - \frac{(-i\pi b^2 \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b^2 \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b^2 \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2/x^2,x)

[Out] -b^2/x\*ln(x^n)^2-(I\*Pi\*b^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*b^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*Pi\*b^2\*csgn(I\*c\*x^n)^3+I\*Pi\*b^2\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*ln(c)\*b^2+2\*b^2\*n+2\*a\*b)/x\*ln(x^n)-1/4\*(-4\*I\*Pi\*b^2\*n\*csgn(I\*c\*

$x^n)^3 - \pi^2 b^2 \operatorname{csgn}(Ic)^2 \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic*x^n)^2 - 4\pi^2 b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)^4 + 2\pi^2 b^2 \operatorname{csgn}(Ic)^2 \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)^3 + 2\pi^2 b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic*x^n)^3 - 4I\pi a b \operatorname{csgn}(Ic*x^n)^3 - 4I\pi b^2 \operatorname{csgn}(Ic*x^n)^3 \ln(c) + 4a^2 + 8b^2 n^2 + 8a b \ln(c) + 8b^2 n \ln(c) + 4b^2 \ln(c)^2 - \pi^2 b^2 \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic*x^n)^4 + 2\pi^2 b^2 \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)^5 + 8a b n - \pi^2 b^2 \operatorname{csgn}(Ic*x^n)^6 + 2\pi^2 b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ic*x^n)^5 - \pi^2 b^2 \operatorname{csgn}(Ic)^2 \operatorname{csgn}(Ic*x^n)^4 + 4I\pi a b \operatorname{csgn}(Ic) \operatorname{csgn}(Ic*x^n)^2 + 4I\pi b^2 n \operatorname{csgn}(Ic*x^n)^2 \operatorname{csgn}(Ic) + 4I\pi b^2 \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)^2 \ln(c) + 4I\pi b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ic*x^n)^2 \ln(c) + 4I\pi a b \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)^2 - 4I\pi b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n) \ln(c) - 4I\pi b^2 n \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n) \operatorname{csgn}(Ic) + 4I\pi b^2 n \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)^2 - 4I\pi a b \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic*x^n)) / x$

**maxima** [A] time = 0.73, size = 70, normalized size = 1.52

$$-2b^2 \left( \frac{n^2}{x} + \frac{n \log(cx^n)}{x} \right) - \frac{b^2 \log(cx^n)^2}{x} - \frac{2abn}{x} - \frac{2ab \log(cx^n)}{x} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^2,x, algorithm="maxima")

[Out]  $-2b^2(n^2/x + n \log(cx^n)/x) - b^2 \log(cx^n)^2/x - 2abn/x - 2ab \log(cx^n)/x - a^2/x$

**mupad** [B] time = 3.61, size = 56, normalized size = 1.22

$$\frac{a^2 + 2abn + 2b^2 n^2}{x} - \frac{b^2 \ln(cx^n)^2}{x} - \frac{2b \ln(cx^n)(a + bn)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^2/x^2,x)

[Out]  $-(a^2 + 2b^2 n^2 + 2abn)/x - (b^2 \log(cx^n)^2)/x - (2b \log(cx^n)(a + bn))/x$

**sympy** [B] time = 0.57, size = 110, normalized size = 2.39

$$\frac{a^2}{x} - \frac{2abn \log(x)}{x} - \frac{2abn}{x} - \frac{2ab \log(c)}{x} - \frac{b^2 n^2 \log(x)^2}{x} - \frac{2b^2 n^2 \log(x)}{x} - \frac{2b^2 n^2}{x} - \frac{2b^2 n \log(c) \log(x)}{x} - \frac{2b^2 n \log(c)}{x} - \frac{b^2 n^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/x\*\*2,x)



```
[Out] -a**2/x - 2*a*b*n*log(x)/x - 2*a*b*n/x - 2*a*b*log(c)/x - b**2*n**2*log(x)*  
*2/x - 2*b**2*n**2*log(x)/x - 2*b**2*n**2/x - 2*b**2*n*log(c)*log(x)/x - 2*  
b**2*n*log(c)/x - b**2*log(c)**2/x
```

$$3.56 \quad \int \frac{(a+b \log(cx^n))^2}{x^3} dx$$

Optimal. Leaf size=52

$$-\frac{bn(a+b \log(cx^n))}{2x^2} - \frac{(a+b \log(cx^n))^2}{2x^2} - \frac{b^2n^2}{4x^2}$$

[Out]  $-1/4*b^2*n^2/x^2-1/2*b*n*(a+b*\ln(c*x^n))/x^2-1/2*(a+b*\ln(c*x^n))^2/x^2$

**Rubi [A]** time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{bn(a+b \log(cx^n))}{2x^2} - \frac{(a+b \log(cx^n))^2}{2x^2} - \frac{b^2n^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/x^3,x]

[Out]  $-(b^2*n^2)/(4*x^2) - (b*n*(a + b*Log[c*x^n]))/(2*x^2) - (a + b*Log[c*x^n])^2/(2*x^2)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{x^3} dx &= -\frac{(a+b \log(cx^n))^2}{2x^2} + (bn) \int \frac{a+b \log(cx^n)}{x^3} dx \\ &= -\frac{b^2n^2}{4x^2} - \frac{bn(a+b \log(cx^n))}{2x^2} - \frac{(a+b \log(cx^n))^2}{2x^2} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 0.79

$$\frac{2(a + b \log(cx^n))^2 + bn(2a + 2b \log(cx^n) + bn)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/x^3,x]

[Out] -1/4\*(2\*(a + b\*Log[c\*x^n])^2 + b\*n\*(2\*a + b\*n + 2\*b\*Log[c\*x^n]))/x^2

**fricas** [A] time = 0.47, size = 83, normalized size = 1.60

$$\frac{2b^2n^2 \log(x)^2 + b^2n^2 + 2b^2 \log(c)^2 + 2abn + 2a^2 + 2(b^2n + 2ab) \log(c) + 2(b^2n^2 + 2b^2n \log(c) + 2abn) \log(x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*b^2\*n^2\*log(x)^2 + b^2\*n^2 + 2\*b^2\*log(c)^2 + 2\*a\*b\*n + 2\*a^2 + 2\*(b^2\*n + 2\*a\*b)\*log(c) + 2\*(b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*a\*b\*n)\*log(x))/x^2

**giac** [A] time = 0.34, size = 90, normalized size = 1.73

$$\frac{b^2n^2 \log(x)^2}{2x^2} - \frac{(b^2n^2 + 2b^2n \log(c) + 2abn) \log(x)}{2x^2} - \frac{b^2n^2 + 2b^2n \log(c) + 2b^2 \log(c)^2 + 2abn + 4ab \log(c) + 2a^2}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^3,x, algorithm="giac")

[Out] -1/2\*b^2\*n^2\*log(x)^2/x^2 - 1/2\*(b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*a\*b\*n)\*log(x)/x^2 - 1/4\*(b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*b^2\*log(c)^2 + 2\*a\*b\*n + 4\*a\*b\*log(c) + 2\*a^2)/x^2

**maple** [C] time = 0.16, size = 703, normalized size = 13.52

$$\frac{b^2 \ln(x^n)^2}{2x^2} - \frac{(-i\pi b^2 \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b^2 \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b^2 \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2/x^3,x)

[Out] -1/2\*b^2/x^2\*ln(x^n)^2-1/2\*(I\*Pi\*b^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*b^2\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)-I\*Pi\*b^2\*csgn(I\*c\*x^n)^3+I\*Pi\*b^2\*csgn(I

\*c)\*csgn(I\*c\*x^n)^2+2\*b^2\*ln(c)+b^2\*n+2\*a\*b)/x^2\*ln(x^n)-1/8\*(-Pi^2\*b^2\*csgn(I\*c)\*csgn(I\*x^n)^2\*csgn(I\*c\*x^n)^2-4\*Pi^2\*b^2\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^4+2\*Pi^2\*b^2\*csgn(I\*c)^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^3+2\*Pi^2\*b^2\*csgn(I\*c)\*csgn(I\*x^n)^2\*csgn(I\*c\*x^n)^3-4\*I\*Pi\*a\*b\*csgn(I\*c\*x^n)^3-4\*I\*Pi\*b^2\*csgn(I\*c\*x^n)^3\*ln(c)+4\*a^2+2\*b^2\*n^2+8\*a\*b\*ln(c)+4\*b^2\*n\*ln(c)-2\*I\*Pi\*b^2\*n\*csgn(I\*c\*x^n)^3+4\*b^2\*ln(c)^2-Pi^2\*b^2\*csgn(I\*x^n)^2\*csgn(I\*c\*x^n)^4+2\*Pi^2\*b^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^5+4\*a\*b\*n-Pi^2\*b^2\*csgn(I\*c\*x^n)^6+2\*Pi^2\*b^2\*csgn(I\*c)\*csgn(I\*c\*x^n)^5-Pi^2\*b^2\*csgn(I\*c)^2\*csgn(I\*c\*x^n)^4+4\*I\*Pi\*a\*b\*csgn(I\*c)\*csgn(I\*c\*x^n)^2+2\*I\*Pi\*b^2\*n\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2+2\*I\*Pi\*b^2\*n\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+4\*I\*Pi\*b^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2\*ln(c)+4\*I\*Pi\*b^2\*csgn(I\*c)\*csgn(I\*c\*x^n)^2\*ln(c)+4\*I\*Pi\*a\*b\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-4\*I\*Pi\*b^2\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*ln(c)-4\*I\*Pi\*a\*b\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)-2\*I\*Pi\*b^2\*n\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c))/x^2

**maxima** [A] time = 0.60, size = 71, normalized size = 1.37

$$-\frac{1}{4}b^2\left(\frac{n^2}{x^2} + \frac{2n\log(cx^n)}{x^2}\right) - \frac{b^2\log(cx^n)^2}{2x^2} - \frac{abn}{2x^2} - \frac{ab\log(cx^n)}{x^2} - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^3,x, algorithm="maxima")

[Out] -1/4\*b^2\*(n^2/x^2 + 2\*n\*log(c\*x^n)/x^2) - 1/2\*b^2\*log(c\*x^n)^2/x^2 - 1/2\*a\*b\*n/x^2 - a\*b\*log(c\*x^n)/x^2 - 1/2\*a^2/x^2

**mupad** [B] time = 3.43, size = 62, normalized size = 1.19

$$-\frac{\frac{a^2}{2} + \frac{abn}{2} + \frac{b^2n^2}{4}}{x^2} - \frac{\ln(cx^n)\left(\frac{nb^2}{2} + ab\right)}{x^2} - \frac{b^2\ln(cx^n)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^2/x^3,x)

[Out] - (a^2/2 + (b^2\*n^2)/4 + (a\*b\*n)/2)/x^2 - (log(c\*x^n)\*(a\*b + (b^2\*n)/2))/x^2 - (b^2\*log(c\*x^n)^2)/(2\*x^2)

**sympy** [B] time = 1.12, size = 128, normalized size = 2.46

$$-\frac{a^2}{2x^2} - \frac{abn\log(x)}{x^2} - \frac{abn}{2x^2} - \frac{ab\log(c)}{x^2} - \frac{b^2n^2\log(x)^2}{2x^2} - \frac{b^2n^2\log(x)}{2x^2} - \frac{b^2n^2}{4x^2} - \frac{b^2n\log(c)\log(x)}{x^2} - \frac{b^2n\log(c)}{2x^2} - \frac{b^2\log(c)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) - a*b*n*log(x)/x**2 - a*b*n/(2*x**2) - a*b*log(c)/x**2 - b**  
2*n**2*log(x)**2/(2*x**2) - b**2*n**2*log(x)/(2*x**2) - b**2*n**2/(4*x**2)  
- b**2*n*log(c)*log(x)/x**2 - b**2*n*log(c)/(2*x**2) - b**2*log(c)**2/(2*x*  
*2)
```

### 3.57 $\int x^3 (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=77

$$\frac{3}{32}b^2n^2x^4(a + b \log(cx^n)) + \frac{1}{4}x^4(a + b \log(cx^n))^3 - \frac{3}{16}bnx^4(a + b \log(cx^n))^2 - \frac{3}{128}b^3n^3x^4$$

[Out]  $-3/128*b^3*n^3*x^4+3/32*b^2*n^2*x^4*(a+b*\ln(c*x^n))-3/16*b*n*x^4*(a+b*\ln(c*x^n))^2+1/4*x^4*(a+b*\ln(c*x^n))^3$

**Rubi [A]** time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{3}{32}b^2n^2x^4(a + b \log(cx^n)) + \frac{1}{4}x^4(a + b \log(cx^n))^3 - \frac{3}{16}bnx^4(a + b \log(cx^n))^2 - \frac{3}{128}b^3n^3x^4$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*\text{Log}[c*x^n])^3, x]$

[Out]  $(-3*b^3*n^3*x^4)/128 + (3*b^2*n^2*x^4*(a + b*\text{Log}[c*x^n]))/32 - (3*b*n*x^4*(a + b*\text{Log}[c*x^n])^2)/16 + (x^4*(a + b*\text{Log}[c*x^n])^3)/4$

#### Rule 2304

$\text{Int}[(a + \text{Log}[c*(x_)^(n_)]*(b_))*((d_)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^(m+1))/(d*(m+1)^2), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1]$

#### Rule 2305

$\text{Int}[(a + \text{Log}[c*(x_)^(n_)]*(b_))^(p_)*((d_)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^p]/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p-1), x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned} \int x^3 (a + b \log(cx^n))^3 dx &= \frac{1}{4}x^4 (a + b \log(cx^n))^3 - \frac{1}{4}(3bn) \int x^3 (a + b \log(cx^n))^2 dx \\ &= -\frac{3}{16}bnx^4 (a + b \log(cx^n))^2 + \frac{1}{4}x^4 (a + b \log(cx^n))^3 + \frac{1}{8}(3b^2n^2) \int x^3 (a + b \log(cx^n)) dx \\ &= -\frac{3}{128}b^3n^3x^4 + \frac{3}{32}b^2n^2x^4 (a + b \log(cx^n)) - \frac{3}{16}bnx^4 (a + b \log(cx^n))^2 + \frac{1}{4}x^4 (a + b \log(cx^n))^3 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.86

$$\frac{1}{4} \left( x^4 (a + b \log(cx^n))^3 - \frac{3}{32} b n x^4 \left( -4 b n (a + b \log(cx^n)) + 8 (a + b \log(cx^n))^2 + b^2 n^2 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Log[c\*x^n])^3,x]

[Out] (x^4\*(a + b\*Log[c\*x^n])^3 - (3\*b\*n\*x^4\*(b^2\*n^2 - 4\*b\*n\*(a + b\*Log[c\*x^n]) + 8\*(a + b\*Log[c\*x^n])^2))/32)/4

**fricas [B]** time = 0.42, size = 222, normalized size = 2.88

$$\frac{1}{4} b^3 n^3 x^4 \log(x)^3 + \frac{1}{4} b^3 x^4 \log(c)^3 - \frac{3}{16} (b^3 n - 4 a b^2) x^4 \log(c)^2 + \frac{3}{32} (b^3 n^2 - 4 a b^2 n + 8 a^2 b) x^4 \log(c) - \frac{1}{128} (3 b^3 n^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/4\*b^3\*n^3\*x^4\*log(x)^3 + 1/4\*b^3\*x^4\*log(c)^3 - 3/16\*(b^3\*n - 4\*a\*b^2)\*x^4\*log(c)^2 + 3/32\*(b^3\*n^2 - 4\*a\*b^2\*n + 8\*a^2\*b)\*x^4\*log(c) - 1/128\*(3\*b^3\*n^3 - 12\*a\*b^2\*n^2 + 24\*a^2\*b\*n - 32\*a^3)\*x^4 + 3/16\*(4\*b^3\*n^2\*x^4\*log(c) - (b^3\*n^3 - 4\*a\*b^2\*n^2)\*x^4)\*log(x)^2 + 3/32\*(8\*b^3\*n\*x^4\*log(c)^2 - 4\*(b^3\*n^2 - 4\*a\*b^2\*n)\*x^4\*log(c) + (b^3\*n^3 - 4\*a\*b^2\*n^2 + 8\*a^2\*b\*n)\*x^4)\*log(x)

**giac [B]** time = 0.29, size = 262, normalized size = 3.40

$$\frac{1}{4} b^3 n^3 x^4 \log(x)^3 - \frac{3}{16} b^3 n^3 x^4 \log(x)^2 + \frac{3}{4} b^3 n^2 x^4 \log(c) \log(x)^2 + \frac{3}{32} b^3 n^3 x^4 \log(x) - \frac{3}{8} b^3 n^2 x^4 \log(c) \log(x) + \frac{3}{4} b^3 n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 1/4\*b^3\*n^3\*x^4\*log(x)^3 - 3/16\*b^3\*n^3\*x^4\*log(x)^2 + 3/4\*b^3\*n^2\*x^4\*log(c)\*log(x)^2 + 3/32\*b^3\*n^3\*x^4\*log(x) - 3/8\*b^3\*n^2\*x^4\*log(c)\*log(x) + 3/4\*b^3\*n\*x^4\*log(c)^2\*log(x) + 3/4\*a\*b^2\*n^2\*x^4\*log(x)^2 - 3/128\*b^3\*n^3\*x^4 + 3/32\*b^3\*n^2\*x^4\*log(c) - 3/16\*b^3\*n\*x^4\*log(c)^2 + 1/4\*b^3\*x^4\*log(c)^3 - 3/8\*a\*b^2\*n^2\*x^4\*log(x) + 3/2\*a\*b^2\*n\*x^4\*log(c)\*log(x) + 3/32\*a\*b^2\*n^2\*x^4 - 3/8\*a\*b^2\*n\*x^4\*log(c) + 3/4\*a\*b^2\*x^4\*log(c)^2 + 3/4\*a^2\*b\*n\*x^4\*log(x) - 3/16\*a^2\*b\*n\*x^4 + 3/4\*a^2\*b\*x^4\*log(c) + 1/4\*a^3\*x^4

**maple [C]** time = 0.32, size = 2649, normalized size = 34.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3(a+b\ln(cx^n))^3, x)$

[Out]  $\frac{1}{4}b^3x^4\ln(x^n)^3 + \frac{3}{16}x^4b^2(2Ib\pi\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 - 2Ib\pi\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 2Ib\pi\text{csgn}(Icx^n)^3 + 2Ib\pi\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 4b\ln(c) - b^n + 4a)\ln(x^n)^2 + \frac{3}{32}b^2x^4(-2\pi^2b^2\text{csgn}(Ic)^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^2 - 8\pi^2b^2\text{csgn}(Ic)\text{csgn}(Ix^n)\text{csgn}(Icx^n)^4 + 4\pi^2b^2\text{csgn}(Ic)^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^3 + 4\pi^2b^2\text{csgn}(Ic)\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^3 + 8a^2b^2n^2 + 16a^2b\ln(c) - 4b^2n\ln(c) + 8b^2\ln(c)^2 - 2\pi^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 + 4\pi^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 - 4a^2b^2n - 2\pi^2b^2\text{csgn}(Icx^n)^6 + 4\pi^2b^2\text{csgn}(Ic)\text{csgn}(Icx^n)^5 - 2\pi^2b^2\text{csgn}(Ic)^2\text{csgn}(Icx^n)^4 + 2Ib\pi b^2n\text{csgn}(Icx^n)^3 - 8I\ln(c)\pi b^2\text{csgn}(Icx^n)^3 - 8I\pi a^2b\text{csgn}(Icx^n)^3 + 8I\pi a^2b\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 8I\pi a^2b\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 8I\ln(c)\pi b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 8I\ln(c)\pi b^2\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 2I\pi b^2n\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 8I\pi a^2b\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 8I\ln(c)\pi b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 2I\pi b^2n\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 - 2I\pi b^2n\text{csgn}(Icx^n)^2\text{csgn}(Ic))\ln(x^n) + \frac{1}{128}x^4(32a^3 + 4\pi^3b^3\text{csgn}(Icx^n)^9 - 24\pi^2a^2b^2\text{csgn}(Icx^n)^6 - 96I\ln(c)\pi a^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) + 24I\ln(c)\pi b^3n\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) + 24I\pi a^2b^2n\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 24\ln(c)\pi^2b^3\text{csgn}(Icx^n)^6 - 48\ln(c)a^2b^2n + 32\ln(c)^3b^3 + 96\ln(c)^2a^2b^2 + 96\ln(c)a^2b - 24\ln(c)^2b^3n + 12\ln(c)b^3n^2 + 12a^2b^2n^2 - 24a^2b^2n + 6\pi^2b^3n\text{csgn}(Icx^n)^6 - 3b^3n^3 + 6\pi^2b^3n\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 - 12\pi^2b^3n\text{csgn}(Icx^n)^5\text{csgn}(Ic) + 6\pi^2b^3n\text{csgn}(Icx^n)^4\text{csgn}(Ic)^2 - 12\pi^2b^3n\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 - 12I\pi^3b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^8 - 24\ln(c)\pi^2b^3\text{csgn}(Icx^n)^4\text{csgn}(Ic)^2 - 24\pi^2a^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 + 48\pi^2a^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 + 48\pi^2a^2b^2\text{csgn}(Icx^n)^5\text{csgn}(Ic) - 24\pi^2a^2b^2\text{csgn}(Icx^n)^4\text{csgn}(Ic)^2 - 24\ln(c)\pi^2b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 + 48\ln(c)\pi^2b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 + 48\ln(c)\pi^2b^3\text{csgn}(Icx^n)^5\text{csgn}(Ic) - 48I\pi a^2b\text{csgn}(Icx^n)^3 - 4I\pi^3b^3\text{csgn}(Ix^n)^3\text{csgn}(Icx^n)^6 + 12I\pi^3b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^7 - 6I\pi b^3n^2\text{csgn}(Icx^n)^3 - 12I\pi^3b^3\text{csgn}(Icx^n)^8\text{csgn}(Ic) + 12I\pi^3b^3\text{csgn}(Icx^n)^7\text{csgn}(Ic)^2 - 4I\pi^3b^3\text{csgn}(Icx^n)^6\text{csgn}(Ic)^3 - 48I\ln(c)^2\pi b^3\text{csgn}(Icx^n)^3 + 48\pi^2a^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^3\text{csgn}(Ic)^2 - 12\pi^2b^3n\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^3\text{csgn}(Ic) + 6\pi^2b^3n\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^2\text{csgn}(Ic)^2 + 24\pi^2b^3n\text{csgn}(Ix^n)\text{csgn}(Icx^n)^4\text{csgn}(Ic) - 12\pi^2b^3n\text{csgn}(Ix^n)\text{csgn}(Icx^n)^3\text{csgn}(Ic)^2 - 36I\pi^3b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^6\text{csgn}(Ic) + 36I\pi^3b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^5\text{csgn}(Ic)^2 - 24\ln(c)\pi^2b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^2\text{csgn}(Ic)^2 - 96\ln(c)\pi^2b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^4\text{csgn}(Ic) + 48\ln(c)\pi^2b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^3\text{csgn}(Ic)^2 + 48\pi^2a^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^3\text{csgn}(Ic) - 24I\pi$



$i^2 a b^2 \operatorname{csgn}(I x^n)^2 \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c)^2 - 96 \pi^2 a b^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^4 \operatorname{csgn}(I c) + 12 I \pi^3 b^3 \operatorname{csgn}(I x^n)^3 \operatorname{csgn}(I c x^n)^5 \operatorname{csgn}(I c) - 12 I \pi^3 b^3 \operatorname{csgn}(I x^n)^3 \operatorname{csgn}(I c x^n)^4 \operatorname{csgn}(I c)^2 + 4 I \pi^3 b^3 \operatorname{csgn}(I x^n)^3 \operatorname{csgn}(I c x^n)^3 \operatorname{csgn}(I c)^3 + 48 \ln(c) \pi^2 b^3 \operatorname{csgn}(I x^n)^2 \operatorname{csgn}(I c x^n)^3 \operatorname{csgn}(I c) + 96 I \ln(c) \pi a b^2 \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) - 48 I \pi a^2 b \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) - 24 I \ln(c) \pi b^3 n \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) - 24 I \ln(c) \pi b^3 n \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 - 48 I \ln(c)^2 \pi b^3 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) + 96 I \ln(c) \pi a b^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 - 96 I \ln(c) \pi a b^2 \operatorname{csgn}(I c x^n)^3 + 48 I \pi a^2 b \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 + 48 I \pi a^2 b \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) - 12 I \pi^3 b^3 \operatorname{csgn}(I x^n)^2 \operatorname{csgn}(I c x^n)^4 \operatorname{csgn}(I c)^3 + 24 I \ln(c) \pi b^3 n \operatorname{csgn}(I c x^n)^3 + 6 I \pi b^3 n^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 + 6 I \pi b^3 n^2 \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) + 24 I \pi a b^2 n \operatorname{csgn}(I c x^n)^3 + 36 I \pi^3 b^3 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^7 \operatorname{csgn}(I c) - 36 I \pi^3 b^3 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^6 \operatorname{csgn}(I c)^2 + 12 I \pi^3 b^3 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^5 \operatorname{csgn}(I c)^3 + 48 I \ln(c)^2 \pi b^3 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 + 48 I \ln(c)^2 \pi b^3 \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) - 6 I \pi b^3 n^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) - 24 I \pi a b^2 n \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 - 24 I \pi a b^2 n \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c)$

**maxima [A]** time = 0.62, size = 135, normalized size = 1.75

$$\frac{1}{4} b^3 x^4 \log(cx^n)^3 + \frac{3}{4} a b^2 x^4 \log(cx^n)^2 - \frac{3}{16} a^2 b n x^4 + \frac{3}{4} a^2 b x^4 \log(cx^n) + \frac{1}{4} a^3 x^4 + \frac{3}{32} (n^2 x^4 - 4 n x^4 \log(cx^n)) a b^2 - \frac{3}{12} a^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/4\*b^3\*x^4\*log(c\*x^n)^3 + 3/4\*a\*b^2\*x^4\*log(c\*x^n)^2 - 3/16\*a^2\*b\*n\*x^4 + 3/4\*a^2\*b\*x^4\*log(c\*x^n) + 1/4\*a^3\*x^4 + 3/32\*(n^2\*x^4 - 4\*n\*x^4\*log(c\*x^n))\*a\*b^2 - 3/128\*(8\*n\*x^4\*log(c\*x^n)^2 + (n^2\*x^4 - 4\*n\*x^4\*log(c\*x^n))\*n)\*b^3

**mupad [B]** time = 3.66, size = 110, normalized size = 1.43

$$x^4 \left( \frac{a^3}{4} - \frac{3 a^2 b n}{16} + \frac{3 a b^2 n^2}{32} - \frac{3 b^3 n^3}{128} \right) + \frac{x^4 \ln(c x^n) \left( 6 a^2 b - 3 a b^2 n + \frac{3 b^3 n^2}{4} \right)}{8} + \frac{x^4 \ln(c x^n)^2 \left( 3 a b^2 - \frac{3 b^3 n}{4} \right)}{4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b\*log(c\*x^n))^3,x)

[Out] x^4\*(a^3/4 - (3\*b^3\*n^3)/128 + (3\*a\*b^2\*n^2)/32 - (3\*a^2\*b\*n)/16) + (x^4\*log(c\*x^n)\*(6\*a^2\*b + (3\*b^3\*n^2)/4 - 3\*a\*b^2\*n))/8 + (x^4\*log(c\*x^n)^2\*(3\*a\*b^2 - (3\*b^3\*n)/4))/4 + (b^3\*x^4\*log(c\*x^n)^3)/4

sympy [B] time = 4.40, size = 338, normalized size = 4.39

$$\frac{a^3x^4}{4} + \frac{3a^2bnx^4 \log(x)}{4} - \frac{3a^2bnx^4}{16} + \frac{3a^2bx^4 \log(c)}{4} + \frac{3ab^2n^2x^4 \log(x)^2}{4} - \frac{3ab^2n^2x^4 \log(x)}{8} + \frac{3ab^2n^2x^4}{32} + \frac{3ab^2nx^4 \log(c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] a\*\*3\*x\*\*4/4 + 3\*a\*\*2\*b\*n\*x\*\*4\*log(x)/4 - 3\*a\*\*2\*b\*n\*x\*\*4/16 + 3\*a\*\*2\*b\*x\*\*4\*log(c)/4 + 3\*a\*b\*\*2\*n\*\*2\*x\*\*4\*log(x)\*\*2/4 - 3\*a\*b\*\*2\*n\*\*2\*x\*\*4\*log(x)/8 + 3\*a\*b\*\*2\*n\*\*2\*x\*\*4/32 + 3\*a\*b\*\*2\*n\*x\*\*4\*log(c)\*log(x)/2 - 3\*a\*b\*\*2\*n\*x\*\*4\*log(c)/8 + 3\*a\*b\*\*2\*x\*\*4\*log(c)\*\*2/4 + b\*\*3\*n\*\*3\*x\*\*4\*log(x)\*\*3/4 - 3\*b\*\*3\*n\*\*3\*x\*\*4\*log(x)\*\*2/16 + 3\*b\*\*3\*n\*\*3\*x\*\*4\*log(x)/32 - 3\*b\*\*3\*n\*\*3\*x\*\*4/128 + 3\*b\*\*3\*n\*\*2\*x\*\*4\*log(c)\*log(x)\*\*2/4 - 3\*b\*\*3\*n\*\*2\*x\*\*4\*log(c)\*log(x)/8 + 3\*b\*\*3\*n\*\*2\*x\*\*4\*log(c)/32 + 3\*b\*\*3\*n\*x\*\*4\*log(c)\*\*2\*log(x)/4 - 3\*b\*\*3\*n\*x\*\*4\*log(c)\*\*2/16 + b\*\*3\*x\*\*4\*log(c)\*\*3/4

### 3.58 $\int x^2 (a + b \log(cx^n))^3 dx$

Optimal. Leaf size=77

$$\frac{2}{9}b^2n^2x^3(a + b \log(cx^n)) + \frac{1}{3}x^3(a + b \log(cx^n))^3 - \frac{1}{3}bnx^3(a + b \log(cx^n))^2 - \frac{2}{27}b^3n^3x^3$$

[Out]  $-2/27*b^3*n^3*x^3+2/9*b^2*n^2*x^3*(a+b*\ln(c*x^n))-1/3*b*n*x^3*(a+b*\ln(c*x^n))^2+1/3*x^3*(a+b*\ln(c*x^n))^3$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{2}{9}b^2n^2x^3(a + b \log(cx^n)) + \frac{1}{3}x^3(a + b \log(cx^n))^3 - \frac{1}{3}bnx^3(a + b \log(cx^n))^2 - \frac{2}{27}b^3n^3x^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*Log[c\*x^n])^3,x]

[Out]  $(-2*b^3*n^3*x^3)/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/9 - (b*n*x^3*(a + b*Log[c*x^n])^2)/3 + (x^3*(a + b*Log[c*x^n])^3)/3$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \log(cx^n))^3 dx &= \frac{1}{3}x^3 (a + b \log(cx^n))^3 - (bn) \int x^2 (a + b \log(cx^n))^2 dx \\ &= -\frac{1}{3}bnx^3 (a + b \log(cx^n))^2 + \frac{1}{3}x^3 (a + b \log(cx^n))^3 + \frac{1}{3}(2b^2n^2) \int x^2 (a + b \log(cx^n)) dx \\ &= -\frac{2}{27}b^3n^3x^3 + \frac{2}{9}b^2n^2x^3 (a + b \log(cx^n)) - \frac{1}{3}bnx^3 (a + b \log(cx^n))^2 + \frac{1}{3}x^3 (a + b \log(cx^n))^3 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 67, normalized size = 0.87

$$\frac{1}{3} \left( x^3 (a + b \log(cx^n))^3 - bn \left( x^3 (a + b \log(cx^n))^2 + \frac{2}{9} bnx^3 (-3a - 3b \log(cx^n) + bn) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x^n])^3,x]

[Out] (x^3\*(a + b\*Log[c\*x^n])^3 - b\*n\*((2\*b\*n\*x^3\*(-3\*a + b\*n - 3\*b\*Log[c\*x^n]))/9 + x^3\*(a + b\*Log[c\*x^n])^2))/3

**fricas [B]** time = 0.45, size = 224, normalized size = 2.91

$$\frac{1}{3} b^3 n^3 x^3 \log(x)^3 + \frac{1}{3} b^3 x^3 \log(c)^3 - \frac{1}{3} (b^3 n - 3 a b^2) x^3 \log(c)^2 + \frac{1}{9} (2 b^3 n^2 - 6 a b^2 n + 9 a^2 b) x^3 \log(c) - \frac{1}{27} (2 b^3 n^3 - 6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/3\*b^3\*n^3\*x^3\*log(x)^3 + 1/3\*b^3\*x^3\*log(c)^3 - 1/3\*(b^3\*n - 3\*a\*b^2)\*x^3\*log(c)^2 + 1/9\*(2\*b^3\*n^2 - 6\*a\*b^2\*n + 9\*a^2\*b)\*x^3\*log(c) - 1/27\*(2\*b^3\*n^3 - 6\*a\*b^2\*n^2 + 9\*a^2\*b\*n - 9\*a^3)\*x^3 + 1/3\*(3\*b^3\*n^2\*x^3\*log(c) - (b^3\*n^3 - 3\*a\*b^2\*n^2)\*x^3)\*log(x)^2 + 1/9\*(9\*b^3\*n\*x^3\*log(c)^2 - 6\*(b^3\*n^2 - 3\*a\*b^2\*n)\*x^3\*log(c) + (2\*b^3\*n^3 - 6\*a\*b^2\*n^2 + 9\*a^2\*b\*n)\*x^3)\*log(x)

**giac [B]** time = 0.30, size = 256, normalized size = 3.32

$$\frac{1}{3} b^3 n^3 x^3 \log(x)^3 - \frac{1}{3} b^3 n^3 x^3 \log(x)^2 + b^3 n^2 x^3 \log(c) \log(x)^2 + \frac{2}{9} b^3 n^3 x^3 \log(x) - \frac{2}{3} b^3 n^2 x^3 \log(c) \log(x) + b^3 n x^3 \log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 1/3\*b^3\*n^3\*x^3\*log(x)^3 - 1/3\*b^3\*n^3\*x^3\*log(x)^2 + b^3\*n^2\*x^3\*log(c)\*log(x)^2 + 2/9\*b^3\*n^3\*x^3\*log(x) - 2/3\*b^3\*n^2\*x^3\*log(c)\*log(x) + b^3\*n\*x^3\*log(c)^2\*log(x) + a\*b^2\*n^2\*x^3\*log(x)^2 - 2/27\*b^3\*n^3\*x^3 + 2/9\*b^3\*n^2\*x^3\*log(c) - 1/3\*b^3\*n\*x^3\*log(c)^2 + 1/3\*b^3\*x^3\*log(c)^3 - 2/3\*a\*b^2\*n^2\*x^3\*log(x) + 2\*a\*b^2\*n\*x^3\*log(c)\*log(x) + 2/9\*a\*b^2\*n^2\*x^3 - 2/3\*a\*b^2\*n\*x^3\*log(c) + a\*b^2\*x^3\*log(c)^2 + a^2\*b\*n\*x^3\*log(x) - 1/3\*a^2\*b\*n\*x^3 + a^2\*b\*x^3\*log(c) + 1/3\*a^3\*x^3

**maple [C]** time = 0.31, size = 2650, normalized size = 34.42

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*(a+b*\ln(c*x^n))^3,x)$

[Out]  $\frac{1}{3}b^3x^3\ln(x^n)^3 + \frac{1}{6}b^2x^3(3Ib\pi\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 - 3Ib\pi\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 3Ib\pi\text{csgn}(Icx^n)^3 + 3Ib\pi\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 6b*\ln(c) - 2b*n + 6a)*\ln(x^n)^2 + \frac{1}{36}b*x^3(-9\pi^2b^2\text{csgn}(Ic)^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^2 - 36\pi^2b^2\text{csgn}(Ic)\text{csgn}(Ix^n)\text{csgn}(Icx^n)^4 + 18\pi^2b^2\text{csgn}(Ic)^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^3 + 18\pi^2b^2\text{csgn}(Ic)\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^3 + 36a^2 + 8b^2n^2 + 72ab*\ln(c) - 24b^2n*\ln(c) + 36b^2*\ln(c)^2 - 9\pi^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 + 18\pi^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 - 24a*b*n - 9\pi^2b^2\text{csgn}(Icx^n)^6 + 18\pi^2b^2\text{csgn}(Ic)\text{csgn}(Icx^n)^5 - 9\pi^2b^2\text{csgn}(Ic)^2\text{csgn}(Icx^n)^4 + 12I\pi*b^2n*\text{csgn}(Icx^n)^3 - 36I*\ln(c)*\pi*b^2\text{csgn}(Icx^n)^3 - 36I*\pi*a*b*\text{csgn}(Icx^n)^3 - 12I*\pi*b^2n*\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 36I*\ln(c)*\pi*b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 36I*\ln(c)*\pi*b^2\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 36I*\pi*a*b*\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 36I*\pi*a*b*\text{csgn}(Icx^n)^2\text{csgn}(Ic) - 12I*\pi*b^2n*\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 12I*\pi*b^2n*\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 36I*\ln(c)*\pi*b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 36I*\pi*a*b*\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic))*\ln(x^n) + \frac{1}{216}x^3(72a^3 + 72I*\pi*a*b^2n*\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 216I*\ln(c)*\pi*a*b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) + 72I*\ln(c)*\pi*b^3n*\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 54\pi^2a*b^2\text{csgn}(Icx^n)^6 - 54\pi^2b^3\text{csgn}(Icx^n)^6*\ln(c) + 9I\pi^3b^3\text{csgn}(Icx^n)^9 - 144a*b^2n*\ln(c) + 72b^3*\ln(c)^3 + 216a*b^2*\ln(c)^2 + 216a^2*b*\ln(c) - 72b^3n*\ln(c)^2 + 48b^3n^2*\ln(c) + 48a*b^2n^2 - 72a^2*b*n + 18\pi^2b^3n*\text{csgn}(Icx^n)^6 - 16b^3n^3 - 108I*\pi*a^2*b*\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 216I*\ln(c)*\pi*a*b^2\text{csgn}(Icx^n)^3 + 108I*\pi*a^2*b*\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 108I*\pi*a^2*b*\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 18\pi^2b^3n*\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 - 36\pi^2b^3n*\text{csgn}(Ic)\text{csgn}(Icx^n)^5 + 18\pi^2b^3n*\text{csgn}(Ic)^2\text{csgn}(Icx^n)^4 - 36\pi^2b^3n*\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 - 81I*\pi^3b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^6\text{csgn}(Ic)^2 + 27I*\pi^3b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5\text{csgn}(Ic)^3 + 108I*\ln(c)^2*\pi*b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 108I*\ln(c)^2*\pi*b^3\text{csgn}(Icx^n)^2\text{csgn}(Ic) - 54\pi^2b^3\text{csgn}(Ic)^2\text{csgn}(Icx^n)^4*\ln(c) + 27I*\pi^3b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^7 - 54\pi^2a*b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4 + 108\pi^2a*b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 - 24I*\pi*b^3n^2\text{csgn}(Icx^n)^3 + 108\pi^2a*b^2\text{csgn}(Ic)\text{csgn}(Icx^n)^5 - 54\pi^2a*b^2\text{csgn}(Ic)^2\text{csgn}(Icx^n)^4 - 9I*\pi^3b^3\text{csgn}(Icx^n)^6\text{csgn}(Ic)^3 - 108I*\ln(c)^2*\pi*b^3\text{csgn}(Icx^n)^3 - 54\pi^2b^3\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4*\ln(c) - 108I*\pi*a^2*b*\text{csgn}(Icx^n)^3 + 108\pi^2b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5*\ln(c) + 108\pi^2b^3\text{csgn}(Ic)\text{csgn}(Icx^n)^5*\ln(c) - 9I*\pi^3b^3\text{csgn}(Ix^n)^3\text{csgn}(Icx^n)^6 - 27I*\pi^3b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)^8 - 27I*\pi^3b^3\text{csgn}(Icx^n)^8\text{csgn}(Ic) + 27I*\pi^3b^3\text{csgn}(Icx^n)^7\text{csgn}(Ic)^2 - 72I*\ln(c)*\pi*b^3n*\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 - 108I*\ln(c)^2*\pi*b^3\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) + 216I*\ln(c)*\pi*a*b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 216I*\ln(c)*\pi*a*b^2\text{csgn}(Icx^n)^2\text{csgn}(Ic)$

$I*c)+108*Pi^2*a*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-36*Pi^2*b^3*n*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+18*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+72*Pi^2*b^3*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-36*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-54*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*\ln(c)-216*Pi^2*b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4*\ln(c)+108*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3*\ln(c)+108*Pi^2*a*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-54*Pi^2*a*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-216*Pi^2*a*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+108*Pi^2*b^3*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3*\ln(c)-24*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-72*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-72*I*Pi*a*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-72*I*\ln(c)*Pi*b^3*n*csgn(I*c*x^n)^2*csgn(I*c)-27*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2+9*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^3*csgn(I*c)^3+24*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2+24*I*Pi*b^3*n^2*csgn(I*c*x^n)^2*csgn(I*c)+72*I*Pi*a*b^2*n*csgn(I*c*x^n)^3+81*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^7*csgn(I*c)+27*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)-81*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^6*csgn(I*c)+81*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^5*csgn(I*c)^2-27*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4*csgn(I*c)^3+72*I*\ln(c)*Pi*b^3*n*csgn(I*c*x^n)^3$

**maxima [A]** time = 0.68, size = 134, normalized size = 1.74

$$\frac{1}{3}b^3x^3 \log(cx^n)^3 + ab^2x^3 \log(cx^n)^2 - \frac{1}{3}a^2bnx^3 + a^2bx^3 \log(cx^n) + \frac{1}{3}a^3x^3 + \frac{2}{9}(n^2x^3 - 3nx^3 \log(cx^n))ab^2 - \frac{1}{27}(9nx^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $\frac{1}{3}b^3x^3 \log(cx^n)^3 + a^2bx^3 \log(cx^n)^2 - \frac{1}{3}a^2bnx^3 + a^2bx^3 \log(cx^n) + \frac{1}{3}a^3x^3 + \frac{2}{9}(n^2x^3 - 3nx^3 \log(cx^n))ab^2 - \frac{1}{27}(9nx^3 \log(cx^n)^2 + 2(n^2x^3 - 3nx^3 \log(cx^n))n)b^3$

**mupad [B]** time = 3.37, size = 108, normalized size = 1.40

$$x^3 \left( \frac{a^3}{3} - \frac{a^2bn}{3} + \frac{2ab^2n^2}{9} - \frac{2b^3n^3}{27} \right) + \frac{x^3 \ln(cx^n) \left( 3a^2b - 2ab^2n + \frac{2b^3n^2}{3} \right)}{3} + x^3 \ln(cx^n)^2 \left( ab^2 - \frac{b^3n}{3} \right) + \frac{b^3x^3 \ln^3(cx^n)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*log(c\*x^n))^3,x)

[Out]  $x^3 \left( \frac{a^3}{3} - \frac{(2b^3n^3)}{27} + \frac{(2a^2bn^2)}{9} - \frac{(a^2bn)}{3} \right) + (x^3 \log(cx^n) \left( 3a^2b + \frac{(2b^3n^2)}{3} - 2a^2bn \right) / 3 + x^3 \log(cx^n)^2 (ab^2 - (b^3n)/3) + (b^3x^3 \log(cx^n)^3) / 3$

sympy [B] time = 2.89, size = 311, normalized size = 4.04

$$\frac{a^3x^3}{3} + a^2bnx^3 \log(x) - \frac{a^2bnx^3}{3} + a^2bx^3 \log(c) + ab^2n^2x^3 \log(x)^2 - \frac{2ab^2n^2x^3 \log(x)}{3} + \frac{2ab^2n^2x^3}{9} + 2ab^2nx^3 \log(c) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] a\*\*3\*x\*\*3/3 + a\*\*2\*b\*n\*x\*\*3\*log(x) - a\*\*2\*b\*n\*x\*\*3/3 + a\*\*2\*b\*x\*\*3\*log(c) + a\*b\*\*2\*n\*\*2\*x\*\*3\*log(x)\*\*2 - 2\*a\*b\*\*2\*n\*\*2\*x\*\*3\*log(x)/3 + 2\*a\*b\*\*2\*n\*\*2\*x\*\*3/9 + 2\*a\*b\*\*2\*n\*x\*\*3\*log(c)\*log(x) - 2\*a\*b\*\*2\*n\*x\*\*3\*log(c)/3 + a\*b\*\*2\*x\*\*3\*log(c)\*\*2 + b\*\*3\*n\*\*3\*x\*\*3\*log(x)\*\*3/3 - b\*\*3\*n\*\*3\*x\*\*3\*log(x)\*\*2/3 + 2\*b\*\*3\*n\*\*3\*x\*\*3\*log(x)/9 - 2\*b\*\*3\*n\*\*3\*x\*\*3/27 + b\*\*3\*n\*\*2\*x\*\*3\*log(c)\*log(x)\*\*2 - 2\*b\*\*3\*n\*\*2\*x\*\*3\*log(c)\*log(x)/3 + 2\*b\*\*3\*n\*\*2\*x\*\*3\*log(c)/9 + b\*\*3\*n\*x\*\*3\*log(c)\*\*2\*log(x) - b\*\*3\*n\*x\*\*3\*log(c)\*\*2/3 + b\*\*3\*x\*\*3\*log(c)\*\*3/3

### 3.59 $\int x (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=77

$$\frac{3}{4}b^2n^2x^2(a + b \log(cx^n)) - \frac{3}{4}bnx^2(a + b \log(cx^n))^2 + \frac{1}{2}x^2(a + b \log(cx^n))^3 - \frac{3}{8}b^3n^3x^2$$

[Out]  $-3/8*b^3*n^3*x^2+3/4*b^2*n^2*x^2*(a+b*\ln(c*x^n))-3/4*b*n*x^2*(a+b*\ln(c*x^n))^2+1/2*x^2*(a+b*\ln(c*x^n))^3$

**Rubi [A]** time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2305, 2304}

$$\frac{3}{4}b^2n^2x^2(a + b \log(cx^n)) - \frac{3}{4}bnx^2(a + b \log(cx^n))^2 + \frac{1}{2}x^2(a + b \log(cx^n))^3 - \frac{3}{8}b^3n^3x^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{Log}[c*x^n])^3, x]$

[Out]  $(-3*b^3*n^3*x^2)/8 + (3*b^2*n^2*x^2*(a + b*\text{Log}[c*x^n]))/4 - (3*b*n*x^2*(a + b*\text{Log}[c*x^n])^2)/4 + (x^2*(a + b*\text{Log}[c*x^n])^3)/2$

#### Rule 2304

$\text{Int}[(a + \text{Log}[c * (x)^n]) * (b * (d * (x))^m), x\_Symbol] \rightarrow \text{Simp}[(d * x)^{m+1} * (a + b * \text{Log}[c * x^n]) / (d * (m+1)), x] - \text{Simp}[(b * n * (d * x)^{m+1}) / (d * (m+1)^2), x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x \} \&\& \text{NeQ}[m, -1]$

#### Rule 2305

$\text{Int}[(a + \text{Log}[c * (x)^n]) * (b * (d * (x))^m)^p, x\_Symbol] \rightarrow \text{Simp}[(d * x)^{m+1} * (a + b * \text{Log}[c * x^n])^p / (d * (m+1)), x] - \text{Dist}[(b * n * p) / (m+1), \text{Int}[(d * x)^m * (a + b * \text{Log}[c * x^n])^{p-1}, x], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x \} \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned} \int x (a + b \log(cx^n))^3 dx &= \frac{1}{2}x^2(a + b \log(cx^n))^3 - \frac{1}{2}(3bn) \int x (a + b \log(cx^n))^2 dx \\ &= -\frac{3}{4}bnx^2(a + b \log(cx^n))^2 + \frac{1}{2}x^2(a + b \log(cx^n))^3 + \frac{1}{2}(3b^2n^2) \int x (a + b \log(cx^n)) dx \\ &= -\frac{3}{8}b^3n^3x^2 + \frac{3}{4}b^2n^2x^2(a + b \log(cx^n)) - \frac{3}{4}bnx^2(a + b \log(cx^n))^2 + \frac{1}{2}x^2(a + b \log(cx^n)) \end{aligned}$$



**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.78

$$\frac{1}{8}x^2 \left( 4(a + b \log(cx^n))^3 - 3bn \left( 2(a + b \log(cx^n))^2 + bn(-2a - 2b \log(cx^n) + bn) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n])^3,x]

[Out] (x^2\*(4\*(a + b\*Log[c\*x^n])^3 - 3\*b\*n\*(b\*n\*(-2\*a + b\*n - 2\*b\*Log[c\*x^n]) + 2\*(a + b\*Log[c\*x^n])^2)))/8

**fricas [B]** time = 0.44, size = 222, normalized size = 2.88

$$\frac{1}{2}b^3n^3x^2 \log(x)^3 + \frac{1}{2}b^3x^2 \log(c)^3 - \frac{3}{4}(b^3n - 2ab^2)x^2 \log(c)^2 + \frac{3}{4}(b^3n^2 - 2ab^2n + 2a^2b)x^2 \log(c) - \frac{1}{8}(3b^3n^3 - 6a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*b^3\*n^3\*x^2\*log(x)^3 + 1/2\*b^3\*x^2\*log(c)^3 - 3/4\*(b^3\*n - 2\*a\*b^2)\*x^2\*log(c)^2 + 3/4\*(b^3\*n^2 - 2\*a\*b^2\*n + 2\*a^2\*b)\*x^2\*log(c) - 1/8\*(3\*b^3\*n^3 - 6\*a\*b^2\*n^2 + 6\*a^2\*b\*n - 4\*a^3)\*x^2 + 3/4\*(2\*b^3\*n^2\*x^2\*log(c) - (b^3\*n^3 - 2\*a\*b^2\*n^2)\*x^2)\*log(x)^2 + 3/4\*(2\*b^3\*n\*x^2\*log(c)^2 - 2\*(b^3\*n^2 - 2\*a\*b^2\*n)\*x^2\*log(c) + (b^3\*n^3 - 2\*a\*b^2\*n^2 + 2\*a^2\*b\*n)\*x^2)\*log(x)

**giac [B]** time = 0.37, size = 262, normalized size = 3.40

$$\frac{1}{2}b^3n^3x^2 \log(x)^3 - \frac{3}{4}b^3n^3x^2 \log(x)^2 + \frac{3}{2}b^3n^2x^2 \log(c) \log(x)^2 + \frac{3}{4}b^3n^3x^2 \log(x) - \frac{3}{2}b^3n^2x^2 \log(c) \log(x) + \frac{3}{2}b^3nx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 1/2\*b^3\*n^3\*x^2\*log(x)^3 - 3/4\*b^3\*n^3\*x^2\*log(x)^2 + 3/2\*b^3\*n^2\*x^2\*log(c)\*log(x)^2 + 3/4\*b^3\*n^3\*x^2\*log(x) - 3/2\*b^3\*n^2\*x^2\*log(c)\*log(x) + 3/2\*b^3\*n\*x^2\*log(c)^2\*log(x) + 3/2\*a\*b^2\*n^2\*x^2\*log(x)^2 - 3/8\*b^3\*n^3\*x^2 + 3/4\*b^3\*n^2\*x^2\*log(c) - 3/4\*b^3\*n\*x^2\*log(c)^2 + 1/2\*b^3\*x^2\*log(c)^3 - 3/2\*a\*b^2\*n^2\*x^2\*log(x) + 3\*a\*b^2\*n\*x^2\*log(c)\*log(x) + 3/4\*a\*b^2\*n^2\*x^2 - 3/2\*a\*b^2\*n\*x^2\*log(c) + 3/2\*a\*b^2\*x^2\*log(c)^2 + 3/2\*a^2\*b\*n\*x^2\*log(x) - 3/4\*a^2\*b\*n\*x^2 + 3/2\*a^2\*b\*x^2\*log(c) + 1/2\*a^3\*x^2

**maple [C]** time = 0.32, size = 2650, normalized size = 34.42

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x*(a+b*\ln(c*x^n))^3,x)$

[Out]  $\frac{1}{2}x^2b^3\ln(x^n)^3 + \frac{3}{4}x^2b^2(I\pi*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 - I\pi*b*\text{csgn}(I*c)*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n) - I\pi*b*\text{csgn}(I*c*x^n)^3 + I\pi*b*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^2 + 2*b*\ln(c) - b*n + 2*a)*\ln(x^n)^2 + \frac{3}{8}b*x^2*(-\pi^2*b^2*\text{csgn}(I*c)^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^2 - 4*\pi^2*b^2*\text{csgn}(I*c)*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^4 + 2*\pi^2*b^2*\text{csgn}(I*c)^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^3 + 2*\pi^2*b^2*\text{csgn}(I*c)*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^3 - 4*I\pi*a*b*\text{csgn}(I*c*x^n)^3 - 4*I\pi*b^2*\text{csgn}(I*c*x^n)^3*\ln(c) + 4*a^2 + 2*b^2*n^2 + 8*a*b*\ln(c) - 4*b^2*n*\ln(c) + 4*b^2*\ln(c)^2 - \pi^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4 + 2*\pi^2*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5 - 4*a*b*n - \pi^2*b^2*\text{csgn}(I*c*x^n)^6 + 2*\pi^2*b^2*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^5 - \pi^2*b^2*\text{csgn}(I*c)^2*\text{csgn}(I*c*x^n)^4 + 4*I\pi*a*b*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^2 + 4*I\pi*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2*\ln(c) + 4*I\pi*b^2*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^2*\ln(c) + 4*I\pi*a*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 - 4*I\pi*b^2*\text{csgn}(I*c)*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\ln(c) + 2*I\pi*b^2*n*\text{csgn}(I*c*x^n)^3 - 4*I\pi*a*b*\text{csgn}(I*c)*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n) + 2*I\pi*b^2*n*\text{csgn}(I*c)*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n) - 2*I\pi*b^2*n*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^2)*\ln(x^n) + \frac{1}{16}x^2*(8*a^3 + 12*I\pi*a*b^2*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c) - 24*I*\ln(c)*\pi*a*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c) - 6*\pi^2*a*b^2*\text{csgn}(I*c*x^n)^6 - 6*\pi^2*b^3*\text{csgn}(I*c*x^n)^6*\ln(c) - 24*a*b^2*n*\ln(c) + 8*b^3*\ln(c)^3 + 24*a*b^2*\ln(c)^2 + 24*a^2*b*\ln(c) - 12*b^3*n*\ln(c)^2 + 12*b^3*n^2*\ln(c) + I\pi^3*b^3*\text{csgn}(I*c*x^n)^9 + 12*a*b^2*n^2 - 12*a^2*b*n + 3*\pi^2*b^3*n*\text{csgn}(I*c*x^n)^6 - 6*b^3*n^3 + 12*I*\ln(c)*\pi*b^3*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c) + 24*I*\ln(c)*\pi*a*b^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c) - 12*I\pi*a^2*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c) - 24*I*\ln(c)*\pi*a*b^2*\text{csgn}(I*c*x^n)^3 + 12*I\pi*a^2*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 + 3*\pi^2*b^3*n*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4 - 6*\pi^2*b^3*n*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^5 + 3*\pi^2*b^3*n*\text{csgn}(I*c)^2*\text{csgn}(I*c*x^n)^4 - 6*\pi^2*b^3*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5 + 9*I\pi^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^7*\text{csgn}(I*c) - 9*I\pi^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c)^2 + 3*I\pi^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)^3 + 12*I*\ln(c)^2*\pi*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 + 12*I*\ln(c)^2*\pi*b^3*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c) - 6*\pi^2*b^3*\text{csgn}(I*c)^2*\text{csgn}(I*c*x^n)^4*\ln(c) - 6*\pi^2*a*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4 + 12*\pi^2*a*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5 + 12*\pi^2*a*b^2*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^5 - 6*\pi^2*a*b^2*\text{csgn}(I*c)^2*\text{csgn}(I*c*x^n)^4 - 6*\pi^2*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4*\ln(c) + 12*\pi^2*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5*\ln(c) + 12*\pi^2*b^3*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^5*\ln(c) - 12*I\pi*a^2*b*\text{csgn}(I*c*x^n)^3 - I\pi^3*b^3*\text{csgn}(I*x^n)^3*\text{csgn}(I*c*x^n)^6 + 3*I\pi^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^7 - 3*I\pi^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^8 - 3*I\pi^3*b^3*\text{csgn}(I*c*x^n)^8*\text{csgn}(I*c) + 3*I\pi^3*b^3*\text{csgn}(I*c*x^n)^7*\text{csgn}(I*c)^2 - I\pi^3*b^3*\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c)^3 - 12*I*\ln(c)^2*\pi*b^3*\text{csgn}(I*c*x^n)^3 - 6*I\pi*b^3*n^2*\text{csgn}(I*c*x^n)^3 - 12*I*\ln(c)*\pi*b^3*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 - 12*I*\ln(c)^2*\pi*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c) + 24*I*\ln(c)*\pi*a*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 + 12*\pi^2*a*b^2*\text{csgn}(I*c)^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^3 - 6*\pi^2*b^3*n*\text{csgn}($

$I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+3*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+12*Pi^2*b^3*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-6*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-6*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*\ln(c)-24*Pi^2*b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4*\ln(c)+12*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3*\ln(c)+12*Pi^2*a*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-6*Pi^2*a*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-24*Pi^2*a*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+12*Pi^2*b^3*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3*\ln(c)-12*I*\ln(c)*Pi*b^3*n*csgn(I*c*x^n)^2*csgn(I*c)-12*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-12*I*Pi*a*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)+12*I*Pi*a^2*b*csgn(I*c*x^n)^2*csgn(I*c)+I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^3*csgn(I*c)^3+3*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)-3*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2-9*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^6*csgn(I*c)+9*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^5*csgn(I*c)^2-3*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4*csgn(I*c)^3+12*I*\ln(c)*Pi*b^3*n*csgn(I*c*x^n)^3+12*I*Pi*a*b^2*n*csgn(I*c*x^n)^3+6*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2+6*I*Pi*b^3*n^2*csgn(I*c)*csgn(I*c*x^n)^2-6*I*Pi*b^3*n^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)$

**maxima [A]** time = 0.69, size = 135, normalized size = 1.75

$$\frac{1}{2} b^3 x^2 \log(cx^n)^3 + \frac{3}{2} ab^2 x^2 \log(cx^n)^2 - \frac{3}{4} a^2 b n x^2 + \frac{3}{2} a^2 b x^2 \log(cx^n) + \frac{1}{2} a^3 x^2 + \frac{3}{4} (n^2 x^2 - 2 n x^2 \log(cx^n)) ab^2 - \frac{3}{8} (2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2} b^3 x^2 \log(c x^n)^3 + \frac{3}{2} a b^2 x^2 \log(c x^n)^2 - \frac{3}{4} a^2 b n x^2 + \frac{3}{2} a^2 b x^2 \log(c x^n) + \frac{1}{2} a^3 x^2 + \frac{3}{4} (n^2 x^2 - 2 n x^2 \log(c x^n)) a b^2 - \frac{3}{8} (2 n x^2 \log(c x^n)^2 + (n^2 x^2 - 2 n x^2 \log(c x^n)) n) b^3$

**mupad [B]** time = 3.44, size = 110, normalized size = 1.43

$$x^2 \left( \frac{a^3}{2} - \frac{3 a^2 b n}{4} + \frac{3 a b^2 n^2}{4} - \frac{3 b^3 n^3}{8} \right) + \frac{x^2 \ln(c x^n) \left( 3 a^2 b - 3 a b^2 n + \frac{3 b^3 n^2}{2} \right)}{2} + \frac{x^2 \ln(c x^n)^2 \left( 3 a b^2 - \frac{3 b^3 n}{2} \right)}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*log(c\*x^n))^3,x)

[Out]  $x^2*(a^3/2 - (3*b^3*n^3)/8 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/4) + (x^2*\log(c*x^n)*(3*a^2*b + (3*b^3*n^2)/2 - 3*a*b^2*n))/2 + (x^2*\log(c*x^n)^2*(3*a*b^2 - (3*b^3*n)/2))/2 + (b^3*x^2*\log(c*x^n)^3)/2$

sympy [B] time = 1.78, size = 337, normalized size = 4.38

$$\frac{a^3x^2}{2} + \frac{3a^2bnx^2 \log(x)}{2} - \frac{3a^2bnx^2}{4} + \frac{3a^2bx^2 \log(c)}{2} + \frac{3ab^2n^2x^2 \log(x)^2}{2} - \frac{3ab^2n^2x^2 \log(x)}{2} + \frac{3ab^2n^2x^2}{4} + 3ab^2nx^2 \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] a\*\*3\*x\*\*2/2 + 3\*a\*\*2\*b\*n\*x\*\*2\*log(x)/2 - 3\*a\*\*2\*b\*n\*x\*\*2/4 + 3\*a\*\*2\*b\*x\*\*2\*log(c)/2 + 3\*a\*b\*\*2\*n\*\*2\*x\*\*2\*log(x)\*\*2/2 - 3\*a\*b\*\*2\*n\*\*2\*x\*\*2\*log(x)/2 + 3\*a\*b\*\*2\*n\*\*2\*x\*\*2/4 + 3\*a\*b\*\*2\*n\*x\*\*2\*log(c)\*log(x) - 3\*a\*b\*\*2\*n\*x\*\*2\*log(c)/2 + 3\*a\*b\*\*2\*x\*\*2\*log(c)\*\*2/2 + b\*\*3\*n\*\*3\*x\*\*2\*log(x)\*\*3/2 - 3\*b\*\*3\*n\*\*3\*x\*\*2\*log(x)\*\*2/4 + 3\*b\*\*3\*n\*\*3\*x\*\*2\*log(x)/4 - 3\*b\*\*3\*n\*\*3\*x\*\*2/8 + 3\*b\*\*3\*n\*\*2\*x\*\*2\*log(c)\*log(x)\*\*2/2 - 3\*b\*\*3\*n\*\*2\*x\*\*2\*log(c)\*log(x)/2 + 3\*b\*\*3\*n\*\*2\*x\*\*2\*log(c)/4 + 3\*b\*\*3\*n\*x\*\*2\*log(c)\*\*2\*log(x)/2 - 3\*b\*\*3\*n\*x\*\*2\*log(c)\*2/4 + b\*\*3\*x\*\*2\*log(c)\*\*3/2

### 3.60 $\int (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=66

$$6ab^2n^2x - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + 6b^3n^2x \log(cx^n) - 6b^3n^3x$$

[Out]  $6*a*b^2*n^2*x - 6*b^3*n^3*x + 6*b^3*n^2*x*\ln(c*x^n) - 3*b*n*x*(a+b*\ln(c*x^n))^2 + x*(a+b*\ln(c*x^n))^3$

**Rubi [A]** time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2296, 2295}

$$6ab^2n^2x - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + 6b^3n^2x \log(cx^n) - 6b^3n^3x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3, x]

[Out]  $6*a*b^2*n^2*x - 6*b^3*n^3*x + 6*b^3*n^2*x*\text{Log}[c*x^n] - 3*b*n*x*(a + b*\text{Log}[c*x^n])^2 + x*(a + b*\text{Log}[c*x^n])^3$

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rubi steps

$$\begin{aligned} \int (a + b \log(cx^n))^3 dx &= x(a + b \log(cx^n))^3 - (3bn) \int (a + b \log(cx^n))^2 dx \\ &= -3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + (6b^2n^2) \int (a + b \log(cx^n)) dx \\ &= 6ab^2n^2x - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + (6b^3n^2) \int \log(cx^n) dx \\ &= 6ab^2n^2x - 6b^3n^3x + 6b^3n^2x \log(cx^n) - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 0.76

$$x \left( (a + b \log(cx^n))^3 - 3bn \left( (a + b \log(cx^n))^2 - 2bn(a + b \log(cx^n) - bn) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3,x]

[Out] x\*((a + b\*Log[c\*x^n])^3 - 3\*b\*n\*((a + b\*Log[c\*x^n])^2 - 2\*b\*n\*(a - b\*n + b\*Log[c\*x^n])))

**fricas [B]** time = 0.46, size = 198, normalized size = 3.00

$$b^3 n^3 x \log(x)^3 + b^3 x \log(c)^3 - 3(b^3 n - ab^2)x \log(c)^2 + 3(2b^3 n^2 - 2ab^2 n + a^2 b)x \log(c) + 3(b^3 n^2 x \log(c) - (b^3 n^3 - a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] b^3\*n^3\*x\*log(x)^3 + b^3\*x\*log(c)^3 - 3\*(b^3\*n - a\*b^2)\*x\*log(c)^2 + 3\*(2\*b^3\*n^2 - 2\*a\*b^2\*n + a^2\*b)\*x\*log(c) + 3\*(b^3\*n^2\*x\*log(c) - (b^3\*n^3 - a\*b^2\*n^2)\*x)\*log(x)^2 - (6\*b^3\*n^3 - 6\*a\*b^2\*n^2 + 3\*a^2\*b\*n - a^3)\*x + 3\*(b^3\*n\*x\*log(c)^2 - 2\*(b^3\*n^2 - a\*b^2\*n)\*x\*log(c) + (2\*b^3\*n^3 - 2\*a\*b^2\*n^2 + a^2\*b\*n)\*x)\*log(x)

**giac [B]** time = 0.25, size = 219, normalized size = 3.32

$$b^3 n^3 x \log(x)^3 - 3 b^3 n^3 x \log(x)^2 + 3 b^3 n^2 x \log(c) \log(x)^2 + 6 b^3 n^3 x \log(x) - 6 b^3 n^2 x \log(c) \log(x) + 3 b^3 n x \log(c)^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] b^3\*n^3\*x\*log(x)^3 - 3\*b^3\*n^3\*x\*log(x)^2 + 3\*b^3\*n^2\*x\*log(c)\*log(x)^2 + 6\*b^3\*n^3\*x\*log(x) - 6\*b^3\*n^2\*x\*log(c)\*log(x) + 3\*b^3\*n\*x\*log(c)^2\*log(x) + 3\*a\*b^2\*n^2\*x\*log(x)^2 - 6\*b^3\*n^3\*x + 6\*b^3\*n^2\*x\*log(c) - 3\*b^3\*n\*x\*log(c)^2 + b^3\*x\*log(c)^3 - 6\*a\*b^2\*n^2\*x\*log(x) + 6\*a\*b^2\*n\*x\*log(c)\*log(x) + 6\*a\*b^2\*n^2\*x - 6\*a\*b^2\*n\*x\*log(c) + 3\*a\*b^2\*x\*log(c)^2 + 3\*a^2\*b\*n\*x\*log(x) - 3\*a^2\*b\*n\*x + 3\*a^2\*b\*x\*log(c) + a^3\*x

**maple [C]** time = 0.29, size = 2641, normalized size = 40.02

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^3,x)

```
[Out] x*b^3*ln(x^n)^3+3/2*b^2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)-I*Pi*b*csgn(I*c*x^n)^3+I*Pi*b*csgn(I*c)*csgn(I*
c*x^n)^2+2*b*ln(c)-2*b*n+2*a)*x*ln(x^n)^2+3/4*b*(-Pi^2*b^2*csgn(I*c)^2*csgn
(I*x^n)^2*csgn(I*c*x^n)^2-4*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+
2*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+2*Pi^2*b^2*csgn(I*c)*csg
n(I*x^n)^2*csgn(I*c*x^n)^3-4*I*Pi*a*b*csgn(I*c*x^n)^3-4*I*Pi*b^2*csgn(I*c*x
^n)^3*ln(c)+4*a^2+8*b^2*n^2+8*a*b*ln(c)-8*b^2*n*ln(c)+4*b^2*ln(c)^2-Pi^2*b^
2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-8*a*
b*n-Pi^2*b^2*csgn(I*c*x^n)^6+2*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5-Pi^2*b^2*
csgn(I*c)^2*csgn(I*c*x^n)^4+4*I*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2+4*I*Pi*b^2
*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+4*I*Pi*b^2*n*csgn(I*c*x^n)^3+4*I*Pi*
b^2*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(c)+4*I*Pi*b^2*csgn(I*c)*csgn(I*c*x^n)^2*
ln(c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*Pi*b^2*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)*ln(c)-4*I*Pi*a*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*P
i*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c))
*x*ln(x^n)+1/8*(8*a^3-24*I*Pi*a*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(
c)-6*Pi^2*a*b^2*csgn(I*c*x^n)^6+24*I*Pi*b^3*n*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)*ln(c)+24*I*Pi*a*b^2*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-6*Pi^2*b^3
*csgn(I*c*x^n)^6*ln(c)-48*a*b^2*n*ln(c)+8*b^3*ln(c)^3+24*a*b^2*ln(c)^2+24*a
^2*b*ln(c)-24*b^3*n*ln(c)^2+48*b^3*n^2*ln(c)+I*Pi^3*b^3*csgn(I*c*x^n)^9+48*
a*b^2*n^2-24*a^2*b*n+6*Pi^2*b^3*n*csgn(I*c*x^n)^6-48*b^3*n^3+24*I*Pi*a*b^2*
csgn(I*c)*csgn(I*c*x^n)^2*ln(c)-12*I*Pi*a^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)-24*I*Pi*a*b^2*csgn(I*c*x^n)^3*ln(c)+12*I*Pi*a^2*b*csgn(I*x^n)*csgn(I
*c*x^n)^2+6*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^4-12*Pi^2*b^3*n*csgn(I*c
)*csgn(I*c*x^n)^5+6*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*c*x^n)^4-12*Pi^2*b^3*n*cs
gn(I*x^n)*csgn(I*c*x^n)^5+9*I*Pi^3*b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^
7-9*I*Pi^3*b^3*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^6+3*I*Pi^3*b^3*csgn(I*
c)^3*csgn(I*x^n)*csgn(I*c*x^n)^5+12*I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2*ln
(c)^2+12*I*Pi*b^3*csgn(I*c)*csgn(I*c*x^n)^2*ln(c)^2-6*Pi^2*b^3*csgn(I*c)^2*
csgn(I*c*x^n)^4*ln(c)-6*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*Pi^2*a*
b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-24*I*Pi*b^3*n^2*csgn(I*c*x^n)^3+12*Pi^2*a*b
^2*csgn(I*c)*csgn(I*c*x^n)^5-6*Pi^2*a*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-6*Pi^
2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4*ln(c)+12*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*
x^n)^5*ln(c)+12*Pi^2*b^3*csgn(I*c)*csgn(I*c*x^n)^5*ln(c)-12*I*Pi*a^2*b*csgn
(I*c*x^n)^3-I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^6+3*I*Pi^3*b^3*csgn(I*x
^n)^2*csgn(I*c*x^n)^7-3*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^8-3*I*Pi^3*b^3*
csgn(I*c)*csgn(I*c*x^n)^8+3*I*Pi^3*b^3*csgn(I*c)^2*csgn(I*c*x^n)^7-I*Pi^3*b
^3*csgn(I*c)^3*csgn(I*c*x^n)^6-12*I*Pi*b^3*csgn(I*c*x^n)^3*ln(c)^2-12*I*Pi*
b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(c)^2+24*I*Pi*a*b^2*csgn(I*x^n)*c
sgn(I*c*x^n)^2*ln(c)+12*Pi^2*a*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-
12*Pi^2*b^3*n*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+6*Pi^2*b^3*n*csgn(I*c
)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+24*Pi^2*b^3*n*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)^4-12*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-6*Pi^2*b^3
*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*ln(c)-24*Pi^2*b^3*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)^4*ln(c)+12*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*
```

$$x^n)^3 \ln(c) + 12\pi^2 a b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic x^n)^3 - 6\pi^2 a b^2 \operatorname{csgn}(Ic)^2 \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic x^n)^2 - 24\pi^2 a b^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic x^n)^4 + 12\pi^2 b^3 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic x^n)^3 \ln(c) - 24\pi^2 b^3 n^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic x^n) - 24\pi^2 b^3 n \operatorname{csgn}(Ic) \operatorname{csgn}(Ic x^n)^2 \ln(c) - 24\pi^2 b^3 n \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic x^n)^2 \ln(c) + 12\pi^2 a^2 b \operatorname{csgn}(Ic) \operatorname{csgn}(Ic x^n)^2 + \pi^3 b^3 \operatorname{csgn}(Ic)^3 \operatorname{csgn}(Ix^n)^3 \operatorname{csgn}(Ic x^n)^3 + 3\pi^3 b^3 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n)^3 \operatorname{csgn}(Ic x^n)^5 - 3\pi^3 b^3 \operatorname{csgn}(Ic)^2 \operatorname{csgn}(Ix^n)^3 \operatorname{csgn}(Ic x^n)^4 - 9\pi^3 b^3 \operatorname{csgn}(Ic) \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic x^n)^6 + 9\pi^3 b^3 \operatorname{csgn}(Ic)^2 \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic x^n)^5 - 3\pi^3 b^3 \operatorname{csgn}(Ic)^3 \operatorname{csgn}(Ix^n)^2 \operatorname{csgn}(Ic x^n)^4 + 24\pi^2 a b^3 n^2 \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic x^n)^2 + 24\pi^2 a b^3 n^2 \operatorname{csgn}(Ic) \operatorname{csgn}(Ic x^n)^2 + 24\pi^2 a b^3 n \operatorname{csgn}(Ic x^n)^3 \ln(c) + 24\pi^2 a^2 b^2 n \operatorname{csgn}(Ic x^n)^3 - 24\pi^2 a^2 b^2 n \operatorname{csgn}(Ix^n) \operatorname{csgn}(Ic x^n)^2 - 24\pi^2 a^2 b^2 n \operatorname{csgn}(Ic) \operatorname{csgn}(Ic x^n)^2) * x$$

**maxima** [A] time = 0.59, size = 113, normalized size = 1.71

$$b^3 x \log(cx^n)^3 + 3ab^2 x \log(cx^n)^2 - 3a^2 b n x + 3a^2 b x \log(cx^n) + 6(n^2 x - n x \log(cx^n)) a b^2 - 3(n x \log(cx^n))^2 + 2(n^2 x -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $b^3 x \log(cx^n)^3 + 3a^2 b^2 x \log(cx^n)^2 - 3a^2 b n x + 3a^2 b x \log(cx^n) + 6(n^2 x - n x \log(cx^n)) a b^2 - 3(n x \log(cx^n))^2 + 2(n^2 x - n x \log(cx^n)) n b^3 + a^3 x$

**mupad** [B] time = 3.66, size = 94, normalized size = 1.42

$$x(a^3 - 3a^2 b n + 6a b^2 n^2 - 6b^3 n^3) + x \ln(cx^n) (3a^2 b - 6a b^2 n + 6b^3 n^2) + b^3 x \ln(cx^n)^3 + 3b^2 x \ln(cx^n)^2 (a -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^3,x)

[Out]  $x(a^3 - 6b^3 n^3 + 6a^2 b^2 n^2 - 3a^2 b n) + x \log(cx^n) (3a^2 b + 6b^3 n^2 - 6a^2 b n) + b^3 x \log(cx^n)^3 + 3b^2 x \log(cx^n)^2 (a - b n)$

**sympy** [B] time = 1.04, size = 270, normalized size = 4.09

$$a^3 x + 3a^2 b n x \log(x) - 3a^2 b n x + 3a^2 b x \log(c) + 3a b^2 n^2 x \log(x)^2 - 6a b^2 n^2 x \log(x) + 6a b^2 n^2 x + 6a b^2 n x \log(c) \log(x) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3,x)



```
[Out] a**3*x + 3*a**2*b*n*x*log(x) - 3*a**2*b*n*x + 3*a**2*b*x*log(c) + 3*a*b**2*  
n**2*x*log(x)**2 - 6*a*b**2*n**2*x*log(x) + 6*a*b**2*n**2*x + 6*a*b**2*n*x*  
log(c)*log(x) - 6*a*b**2*n*x*log(c) + 3*a*b**2*x*log(c)**2 + b**3*n**3*x*lo  
g(x)**3 - 3*b**3*n**3*x*log(x)**2 + 6*b**3*n**3*x*log(x) - 6*b**3*n**3*x +  
3*b**3*n**2*x*log(c)*log(x)**2 - 6*b**3*n**2*x*log(c)*log(x) + 6*b**3*n**2*  
x*log(c) + 3*b**3*n*x*log(c)**2*log(x) - 3*b**3*n*x*log(c)**2 + b**3*x*log(  
c)**3
```

$$3.61 \quad \int \frac{(a+b \log(cx^n))^3}{x} dx$$

Optimal. Leaf size=22

$$\frac{(a + b \log (cx^n))^4}{4bn}$$

[Out] 1/4\*(a+b\*ln(c\*x^n))^4/b/n

**Rubi [A]** time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$\frac{(a + b \log (cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x, x]

[Out] (a + b\*Log[c\*x^n])^4/(4\*b\*n)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \log (cx^n))^3}{x} dx &= \frac{\text{Subst} \left( \int x^3 dx, x, a + b \log (cx^n) \right)}{bn} \\ &= \frac{(a + b \log (cx^n))^4}{4bn} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$\frac{(a + b \log(cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3/x,x]

[Out] (a + b\*Log[c\*x^n])^4/(4\*b\*n)

**fricas [B]** time = 0.44, size = 100, normalized size = 4.55

$$\frac{1}{4} b^3 n^3 \log(x)^4 + (b^3 n^2 \log(c) + ab^2 n^2) \log(x)^3 + \frac{3}{2} (b^3 n \log(c)^2 + 2 ab^2 n \log(c) + a^2 b n) \log(x)^2 + (b^3 \log(c)^3 + 3 a b^2 \log(c)^2 + 3 a^2 b \log(c) + a^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x,x, algorithm="fricas")

[Out] 1/4\*b^3\*n^3\*log(x)^4 + (b^3\*n^2\*log(c) + a\*b^2\*n^2)\*log(x)^3 + 3/2\*(b^3\*n\*log(c)^2 + 2\*a\*b^2\*n\*log(c) + a^2\*b\*n)\*log(x)^2 + (b^3\*log(c)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3)\*log(x)

**giac [B]** time = 0.27, size = 114, normalized size = 5.18

$$\frac{1}{4} b^3 n^3 \log(x)^4 + b^3 n^2 \log(c) \log(x)^3 + \frac{3}{2} b^3 n \log(c)^2 \log(x)^2 + ab^2 n^2 \log(x)^3 + b^3 \log(c)^3 \log(x) + 3 ab^2 n \log(c) \log(x) + (3 a^2 b \log(c) + a^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x,x, algorithm="giac")

[Out] 1/4\*b^3\*n^3\*log(x)^4 + b^3\*n^2\*log(c)\*log(x)^3 + 3/2\*b^3\*n\*log(c)^2\*log(x)^2 + a\*b^2\*n^2\*log(x)^3 + b^3\*log(c)^3\*log(x) + 3\*a\*b^2\*n\*log(c)\*log(x)^2 + 3\*a\*b^2\*log(c)^2\*log(x) + 3/2\*a^2\*b\*n\*log(x)^2 + 3\*a^2\*b\*log(c)\*log(x) + a^3\*log(x)

**maple [B]** time = 0.02, size = 75, normalized size = 3.41

$$\frac{b^3 \ln(cx^n)^4}{4n} + \frac{ab^2 \ln(cx^n)^3}{n} + \frac{3a^2b \ln(cx^n)^2}{2n} + \frac{a^3 \ln(cx^n)}{n} + \frac{a^4}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^3/x,x)

[Out]  $1/4/n*b^3*\ln(c*x^n)^4+1/n*b^2*\ln(c*x^n)^3*a+3/2/n*b*\ln(c*x^n)^2*a^2+1/n*\ln(c*x^n)*a^3+1/4/n/b*a^4$

**maxima** [A] time = 0.56, size = 20, normalized size = 0.91

$$\frac{(b \log(cx^n) + a)^4}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^3/x,x, algorithm="maxima")`

[Out]  $1/4*(b*\log(c*x^n) + a)^4/(b*n)$

**mupad** [B] time = 3.37, size = 56, normalized size = 2.55

$$a^3 \ln(x) + \frac{b^3 \ln(cx^n)^4}{4n} + \frac{3a^2 b \ln(cx^n)^2}{2n} + \frac{a b^2 \ln(cx^n)^3}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))^3/x,x)`

[Out]  $a^3*\log(x) + (b^3*\log(c*x^n)^4)/(4*n) + (3*a^2*b*\log(c*x^n)^2)/(2*n) + (a*b^2*\log(c*x^n)^3)/n$

**sympy** [B] time = 28.94, size = 92, normalized size = 4.18

$$\begin{cases} \frac{a^3 \log(cx^n) + \frac{3a^2 b \log(cx^n)^2}{2} + ab^2 \log(cx^n)^3 + \frac{b^3 \log(cx^n)^4}{4}}{n} & \text{for } n \neq 0 \\ (a^3 + 3a^2 b \log(c) + 3ab^2 \log(c)^2 + b^3 \log(c)^3) \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**3/x,x)`

[Out] `Piecewise(((a**3*log(c*x**n) + 3*a**2*b*log(c*x**n)**2/2 + a*b**2*log(c*x**n)**3 + b**3*log(c*x**n)**4/4)/n, Ne(n, 0)), ((a**3 + 3*a**2*b*log(c) + 3*a*b**2*log(c)**2 + b**3*log(c)**3)*log(x), True))`

$$3.62 \quad \int \frac{(a+b \log(cx^n))^3}{x^2} dx$$

Optimal. Leaf size=69

$$\frac{6b^2n^2(a+b \log(cx^n))}{x} - \frac{3bn(a+b \log(cx^n))^2}{x} - \frac{(a+b \log(cx^n))^3}{x} - \frac{6b^3n^3}{x}$$

[Out]  $-6*b^3*n^3/x-6*b^2*n^2*(a+b*\ln(c*x^n))/x-3*b*n*(a+b*\ln(c*x^n))^2/x-(a+b*\ln(c*x^n))^3/x$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{6b^2n^2(a+b \log(cx^n))}{x} - \frac{3bn(a+b \log(cx^n))^2}{x} - \frac{(a+b \log(cx^n))^3}{x} - \frac{6b^3n^3}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x^2,x]

[Out]  $(-6*b^3*n^3)/x - (6*b^2*n^2*(a + b*Log[c*x^n]))/x - (3*b*n*(a + b*Log[c*x^n])^2)/x - (a + b*Log[c*x^n])^3/x$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \log(cx^n))^3}{x^2} dx &= -\frac{(a + b \log(cx^n))^3}{x} + (3bn) \int \frac{(a + b \log(cx^n))^2}{x^2} dx \\
&= -\frac{3bn(a + b \log(cx^n))^2}{x} - \frac{(a + b \log(cx^n))^3}{x} + (6b^2n^2) \int \frac{a + b \log(cx^n)}{x^2} dx \\
&= -\frac{6b^3n^3}{x} - \frac{6b^2n^2(a + b \log(cx^n))}{x} - \frac{3bn(a + b \log(cx^n))^2}{x} - \frac{(a + b \log(cx^n))^3}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 0.75

$$\frac{(a + b \log(cx^n))^3 + 3bn \left( (a + b \log(cx^n))^2 + 2bn(a + b \log(cx^n) + bn) \right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3/x^2,x]

[Out] -(((a + b\*Log[c\*x^n])^3 + 3\*b\*n\*((a + b\*Log[c\*x^n])^2 + 2\*b\*n\*(a + b\*n + b\*Log[c\*x^n])))/x)

**fricas [B]** time = 0.41, size = 180, normalized size = 2.61

$$\frac{b^3n^3 \log(x)^3 + 6b^3n^3 + b^3 \log(c)^3 + 6ab^2n^2 + 3a^2bn + a^3 + 3(b^3n + ab^2) \log(c)^2 + 3(b^3n^3 + b^3n^2 \log(c) + ab^2 \log(c)^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^2,x, algorithm="fricas")

[Out] -(b^3\*n^3\*log(x)^3 + 6\*b^3\*n^3 + b^3\*log(c)^3 + 6\*a\*b^2\*n^2 + 3\*a^2\*b\*n + a^3 + 3\*(b^3\*n + a\*b^2)\*log(c)^2 + 3\*(b^3\*n^3 + b^3\*n^2\*log(c) + a\*b^2\*n^2)\*log(x)^2 + 3\*(2\*b^3\*n^2 + 2\*a\*b^2\*n + a^2\*b)\*log(c) + 3\*(2\*b^3\*n^3 + b^3\*n\*log(c)^2 + 2\*a\*b^2\*n^2 + a^2\*b\*n + 2\*(b^3\*n^2 + a\*b^2\*n)\*log(c))\*log(x))/x

**giac [B]** time = 0.28, size = 197, normalized size = 2.86

$$\frac{b^3n^3 \log(x)^3}{x} - \frac{3(b^3n^3 + b^3n^2 \log(c) + ab^2n^2) \log(x)^2}{x} - \frac{3(2b^3n^3 + 2b^3n^2 \log(c) + b^3n \log(c)^2 + 2ab^2n^2 + 2ab^2 \log(c)^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^2,x, algorithm="giac")

```
[Out] -b^3*n^3*log(x)^3/x - 3*(b^3*n^3 + b^3*n^2*log(c) + a*b^2*n^2)*log(x)^2/x -
3*(2*b^3*n^3 + 2*b^3*n^2*log(c) + b^3*n*log(c)^2 + 2*a*b^2*n^2 + 2*a*b^2*n
*log(c) + a^2*b*n)*log(x)/x - (6*b^3*n^3 + 6*b^3*n^2*log(c) + 3*b^3*n*log(c)
)^2 + b^3*log(c)^3 + 6*a*b^2*n^2 + 6*a*b^2*n*log(c) + 3*a*b^2*log(c)^2 + 3*
a^2*b*n + 3*a^2*b*log(c) + a^3)/x
```

**maple [C]** time = 0.28, size = 2674, normalized size = 38.75

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3/x^2,x)
```

```
[Out] -b^3/x*ln(x^n)^3-3/2*(I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b^3*csgn(I*
x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b^3*csgn(I*c*x^n)^3+I*Pi*b^3*csgn(I*c*x^n
)^2*csgn(I*c)+2*ln(c)*b^3+2*b^3*n+2*a*b^2)/x*ln(x^n)^2-3/4*(-4*I*Pi*a*b^2*c
sgn(I*c*x^n)^3+2*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^3*
csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2+4*b*a^2+4*ln(c)^2*b^3-Pi^2*b^3*cs
gn(I*c*x^n)^4*csgn(I*c)^2-Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^3
*csgn(I*x^n)*csgn(I*c*x^n)^5+2*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)+8*a*b^2*n
+8*ln(c)*a*b^2+8*n*ln(c)*b^3-Pi^2*b^3*csgn(I*c*x^n)^6+8*b^3*n^2-4*I*n*Pi*b^
3*csgn(I*c*x^n)^3-4*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+2*Pi^2*b
^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-4*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^3
+4*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*ln(c)*Pi*b^3*csgn(I*c*x^n
)^2*csgn(I*c)+4*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*a*b^2*csgn(I*
c*x^n)^2*csgn(I*c)-4*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*n*P
i*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*c)+4*I*n*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+4*I*n*Pi*b^3*cs
gn(I*x^n)*csgn(I*c*x^n)^2)/x*ln(x^n)-1/8*(8*a^3-24*I*Pi*a*b^2*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)*ln(c)-6*Pi^2*a*b^2*csgn(I*c*x^n)^6-6*Pi^2*b^3*csgn(I*
c*x^n)^6*ln(c)+48*a*b^2*n*ln(c)+8*b^3*ln(c)^3+24*a*b^2*ln(c)^2+24*a^2*b*ln(
c)+24*b^3*n*ln(c)^2+48*b^3*n^2*ln(c)+I*Pi^3*b^3*csgn(I*c*x^n)^9+48*a*b^2*n^
2+24*a^2*b*n-6*Pi^2*b^3*n*csgn(I*c*x^n)^6+48*b^3*n^3-24*I*Pi*a*b^2*n*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)-24*I*n*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)
*csgn(I*c)+24*I*Pi*a*b^2*csgn(I*c)*csgn(I*c*x^n)^2*ln(c)-12*I*Pi*a^2*b*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+24*I*ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n
)^2+24*I*n*ln(c)*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)-24*I*Pi*a*b^2*csgn(I*c*x
^n)^3*ln(c)+12*I*Pi*a^2*b*csgn(I*x^n)*csgn(I*c*x^n)^2-6*Pi^2*b^3*n*csgn(I*x
^n)^2*csgn(I*c*x^n)^4+12*Pi^2*b^3*n*csgn(I*c)*csgn(I*c*x^n)^5-6*Pi^2*b^3*n*
csgn(I*c)^2*csgn(I*c*x^n)^4+12*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5+9*I*P
i^3*b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^7-9*I*Pi^3*b^3*csgn(I*c)^2*csgn
(I*x^n)*csgn(I*c*x^n)^6+3*I*Pi^3*b^3*csgn(I*c)^3*csgn(I*x^n)*csgn(I*c*x^n)^
5+12*I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(c)^2+12*I*Pi*b^3*csgn(I*c)*csg
n(I*c*x^n)^2*ln(c)^2-6*Pi^2*b^3*csgn(I*c)^2*csgn(I*c*x^n)^4*ln(c)-6*Pi^2*a*
b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5
```

$$\begin{aligned}
& -24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c * x^n)^3 + 12 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^5 - 6 * \\
& \text{Pi}^2 * a * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * c * x^n)^4 - 6 * \text{Pi}^2 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 * \ln(c) + 12 * \text{Pi}^2 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^5 * \ln(c) + 12 * \text{Pi}^2 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^5 * \ln(c) - 12 * I * \text{Pi} * a^2 * b * \text{csgn}(I * c * x^n)^3 - I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^6 + 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^7 - 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^8 - 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^8 + 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * c * x^n)^7 - I * \text{Pi}^3 * b^3 * \text{csgn}(I * c)^3 * \text{csgn}(I * c * x^n)^6 - 12 * I * \text{Pi} * b^3 * \text{csgn}(I * c * x^n)^3 * \ln(c)^2 - 24 * I * \ln(c) * \text{Pi} * b^3 * n * \text{csgn}(I * c * x^n)^3 - 24 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * c * x^n)^3 + 24 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 24 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) - 12 * I * \text{Pi} * b^3 * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \ln(c)^2 + 24 * I * \text{Pi} * a * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 * \ln(c) + 12 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^3 + 12 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^3 - 6 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 - 24 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^4 + 12 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^3 - 6 * \text{Pi}^2 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 * \ln(c) - 24 * \text{Pi}^2 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^4 * \ln(c) + 12 * \text{Pi}^2 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^3 * \ln(c) + 12 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^3 - 6 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 - 24 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^4 + 12 * \text{Pi}^2 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^3 * \ln(c) - 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) + 12 * I * \text{Pi} * a^2 * b * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^2 + I * \text{Pi}^3 * b^3 * \text{csgn}(I * c)^3 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^3 + 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^5 - 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^4 - 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^6 + 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^5 - 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c)^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 + 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^2) / x
\end{aligned}$$

**maxima** [A] time = 0.61, size = 133, normalized size = 1.93

$$-\frac{b^3 \log(cx^n)^3}{x} - 3 \left( 2n \left( \frac{n^2}{x} + \frac{n \log(cx^n)}{x} \right) + \frac{n \log(cx^n)^2}{x} \right) b^3 - 6ab^2 \left( \frac{n^2}{x} + \frac{n \log(cx^n)}{x} \right) - \frac{3ab^2 \log(cx^n)^2}{x} - \frac{3a^2bn}{x} - \frac{3a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^2,x, algorithm="maxima")

[Out]  $-b^3 * \log(c * x^n)^3 / x - 3 * (2 * n * (n^2 / x + n * \log(c * x^n) / x) + n * \log(c * x^n)^2 / x) * b^3 - 6 * a * b^2 * (n^2 / x + n * \log(c * x^n) / x) - 3 * a * b^2 * \log(c * x^n)^2 / x - 3 * a^2 * b * n / x - 3 * a^2 * b * \log(c * x^n) / x - a^3 / x$

**mupad** [B] time = 3.37, size = 104, normalized size = 1.51

$$\frac{a^3 + 3a^2bn + 6ab^2n^2 + 6b^3n^3}{x} - \frac{\ln(cx^n) (3a^2b + 6ab^2n + 6b^3n^2)}{x} - \frac{b^3 \ln(cx^n)^3}{x} - \frac{3b^2 \ln(cx^n)^2 (a + bn)}{x}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))^3/x^2,x)`

[Out]  $-(a^3 + 6*b^3*n^3 + 6*a*b^2*n^2 + 3*a^2*b*n)/x - (\log(c*x^n)*(3*a^2*b + 6*b^3*n^2 + 6*a*b^2*n))/x - (b^3*\log(c*x^n)^3)/x - (3*b^2*\log(c*x^n)^2*(a + b*n))/x$

**sympy** [B] time = 1.03, size = 272, normalized size = 3.94

$$\frac{a^3}{x} - \frac{3a^2bn \log(x)}{x} - \frac{3a^2bn}{x} - \frac{3a^2b \log(c)}{x} - \frac{3ab^2n^2 \log(x)^2}{x} - \frac{6ab^2n^2 \log(x)}{x} - \frac{6ab^2n^2}{x} - \frac{6ab^2n \log(c) \log(x)}{x} - \frac{6ab^2n \log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**3/x**2,x)`

[Out]  $-a**3/x - 3*a**2*b*n*\log(x)/x - 3*a**2*b*n/x - 3*a**2*b*\log(c)/x - 3*a*b**2*n**2*\log(x)**2/x - 6*a*b**2*n**2*\log(x)/x - 6*a*b**2*n**2/x - 6*a*b**2*n*\log(c)*\log(x)/x - 6*a*b**2*n*\log(c)/x - 3*a*b**2*\log(c)**2/x - b**3*n**3*\log(x)**3/x - 3*b**3*n**3*\log(x)**2/x - 6*b**3*n**3*\log(x)/x - 6*b**3*n**3/x - 3*b**3*n**2*\log(c)*\log(x)**2/x - 6*b**3*n**2*\log(c)*\log(x)/x - 6*b**3*n**2*\log(c)/x - 3*b**3*n*\log(c)**2*\log(x)/x - 3*b**3*n*\log(c)**2/x - b**3*\log(c)**3/x$

$$3.63 \quad \int \frac{(a+b \log(cx^n))^3}{x^3} dx$$

Optimal. Leaf size=77

$$-\frac{3b^2n^2(a+b \log(cx^n))}{4x^2} - \frac{3bn(a+b \log(cx^n))^2}{4x^2} - \frac{(a+b \log(cx^n))^3}{2x^2} - \frac{3b^3n^3}{8x^2}$$

[Out]  $-3/8*b^3*n^3/x^2-3/4*b^2*n^2*(a+b*\ln(c*x^n))/x^2-3/4*b*n*(a+b*\ln(c*x^n))^2/x^2-1/2*(a+b*\ln(c*x^n))^3/x^2$

**Rubi [A]** time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{3b^2n^2(a+b \log(cx^n))}{4x^2} - \frac{3bn(a+b \log(cx^n))^2}{4x^2} - \frac{(a+b \log(cx^n))^3}{2x^2} - \frac{3b^3n^3}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x^3, x]

[Out]  $(-3*b^3*n^3)/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n]))/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2)/(4*x^2) - (a + b*Log[c*x^n])^3/(2*x^2)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \log(cx^n))^3}{x^3} dx &= -\frac{(a + b \log(cx^n))^3}{2x^2} + \frac{1}{2}(3bn) \int \frac{(a + b \log(cx^n))^2}{x^3} dx \\
&= -\frac{3bn(a + b \log(cx^n))^2}{4x^2} - \frac{(a + b \log(cx^n))^3}{2x^2} + \frac{1}{2}(3b^2n^2) \int \frac{a + b \log(cx^n)}{x^3} dx \\
&= -\frac{3b^3n^3}{8x^2} - \frac{3b^2n^2(a + b \log(cx^n))}{4x^2} - \frac{3bn(a + b \log(cx^n))^2}{4x^2} - \frac{(a + b \log(cx^n))^3}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 60, normalized size = 0.78

$$\frac{4(a + b \log(cx^n))^3 + 3bn(2(a + b \log(cx^n))^2 + bn(2a + 2b \log(cx^n) + bn))}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3/x^3,x]

[Out] -1/8\*(4\*(a + b\*Log[c\*x^n])^3 + 3\*b\*n\*(2\*(a + b\*Log[c\*x^n])^2 + b\*n\*(2\*a + b\*n + 2\*b\*Log[c\*x^n]))) / x^2

**fricas [B]** time = 0.42, size = 189, normalized size = 2.45

$$\frac{4b^3n^3 \log(x)^3 + 3b^3n^3 + 4b^3 \log(c)^3 + 6ab^2n^2 + 6a^2bn + 4a^3 + 6(b^3n + 2ab^2) \log(c)^2 + 6(b^3n^3 + 2b^3n^2 \log(c) + 2ab^2n^2) \log(x)^2 + 6(b^3n^3 + 2b^3n^2 \log(c) + 2ab^2n^2) \log(x)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^3,x, algorithm="fricas")

[Out] -1/8\*(4\*b^3\*n^3\*log(x)^3 + 3\*b^3\*n^3 + 4\*b^3\*log(c)^3 + 6\*a\*b^2\*n^2 + 6\*a^2\*b\*n + 4\*a^3 + 6\*(b^3\*n + 2\*a\*b^2)\*log(c)^2 + 6\*(b^3\*n^3 + 2\*b^3\*n^2\*log(c) + 2\*a\*b^2\*n^2)\*log(x)^2 + 6\*(b^3\*n^3 + 2\*b^3\*n^2\*log(c) + 2\*a\*b^2\*n^2)\*log(x) + 6\*(b^3\*n^3 + 2\*b^3\*n^2\*log(c) + 2\*a\*b^2\*n^2)\*log(c)\*log(x)) / x^2

**giac [B]** time = 0.35, size = 203, normalized size = 2.64

$$\frac{b^3n^3 \log(x)^3}{2x^2} - \frac{3(b^3n^3 + 2b^3n^2 \log(c) + 2ab^2n^2) \log(x)^2}{4x^2} - \frac{3(b^3n^3 + 2b^3n^2 \log(c) + 2b^3n \log(c)^2 + 2ab^2n^2 + 2a^2bn + 4a^3) \log(x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.



$n)^2 \operatorname{csgn}(I * c * x^n)^4 + 12 * \pi^2 * a * b^2 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^5 + 12 * \pi^2 * a * b^2 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * c * x^n)^5 - 6 * \pi^2 * a * b^2 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * c * x^n)^4 - 6 * \pi^2 * b^3 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^4 * \ln(c) + 12 * \pi^2 * b^3 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^5 * \ln(c) + 12 * \pi^2 * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * c * x^n)^5 * \ln(c) - 12 * I * \pi * a^2 * b * \operatorname{csgn}(I * c * x^n)^3 - I * \pi^3 * b^3 * \operatorname{csgn}(I * x^n)^3 * \operatorname{csgn}(I * c * x^n)^6 + 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^7 - 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^8 - 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * c * x^n)^8 + 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * c * x^n)^7 - I * \pi^3 * b^3 * \operatorname{csgn}(I * c)^3 * \operatorname{csgn}(I * c * x^n)^6 - 12 * I * \pi * b^3 * \operatorname{csgn}(I * c * x^n)^3 * \ln(c)^2 - 6 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I * c * x^n)^3 - 12 * I * \ln(c) * \pi * b^3 * n * \operatorname{csgn}(I * c * x^n)^3 + 12 * I * \pi * a * b^2 * n * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^2 + 12 * I * \pi * a * b^2 * n * \operatorname{csgn}(I * c * x^n)^2 * \operatorname{csgn}(I * c) + 12 * I * \ln(c) * \pi * b^3 * n * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^2 - 12 * I * \pi * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n) * \ln(c)^2 + 24 * I * \pi * a * b^2 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^2 * \ln(c) + 12 * I * n * \ln(c) * \pi * b^3 * \operatorname{csgn}(I * c * x^n)^2 * \operatorname{csgn}(I * c) + 12 * \pi^2 * a * b^2 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^3 + 6 * \pi^2 * b^3 * n * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^3 - 3 * \pi^2 * b^3 * n * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^2 - 12 * \pi^2 * b^3 * n * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^4 + 6 * \pi^2 * b^3 * n * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^3 - 6 * \pi^2 * b^3 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^2 * \ln(c) - 24 * \pi^2 * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^4 * \ln(c) + 12 * \pi^2 * b^3 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^3 * \ln(c) + 12 * \pi^2 * a * b^2 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^3 - 6 * \pi^2 * a * b^2 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^2 - 24 * \pi^2 * a * b^2 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^4 + 12 * \pi^2 * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^3 * \ln(c) + 12 * I * \pi * a^2 * b * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * c * x^n)^2 + I * \pi^3 * b^3 * \operatorname{csgn}(I * c)^3 * \operatorname{csgn}(I * x^n)^3 * \operatorname{csgn}(I * c * x^n)^3 + 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n)^3 * \operatorname{csgn}(I * c * x^n)^5 - 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n)^3 * \operatorname{csgn}(I * c * x^n)^4 - 9 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^6 + 9 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c)^2 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^5 - 3 * I * \pi^3 * b^3 * \operatorname{csgn}(I * c)^3 * \operatorname{csgn}(I * x^n)^2 * \operatorname{csgn}(I * c * x^n)^4 + 6 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n)^2 + 6 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * c * x^n)^2 - 6 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I * c) * \operatorname{csgn}(I * x^n) * \operatorname{csgn}(I * c * x^n) - 12 * I * \pi * a * b^2 * n * \operatorname{csgn}(I * c * x^n)^3) / x^2$

**maxima** [A] time = 0.60, size = 135, normalized size = 1.75

$$-\frac{3}{8} \left( n \left( \frac{n^2}{x^2} + \frac{2n \log(cx^n)}{x^2} \right) + \frac{2n \log(cx^n)^2}{x^2} \right) b^3 - \frac{3}{4} ab^2 \left( \frac{n^2}{x^2} + \frac{2n \log(cx^n)}{x^2} \right) - \frac{b^3 \log(cx^n)^3}{2x^2} - \frac{3ab^2 \log(cx^n)^2}{2x^2} - \frac{3a^2 b \log(cx^n)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^3,x, algorithm="maxima")

[Out]  $-3/8 * (n * (n^2/x^2 + 2*n*log(c*x^n)/x^2) + 2*n*log(c*x^n)^2/x^2) * b^3 - 3/4 * a * b^2 * (n^2/x^2 + 2*n*log(c*x^n)/x^2) - 1/2 * b^3 * log(c*x^n)^3/x^2 - 3/2 * a * b^2 * log(c*x^n)^2/x^2 - 3/4 * a^2 * b * n/x^2 - 3/2 * a^2 * b * log(c*x^n)/x^2 - 1/2 * a^3/x^2$

**mupad [B]** time = 3.68, size = 111, normalized size = 1.44

$$\frac{\frac{a^3}{2} + \frac{3a^2bn}{4} + \frac{3ab^2n^2}{4} + \frac{3b^3n^3}{8}}{x^2} - \frac{\ln(cx^n) \left(3a^2b + 3ab^2n + \frac{3b^3n^2}{2}\right)}{2x^2} - \frac{\ln(cx^n)^2 \left(\frac{3nb^3}{2} + 3ab^2\right)}{2x^2} - \frac{b^3 \ln(cx^n)^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^3/x^3,x)

[Out]  $-\frac{(a^3/2 + (3*b^3*n^3)/8 + (3*a*b^2*n^2)/4 + (3*a^2*b*n)/4)}{x^2} - \frac{(\log(c*x^n)*(3*a^2*b + (3*b^3*n^2)/2 + 3*a*b^2*n))}{(2*x^2)} - \frac{(\log(c*x^n)^2*(3*a*b^2 + (3*b^3*n)/2))}{(2*x^2)} - \frac{(b^3*\log(c*x^n)^3)}{(2*x^2)}$

**sympy [B]** time = 1.16, size = 338, normalized size = 4.39

$$\frac{a^3}{2x^2} - \frac{3a^2bn \log(x)}{2x^2} - \frac{3a^2bn}{4x^2} - \frac{3a^2b \log(c)}{2x^2} - \frac{3ab^2n^2 \log(x)^2}{2x^2} - \frac{3ab^2n^2 \log(x)}{2x^2} - \frac{3ab^2n^2}{4x^2} - \frac{3ab^2n \log(c) \log(x)}{x^2} - \frac{3ab^2n \log(c)}{x^2} - \frac{3ab^2n}{4x^2} - \frac{3a^2b \log(c)}{2x^2} - \frac{3a^2b}{4x^2} - \frac{3a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3/x\*\*3,x)

[Out]  $-a^{**3}/(2*x^{**2}) - 3*a^{**2}*b*n*\log(x)/(2*x^{**2}) - 3*a^{**2}*b*n/(4*x^{**2}) - 3*a^{**2}*b*\log(c)/(2*x^{**2}) - 3*a*b^{**2}*n^{**2}*\log(x)^{**2}/(2*x^{**2}) - 3*a*b^{**2}*n^{**2}*\log(x)/(2*x^{**2}) - 3*a*b^{**2}*n^{**2}/(4*x^{**2}) - 3*a*b^{**2}*n*\log(c)*\log(x)/x^{**2} - 3*a*b^{**2}*n*\log(c)/(2*x^{**2}) - 3*a*b^{**2}*\log(c)^{**2}/(2*x^{**2}) - b^{**3}*n^{**3}*\log(x)^{**3}/(2*x^{**2}) - 3*b^{**3}*n^{**3}*\log(x)^{**2}/(4*x^{**2}) - 3*b^{**3}*n^{**3}*\log(x)/(4*x^{**2}) - 3*b^{**3}*n^{**3}/(8*x^{**2}) - 3*b^{**3}*n^{**2}*\log(c)*\log(x)^{**2}/(2*x^{**2}) - 3*b^{**3}*n^{**2}*\log(c)*\log(x)/(2*x^{**2}) - 3*b^{**3}*n^{**2}*\log(c)/(4*x^{**2}) - 3*b^{**3}*n*\log(c)^{**2}*\log(x)/(2*x^{**2}) - 3*b^{**3}*n*\log(c)^{**2}/(4*x^{**2}) - b^{**3}*\log(c)^{**3}/(2*x^{**2})$

$$3.64 \quad \int \frac{(a+b \log(cx^n))^3}{x^4} dx$$

Optimal. Leaf size=77

$$-\frac{2b^2n^2(a+b \log(cx^n))}{9x^3} - \frac{bn(a+b \log(cx^n))^2}{3x^3} - \frac{(a+b \log(cx^n))^3}{3x^3} - \frac{2b^3n^3}{27x^3}$$

[Out]  $-2/27*b^3*n^3/x^3-2/9*b^2*n^2*(a+b*\ln(c*x^n))/x^3-1/3*b*n*(a+b*\ln(c*x^n))^2/x^3-1/3*(a+b*\ln(c*x^n))^3/x^3$

**Rubi** [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{2b^2n^2(a+b \log(cx^n))}{9x^3} - \frac{bn(a+b \log(cx^n))^2}{3x^3} - \frac{(a+b \log(cx^n))^3}{3x^3} - \frac{2b^3n^3}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x^4,x]

[Out]  $(-2*b^3*n^3)/(27*x^3) - (2*b^2*n^2*(a + b*Log[c*x^n]))/(9*x^3) - (b*n*(a + b*Log[c*x^n])^2)/(3*x^3) - (a + b*Log[c*x^n])^3/(3*x^3)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \log(cx^n))^3}{x^4} dx &= -\frac{(a + b \log(cx^n))^3}{3x^3} + (bn) \int \frac{(a + b \log(cx^n))^2}{x^4} dx \\
&= -\frac{bn(a + b \log(cx^n))^2}{3x^3} - \frac{(a + b \log(cx^n))^3}{3x^3} + \frac{1}{3}(2b^2n^2) \int \frac{a + b \log(cx^n)}{x^4} dx \\
&= -\frac{2b^3n^3}{27x^3} - \frac{2b^2n^2(a + b \log(cx^n))}{9x^3} - \frac{bn(a + b \log(cx^n))^2}{3x^3} - \frac{(a + b \log(cx^n))^3}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.78

$$\frac{9(a + b \log(cx^n))^3 + bn(9(a + b \log(cx^n))^2 + 2bn(3a + 3b \log(cx^n) + bn))}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3/x^4, x]

[Out] -1/27\*(9\*(a + b\*Log[c\*x^n])^3 + b\*n\*(9\*(a + b\*Log[c\*x^n])^2 + 2\*b\*n\*(3\*a + b\*n + 3\*b\*Log[c\*x^n]))) / x^3

**fricas [B]** time = 0.42, size = 191, normalized size = 2.48

$$\frac{9b^3n^3 \log(x)^3 + 2b^3n^3 + 9b^3 \log(c)^3 + 6ab^2n^2 + 9a^2bn + 9a^3 + 9(b^3n + 3ab^2) \log(c)^2 + 9(b^3n^3 + 3b^3n^2 \log(c))}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^4,x, algorithm="fricas")

[Out] -1/27\*(9\*b^3\*n^3\*log(x)^3 + 2\*b^3\*n^3 + 9\*b^3\*log(c)^3 + 6\*a\*b^2\*n^2 + 9\*a^2\*b\*n + 9\*a^3 + 9\*(b^3\*n + 3\*a\*b^2)\*log(c)^2 + 9\*(b^3\*n^3 + 3\*b^3\*n^2\*log(c)) + 3\*a\*b^2\*n^2\*log(x)^2 + 3\*(2\*b^3\*n^2 + 6\*a\*b^2\*n + 9\*a^2\*b)\*log(c) + 3\*(2\*b^3\*n^3 + 9\*b^3\*n\*log(c)^2 + 6\*a\*b^2\*n^2 + 9\*a^2\*b\*n + 6\*(b^3\*n^2 + 3\*a\*b^2\*n)\*log(c))\*log(x))/x^3

**giac [B]** time = 0.30, size = 204, normalized size = 2.65

$$\frac{b^3n^3 \log(x)^3}{3x^3} - \frac{(b^3n^3 + 3b^3n^2 \log(c) + 3ab^2n^2) \log(x)^2}{3x^3} - \frac{(2b^3n^3 + 6b^3n^2 \log(c) + 9b^3n \log(c)^2 + 6ab^2n^2 + 18a^2bn + 9a^3) \log(c)}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*log(c\*x^n))^3/x^4,x, algorithm="giac")

[Out]  $-1/3*b^3*n^3*\log(x)^3/x^3 - 1/3*(b^3*n^3 + 3*b^3*n^2*\log(c) + 3*a*b^2*n^2)*\log(x)^2/x^3 - 1/9*(2*b^3*n^3 + 6*b^3*n^2*\log(c) + 9*b^3*n*\log(c)^2 + 6*a*b^2*n^2 + 18*a*b^2*n*\log(c) + 9*a^2*b*n)*\log(x)/x^3 - 1/27*(2*b^3*n^3 + 6*b^3*n^2*\log(c) + 9*b^3*n*\log(c)^2 + 9*b^3*\log(c)^3 + 6*a*b^2*n^2 + 18*a*b^2*n*\log(c) + 27*a*b^2*\log(c)^2 + 9*a^2*b*n + 27*a^2*b*\log(c) + 9*a^3)/x^3$

**maple [C]** time = 0.29, size = 2674, normalized size = 34.73

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^3/x^4,x)

[Out]  $-1/3*b^3/x^3*\ln(x^n)^3 - 1/6*(3*I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2 - 3*I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) - 3*I*Pi*b^3*csgn(I*c*x^n)^3 + 3*I*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c) + 6*b^3*\ln(c) + 2*b^3*n + 6*a*b^2)/x^3*\ln(x^n)^2 - 1/36*(18*Pi^2*b^3*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3 - 9*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2 + 36*a^2*b + 36*b^3*\ln(c)^2 - 9*Pi^2*b^3*csgn(I*c)^2*csgn(I*c*x^n)^4 - 9*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4 + 18*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5 + 18*Pi^2*b^3*csgn(I*c)*csgn(I*c*x^n)^5 + 24*a*b^2*n + 72*a*b^2*\ln(c) + 24*b^3*n*\ln(c) - 9*Pi^2*b^3*csgn(I*c*x^n)^6 + 8*b^3*n^2 + 36*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2 + 36*I*Pi*a*b^2*csgn(I*c*x^n)^2*csgn(I*c) + 12*I*n*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c) + 12*I*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2 - 12*I*n*Pi*b^3*csgn(I*c*x^n)^3 - 36*I*\ln(c)*Pi*b^3*csgn(I*c*x^n)^3 - 36*Pi^2*b^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4 + 18*Pi^2*b^3*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3 - 36*I*Pi*a*b^2*csgn(I*c*x^n)^3 - 36*I*\ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) - 36*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) - 12*I*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) + 36*I*\ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2 + 36*I*\ln(c)*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c))/x^3*\ln(x^n) - 1/216*(72*a^3 - 72*I*n*\ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) - 72*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) - 216*I*\ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) + 9*I*Pi^3*b^3*csgn(I*c*x^n)^9 - 54*Pi^2*a*b^2*csgn(I*c*x^n)^6 - 54*Pi^2*b^3*csgn(I*c*x^n)^6*\ln(c) + 144*a*b^2*n*\ln(c) + 72*b^3*\ln(c)^3 + 216*a*b^2*\ln(c)^2 + 216*a^2*b*\ln(c) + 72*b^3*n*\ln(c)^2 + 48*b^3*n^2*\ln(c) + 48*a*b^2*n^2 + 72*a^2*b*n - 18*Pi^2*b^3*n*csgn(I*c*x^n)^6 + 16*b^3*n^3 - 9*I*Pi^3*b^3*csgn(I*c*x^n)^6*csgn(I*c)^3 - 108*I*\ln(c)^2*Pi*b^3*csgn(I*c*x^n)^3 - 108*I*Pi*a^2*b*csgn(I*c*x^n)^3 - 9*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^6 + 27*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7 + 216*I*\ln(c)*Pi*a*b^2*csgn(I*c*x^n)^2*csgn(I*c) - 108*I*Pi*a^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c) + 72*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2 + 72*I*Pi*a*b^2*n*csgn(I*c*x^n)^2*csgn(I*c) + 72*I*\ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^2 + 72*I*n*\ln(c)*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c) - 18*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^4 + 36*Pi^2*b^3*n*csgn(I*c)*csgn(I*c*x^n)^5 - 18*Pi^2*b^3*n*csgn(I*c)^2*csgn(I*c*x^n)^4 + 36*$

$$\begin{aligned} & \text{Pi}^2 * b^3 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^5 - 54 * \text{Pi}^2 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * c * x^n) \\ & ^4 * \ln(c) - 54 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 + 108 * \text{Pi}^2 * a * b^2 * \text{csgn}(I \\ & * x^n) * \text{csgn}(I * c * x^n)^5 - 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c * x^n)^3 + 108 * \text{Pi}^2 * a * b^2 * \text{csgn}(I \\ & * c) * \text{csgn}(I * c * x^n)^5 - 54 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * c * x^n)^4 - 54 * \text{Pi}^2 * b^3 * c \\ & \text{sgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 * \ln(c) + 108 * \text{Pi}^2 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^5 \\ & * \ln(c) + 108 * \text{Pi}^2 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^5 * \ln(c) - 108 * I * \ln(c)^2 * \text{Pi} * b^3 * c \\ & \text{sgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) + 216 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c \\ & * x^n)^2 - 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^8 - 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c * x^n) \\ & ^8 * \text{csgn}(I * c) + 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * c * x^n)^7 * \text{csgn}(I * c)^2 - 72 * I * \ln(c) * \text{Pi} * b^3 * \\ & n * \text{csgn}(I * c * x^n)^3 - 72 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * c * x^n)^3 - 81 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) \\ & * \text{csgn}(I * c * x^n)^6 * \text{csgn}(I * c)^2 + 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^5 * \text{csgn} \\ & (I * c)^3 + 108 * I * \ln(c)^2 * \text{Pi} * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 108 * I * \ln(c)^2 * \text{Pi} * b \\ & ^3 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) + 108 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n) * \text{csgn}(I * \\ & c * x^n)^3 + 36 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^3 - 18 * \text{Pi}^2 * b^3 * \\ & n * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 - 72 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c) * \text{csgn}(I * \\ & x^n) * \text{csgn}(I * c * x^n)^4 + 36 * \text{Pi}^2 * b^3 * n * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^3 - \\ & 54 * \text{Pi}^2 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 * \ln(c) - 216 * \text{Pi}^2 * b^3 * c \\ & \text{sgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^4 * \ln(c) + 108 * \text{Pi}^2 * b^3 * \text{csgn}(I * c)^2 * \text{csgn}(I * x \\ & ^n) * \text{csgn}(I * c * x^n)^3 * \ln(c) + 108 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x \\ & ^n)^3 - 54 * \text{Pi}^2 * a * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 - 216 * \text{Pi}^2 * a * b^2 \\ & * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^4 + 108 * \text{Pi}^2 * b^3 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 \\ & * \text{csgn}(I * c * x^n)^3 * \ln(c) - 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) \\ & + 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c) * \text{csgn} \\ & (I * c * x^n)^2 - 81 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^6 * \text{csgn}(I * c) + 81 * I * \text{Pi}^3 \\ & * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^5 * \text{csgn}(I * c)^2 - 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \\ & \text{csgn}(I * c * x^n)^4 * \text{csgn}(I * c)^3 + 81 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^7 * \text{csgn}( \\ & I * c) + 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^5 * \text{csgn}(I * c) - 27 * I * \text{Pi}^3 * b^3 * c \\ & \text{sgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^4 * \text{csgn}(I * c)^2 + 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c \\ & * x^n)^3 * \text{csgn}(I * c)^3 - 216 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * c * x^n)^3 + 108 * I * \text{Pi} * a^2 * b * c \\ & \text{sgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 108 * I * \text{Pi} * a^2 * b * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c)) / x^3 \end{aligned}$$

**maxima** [A] time = 0.59, size = 136, normalized size = 1.77

$$-\frac{1}{27} \left( 2n \left( \frac{n^2}{x^3} + \frac{3n \log(cx^n)}{x^3} \right) + \frac{9n \log(cx^n)^2}{x^3} \right) b^3 - \frac{2}{9} ab^2 \left( \frac{n^2}{x^3} + \frac{3n \log(cx^n)}{x^3} \right) - \frac{b^3 \log(cx^n)^3}{3x^3} - \frac{ab^2 \log(cx^n)^2}{x^3} - \frac{a^2 b}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^4,x, algorithm="maxima")

[Out] -1/27\*(2\*n\*(n^2/x^3 + 3\*n\*log(c\*x^n)/x^3) + 9\*n\*log(c\*x^n)^2/x^3)\*b^3 - 2/9\*a\*b^2\*(n^2/x^3 + 3\*n\*log(c\*x^n)/x^3) - 1/3\*b^3\*log(c\*x^n)^3/x^3 - a\*b^2\*log(c\*x^n)^2/x^3 - 1/3\*a^2\*b\*n/x^3 - a^2\*b\*log(c\*x^n)/x^3 - 1/3\*a^3/x^3

**mupad [B]** time = 3.59, size = 110, normalized size = 1.43

$$\frac{\frac{a^3}{3} + \frac{a^2 b n}{3} + \frac{2 a b^2 n^2}{9} + \frac{2 b^3 n^3}{27}}{x^3} - \frac{\ln(c x^n) \left( 3 a^2 b + 2 a b^2 n + \frac{2 b^3 n^2}{3} \right)}{3 x^3} - \frac{\ln(c x^n)^2 \left( \frac{n b^3}{3} + a b^2 \right)}{x^3} - \frac{b^3 \ln(c x^n)^3}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^3/x^4,x)

[Out]  $-\frac{a^3}{3} - \frac{2 b^3 n^3}{27} + \frac{2 a b^2 n^2}{9} + \frac{a^2 b n}{3} \Big/ x^3 - \frac{\ln(c x^n) (3 a^2 b + 2 a b^2 n + \frac{2 b^3 n^2}{3})}{3 x^3} - \frac{\ln(c x^n)^2 (a b^2 + \frac{b^3 n}{3})}{x^3} - \frac{b^3 \ln(c x^n)^3}{3 x^3}$

**sympy [B]** time = 1.81, size = 313, normalized size = 4.06

$$\frac{a^3}{3x^3} - \frac{a^2 b n \log(x)}{x^3} - \frac{a^2 b n}{3x^3} - \frac{a^2 b \log(c)}{x^3} - \frac{a b^2 n^2 \log(x)^2}{x^3} - \frac{2 a b^2 n^2 \log(x)}{3x^3} - \frac{2 a b^2 n^2}{9x^3} - \frac{2 a b^2 n \log(c) \log(x)}{x^3} - \frac{2 a b^2 n \log(c)}{3x^3} - \frac{2 a b^2 n}{9x^3} - \frac{2 a b^2 \log(x)}{3x^3} - \frac{2 a b^2}{27x^3} - \frac{2 a b n \log(c) \log(x)}{x^3} - \frac{2 a b n \log(c)}{3x^3} - \frac{2 a b n}{9x^3} - \frac{2 a b \log(c)}{3x^3} - \frac{2 a b}{27x^3} - \frac{b^3 n^3 \log(x)^3}{3x^3} - \frac{b^3 n^3 \log(x)^2}{x^3} - \frac{b^3 n^3 \log(x)}{9x^3} - \frac{b^3 n^3 \log(c) \log(x)}{3x^3} - \frac{b^3 n^3 \log(c)}{9x^3} - \frac{b^3 n^3}{27x^3} - \frac{b^3 n^2 \log(x)^2}{x^3} - \frac{2 b^3 n^2 \log(x)}{3x^3} - \frac{2 b^3 n^2}{9x^3} - \frac{2 b^3 n \log(c) \log(x)}{x^3} - \frac{2 b^3 n \log(c)}{3x^3} - \frac{2 b^3 n}{9x^3} - \frac{2 b^3 \log(x)}{3x^3} - \frac{2 b^3}{27x^3} - \frac{b^3 \log(x)}{3x^3} - \frac{b^3}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3/x\*\*4,x)

[Out]  $-a^3/(3*x^3) - a^2*b*n*log(x)/x^3 - a^2*b*n/(3*x^3) - a^2*b*log(c)/x^3 - a*b^2*n^2*log(x)^2/x^3 - 2*a*b^2*n^2*log(x)/(3*x^3) - 2*a*b^2*n^2/(9*x^3) - 2*a*b^2*n*log(c)*log(x)/x^3 - 2*a*b^2*n*log(c)/(3*x^3) - a*b^2*log(c)^2/x^3 - b^3*n^3*log(x)^3/(3*x^3) - b^3*n^3*log(x)^2/(3*x^3) - 2*b^3*n^3*log(x)/(9*x^3) - 2*b^3*n^3/(27*x^3) - b^3*n^2*log(c)*log(x)^2/x^3 - 2*b^3*n^2*log(c)*log(x)/(3*x^3) - 2*b^3*n^2*log(c)/(9*x^3) - b^3*n*log(c)^2*log(x)/x^3 - b^3*n*log(c)^2/(3*x^3) - b^3*log(c)^3/(3*x^3)$

$$3.65 \quad \int \frac{x^3}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=51

$$\frac{x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{bn}$$

[Out]  $x^4 \operatorname{Ei}(4*(a+b*\ln(c*x^n))/b/n)/b/\exp(4*a/b/n)/n/((c*x^n)^(4/n))$

**Rubi [A]** time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(a + b*\operatorname{Log}[c*x^n]), x]$

[Out]  $(x^4*\operatorname{ExpIntegralEi}[(4*(a + b*\operatorname{Log}[c*x^n]))/(b*n)])/(b*E^{((4*a)/(b*n))*n}*(c*x^n)^(4/n))$

Rule 2178

$\operatorname{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^(g*(e - (c*f)/d))*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\amp; \operatorname{!}\$UseGamma == \operatorname{True}$

Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^(n_)]*(b_.)]^(p_)*((d_.)*(x_)^(m_)), x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), \operatorname{Subst}[\operatorname{Int}[E^{((m + 1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x\}$

Rubi steps

$$\int \frac{x^3}{a + b \log(cx^n)} dx = \frac{(x^4 (cx^n)^{-4/n}) \text{Subst} \left( \int \frac{e^{\frac{4x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{n}$$

$$= \frac{e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \text{Ei} \left( \frac{4(a+b \log(cx^n))}{bn} \right)}{bn}$$

**Mathematica** [A] time = 0.06, size = 51, normalized size = 1.00

$$\frac{x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \text{Ei} \left( \frac{4(a+b \log(cx^n))}{bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Log[c\*x^n]),x]

[Out] (x^4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((4\*a)/(b\*n))\*n\*(c\*x^n)^(4/n))

**fricas** [A] time = 0.44, size = 42, normalized size = 0.82

$$\frac{e^{\left(-\frac{4(b \log(c)+a)}{bn}\right)} \log\_integral \left( x^4 e^{\left(\frac{4(b \log(c)+a)}{bn}\right)} \right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(-4\*(b\*log(c) + a)/(b\*n))\*log\_integral(x^4\*e^(4\*(b\*log(c) + a)/(b\*n)))/(b\*n)

**giac** [A] time = 0.34, size = 48, normalized size = 0.94

$$\frac{\text{Ei} \left( \frac{4 \log(c)}{n} + \frac{4a}{bn} + 4 \log(x) \right) e^{\left(-\frac{4a}{bn}\right)}}{bc^{\frac{4}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] Ei(4\*log(c)/n + 4\*a/(b\*n) + 4\*log(x))\*e^(-4\*a/(b\*n))/(b\*c^(4/n)\*n)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \ln(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*ln(c\*x^n)),x)

[Out] int(x^3/(a+b\*ln(c\*x^n)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(x^3/(b\*log(c\*x^n) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b\*log(c\*x^n)),x)

[Out] int(x^3/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(x\*\*3/(a + b\*log(c\*x\*\*n)), x)

$$3.66 \quad \int \frac{x^2}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=51

$$\frac{x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{bn}$$

[Out]  $x^3 \operatorname{Ei}(3(a+b \ln(cx^n))/b/n)/b/\exp(3a/b/n)/n/((cx^n)^{(3/n)})$

**Rubi** [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(a + b \operatorname{Log}[c*x^n]), x]$

[Out]  $(x^3 \operatorname{ExpIntegralEi}[(3(a + b \operatorname{Log}[c*x^n]))/(b*n)])/(b * E^{((3*a)/(b*n))} * n * (c*x^n)^{(3/n)})$

Rule 2178

$\operatorname{Int}[(F_)^((g_.) * ((e_.) + (f_.) * (x_)))/((c_.) + (d_.) * (x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d))} * \operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x$  && !\$UseGamma == True

Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.) * (x_)^{(n_.)}] * (b_.)^{(p_)} * ((d_.) * (x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x)/n} * (a + b*x)^p, x], x, \operatorname{Log}[c*x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x$

Rubi steps

$$\int \frac{x^2}{a + b \log(cx^n)} dx = \frac{(x^3 (cx^n)^{-3/n}) \text{Subst} \left( \int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{n}$$

$$= \frac{e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \text{Ei} \left( \frac{3(a+b \log(cx^n))}{bn} \right)}{bn}$$

**Mathematica** [A] time = 0.06, size = 51, normalized size = 1.00

$$\frac{x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \text{Ei} \left( \frac{3(a+b \log(cx^n))}{bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Log[c\*x^n]),x]

[Out] (x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((3\*a)/(b\*n))\*n\*(c\*x^n)^(3/n))

**fricas** [A] time = 0.41, size = 42, normalized size = 0.82

$$\frac{e^{\left(-\frac{3(b \log(c)+a)}{bn}\right)} \log\_integral \left( x^3 e^{\left(\frac{3(b \log(c)+a)}{bn}\right)} \right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(-3\*(b\*log(c) + a)/(b\*n))\*log\_integral(x^3\*e^(3\*(b\*log(c) + a)/(b\*n)))/(b\*n)

**giac** [A] time = 0.26, size = 48, normalized size = 0.94

$$\frac{\text{Ei} \left( \frac{3 \log(c)}{n} + \frac{3a}{bn} + 3 \log(x) \right) e^{\left(-\frac{3a}{bn}\right)}}{bc^{\frac{3}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^2/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] Ei(3\*log(c)/n + 3\*a/(b\*n) + 3\*log(x))\*e^(-3\*a/(b\*n))/(b\*c^(3/n)\*n)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \ln(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*ln(c\*x^n)+a),x)

[Out] int(x^2/(b\*ln(c\*x^n)+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(x^2/(b\*log(c\*x^n) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b\*log(c\*x^n)),x)

[Out] int(x^2/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(x\*\*2/(a + b\*log(c\*x\*\*n)), x)

$$3.67 \quad \int \frac{x}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=51

$$\frac{x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{bn}$$

[Out]  $x^2 \operatorname{Ei}(2*(a+b*\ln(c*x^n))/b/n)/b/\exp(2*a/b/n)/n/((c*x^n)^(2/n))$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2310, 2178}

$$\frac{x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Log[c\*x^n]), x]

[Out]  $(x^2 \operatorname{ExpIntegralEi}[(2*(a + b*\log(c*x^n)))/(b*n)])/(b*E^{(2*a)/(b*n)}*n*(c*x^n)^(2/n))$

Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{x}{a + b \log(cx^n)} dx = \frac{(x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int \frac{e^{\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{n}$$

$$= \frac{e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \text{Ei} \left( \frac{2(a+b \log(cx^n))}{bn} \right)}{bn}$$

**Mathematica** [A] time = 0.05, size = 51, normalized size = 1.00

$$\frac{x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \text{Ei} \left( \frac{2(a+b \log(cx^n))}{bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Log[c\*x^n]),x]

[Out] (x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((2\*a)/(b\*n))\*n\*(c\*x^n)^(2/n))

**fricas** [A] time = 0.43, size = 42, normalized size = 0.82

$$\frac{e^{\left(-\frac{2(b \log(c)+a)}{bn}\right)} \log\_integral \left( x^2 e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} \right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(-2\*(b\*log(c) + a)/(b\*n))\*log\_integral(x^2\*e^(2\*(b\*log(c) + a)/(b\*n)))/(b\*n)

**giac** [A] time = 0.31, size = 48, normalized size = 0.94

$$\frac{\text{Ei} \left( \frac{2 \log(c)}{n} + \frac{2a}{bn} + 2 \log(x) \right) e^{\left(-\frac{2a}{bn}\right)}}{bc^n n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] Ei(2\*log(c)/n + 2\*a/(b\*n) + 2\*log(x))\*e^(-2\*a/(b\*n))/(b\*c^(2/n)\*n)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x}{b \ln(c x^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*ln(c\*x^n)+a),x)

[Out] int(x/(b\*ln(c\*x^n)+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \log(c x^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(x/(b\*log(c\*x^n) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{a + b \ln(c x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b\*log(c\*x^n)),x)

[Out] int(x/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \log(c x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(x/(a + b\*log(c\*x\*\*n)), x)

$$3.68 \quad \int \frac{1}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=48

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn}$$

[Out]  $x \operatorname{Ei}((a+b \ln(c*x^n))/b/n)/b/\exp(a/b/n)/n/((c*x^n)^{(1/n)})$

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2300, 2178}

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b \operatorname{Log}[c*x^n])^{-1}, x]$

[Out]  $(x \operatorname{ExpIntegralEi}[(a + b \operatorname{Log}[c*x^n])/(b*n)])/(b \operatorname{E}^{(a/(b*n))} * n * (c*x^n)^n^{-1})$

Rule 2178

$\operatorname{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_))) / ((c_.) + (d_.) * (x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g * (e - (c*f)/d))} * \operatorname{ExpIntegralEi}[(f * g * (c + d*x) * \operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\}$  &&  $! \$UseGamma == True$

Rule 2300

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.) * (x_)^{(n_.)}] * (b_.)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[x / (n * (c*x^n)^{(1/n)}), \operatorname{Subst}[\operatorname{Int}[E^{(x/n)} * (a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, n, p, x\}$

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \log(cx^n)} dx &= \frac{(x (cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 48, normalized size = 1.00

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^(-1), x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b\*E^(a/(b\*n))\*n\*(c\*x^n)^n^(-1))

**fricas [A]** time = 0.42, size = 39, normalized size = 0.81

$$\frac{e^{\left(-\frac{b \log(c)+a}{bn}\right)} \log\_integral\left(x e^{\left(\frac{b \log(c)+a}{bn}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] e^(-(b\*log(c) + a)/(b\*n))\*log\_integral(x\*e^((b\*log(c) + a)/(b\*n)))/(b\*n)

**giac [A]** time = 0.29, size = 42, normalized size = 0.88

$$\frac{\operatorname{Ei}\left(\frac{\log(c)}{n} + \frac{a}{bn} + \log(x)\right) e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n)), x, algorithm="giac")

[Out] Ei(log(c)/n + a/(b\*n) + log(x))\*e^(-a/(b\*n))/(b\*c^(1/n)\*n)

**maple [C]** time = 0.50, size = 240, normalized size = 5.00

$$x c^{-\frac{1}{n}} (x^n)^{-\frac{1}{n}} \operatorname{Ei}\left(1, -\ln(x) - \frac{-i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2 - i\pi b \operatorname{csgn}(ic x^n)^3 + 2b \ln}{2bn}\right)$$

*bn*

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*ln(c\*x^n)+a), x)

[Out]  $-1/b/n*x*c^{(-1/n)}*(x^n)^{(-1/n)}*\exp(-1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2 - I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n) - I*Pi*b*csgn(I*c*x^n)^3 + I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2 + 2*a)/b/n)*Ei(1, -\ln(x) - 1/2*(I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2 - I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n) - I*Pi*b*csgn(I*c*x^n)^3 + I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2 + 2*b*\ln(c) + 2*b*(\ln(x^n) - n*\ln(x)) + 2*a)/b/n)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(1/(b\*log(c\*x^n) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*log(c\*x^n)),x)

[Out] int(1/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(a + b\*log(c\*x\*\*n)), x)

$$3.69 \quad \int \frac{1}{x(a+b \log(cx^n))} dx$$

Optimal. Leaf size=18

$$\frac{\log(a + b \log(cx^n))}{bn}$$

[Out]  $\ln(a+b*\ln(c*x^n))/b/n$

**Rubi [A]** time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 29}

$$\frac{\log(a + b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*(a + b*\text{Log}[c*x^n])), x]$

[Out]  $\text{Log}[a + b*\text{Log}[c*x^n]]/(b*n)$

Rule 29

$\text{Int}[(x_)^{(-1)}, x\_Symbol] \text{ :> } \text{Simp}[\text{Log}[x], x]$

Rule 2302

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}/(x_), x\_Symbol] \text{ :> } \text{Dist}[1/(b*n), \text{Subst}[\text{Int}[x^p, x], x, a + b*\text{Log}[c*x^n]], x] \text{ /; } \text{FreeQ}\{a, b, c, n, p\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, a + b \log(cx^n)\right)}{bn} \\ &= \frac{\log(a + b \log(cx^n))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 18, normalized size = 1.00

$$\frac{\log(a + b \log(cx^n))}{bn}$$



Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*Log[c\*x^n])),x]

[Out] Log[a + b\*Log[c\*x^n]]/(b\*n)

**fricas** [A] time = 0.44, size = 19, normalized size = 1.06

$$\frac{\log(bn \log(x) + b \log(c) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] log(b\*n\*log(x) + b\*log(c) + a)/(b\*n)

**giac** [B] time = 0.31, size = 45, normalized size = 2.50

$$\frac{\log\left(\frac{1}{4}\left(\pi bn(\operatorname{sgn}(x) - 1) + \pi b(\operatorname{sgn}(c) - 1)\right)^2 + \left(bn \log(|x|) + b \log(|c|) + a\right)^2\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 1/2\*log(1/4\*(pi\*b\*n\*(sgn(x) - 1) + pi\*b\*(sgn(c) - 1))^2 + (b\*n\*log(abs(x)) + b\*log(abs(c)) + a)^2)/(b\*n)

**maple** [A] time = 0.03, size = 19, normalized size = 1.06

$$\frac{\ln(b \ln(c x^n) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*ln(c\*x^n)+a),x)

[Out] ln(b\*ln(c\*x^n)+a)/b/n

**maxima** [A] time = 0.80, size = 18, normalized size = 1.00

$$\frac{\log(b \log(cx^n) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out]  $\log(b \cdot \log(c \cdot x^n) + a) / (b \cdot n)$

**mupad** [B] time = 3.56, size = 18, normalized size = 1.00

$$\frac{\ln(a + b \ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*log(c*x^n))),x)`

[Out]  $\log(a + b \cdot \log(c \cdot x^n)) / (b \cdot n)$

**sympy** [A] time = 1.09, size = 34, normalized size = 1.89

$$\left\{ \begin{array}{ll} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b \log(c)} & \text{for } n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b} + n \log(x) + \log(c)\right)}{bn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*ln(c*x**n)),x)`

[Out] `Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b*log(c)), Eq(n, 0)), (log(x)/a, Eq(b, 0)), (log(a/b + n*log(x) + log(c))/(b*n), True))`

$$3.70 \quad \int \frac{1}{x^2(a+b \log(cx^n))} dx$$

Optimal. Leaf size=48

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

[Out]  $\exp(a/b/n)*(c*x^n)^{(1/n)}*\operatorname{Ei}((-a-b*\ln(c*x^n))/b/n)/b/n/x$

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^2*(a + b*\operatorname{Log}[c*x^n])), x]$

[Out]  $(E^{(a/(b*n))}*(c*x^n)^{n^{(-1)}}*\operatorname{ExpIntegralEi}[-((a + b*\operatorname{Log}[c*x^n])/(b*n))])/(b*n*x)$

#### Rule 2178

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d)})*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \ \&\& \ !\$UseGamma === \operatorname{True}$

#### Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}[\{a, b, c, d, m, n, p\}, x]$

#### Rubi steps

$$\int \frac{1}{x^2 (a + b \log(cx^n))} dx = \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{a+bx} dx, x, \log(cx^n)\right)}{nx}$$

$$= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

**Mathematica** [A] time = 0.05, size = 48, normalized size = 1.00

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*Log[c\*x^n])),x]

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])/(b\*n\*x)

**fricas** [A] time = 0.42, size = 41, normalized size = 0.85

$$\frac{e^{\left(\frac{b \log(c)+a}{bn}\right)} \log\_integral\left(\frac{e^{\left(-\frac{b \log(c)+a}{bn}\right)}}{x}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^((b\*log(c) + a)/(b\*n))\*log\_integral(e^(-(b\*log(c) + a)/(b\*n))/x)/(b\*n)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^2), x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(c x^n) + a) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*ln(c\*x^n)+a), x)

[Out] int(1/x^2/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(c x^n) + a) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 (a + b \ln(c x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*log(c\*x^n))), x)

[Out] int(1/(x^2\*(a + b\*log(c\*x^n))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \log(c x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral(1/(x\*\*2\*(a + b\*log(c\*x\*\*n))), x)

$$3.71 \quad \int \frac{1}{x^3(a+b \log(cx^n))} dx$$

Optimal. Leaf size=51

$$\frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

[Out]  $\exp(2*a/b/n)*(c*x^n)^{(2/n)}*Ei(-2*(a+b*\ln(c*x^n))/b/n)/b/n/x^2$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(a + b*\text{Log}[c*x^n])), x]$

[Out]  $(E^{((2*a)/(b*n))}*(c*x^n)^{(2/n)}*\text{ExpIntegralEi}[(-2*(a + b*\text{Log}[c*x^n]))/(b*n)])/ (b*n*x^2)$

Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)})*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$   
 $\text{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \text{!}\$UseGamma == True$

Rule 2310

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[E^{((m+1)*x)/n}*(a + b*x)^p, x], x, \text{Log}[c*x^n]], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

Rubi steps

$$\int \frac{1}{x^3 (a + b \log(cx^n))} dx = \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{nx^2}$$

$$= \frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

**Mathematica [A]** time = 0.05, size = 51, normalized size = 1.00

$$\frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)])/ (b\*n\*x^2)

**fricas [A]** time = 0.60, size = 42, normalized size = 0.82

$$\frac{e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} \log\_integral\left(\frac{e^{\left(-\frac{2(b \log(c)+a)}{bn}\right)}}{x^2}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(2\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-2\*(b\*log(c) + a)/(b\*n))/x^2)/(b\*n)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^3), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b\*ln(c\*x^n)+a),x)

[Out] int(1/x^3/(b\*ln(c\*x^n)+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*(a + b\*log(c\*x^n))),x)

[Out] int(1/(x^3\*(a + b\*log(c\*x^n))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(x\*\*3\*(a + b\*log(c\*x\*\*n))), x)



$$3.72 \quad \int \frac{1}{x^4(a+b \log(cx^n))} dx$$

Optimal. Leaf size=51

$$\frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{bnx^3}$$

[Out]  $\exp(3*a/b/n)*(c*x^n)^{(3/n)}*Ei(-3*(a+b*\ln(c*x^n))/b/n)/b/n/x^3$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{bnx^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*(a + b*\text{Log}[c*x^n])),x]$

[Out]  $(E^{((3*a)/(b*n))}*(c*x^n)^{(3/n)}*\text{ExpIntegralEi}[(-3*(a + b*\text{Log}[c*x^n]))/(b*n)])/ (b*n*x^3)$

Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x\_Symbol] :> \text{Simp}[(F^{(g*(e - (c*f)/d)})*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \text{!}\$UseGamma == \text{True}$

Rule 2310

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] :> \text{Dist}[(d*x)^{(m + 1)}/(d*n*(c*x^n)^{(m + 1)/n}), \text{Subst}[\text{Int}[E^{((m + 1)*x)/n}*(a + b*x)^p, x], x, \text{Log}[c*x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

Rubi steps

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx = \frac{(cx^n)^{3/n} \text{Subst} \left( \int \frac{e^{-\frac{3x}{n}}}{a+bx} dx, x, \log(cx^n) \right)}{nx^3}$$

$$= \frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right)}{bnx^3}$$

**Mathematica** [A] time = 0.06, size = 51, normalized size = 1.00

$$\frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right)}{bnx^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*n\*x^3)

**fricas** [A] time = 0.53, size = 42, normalized size = 0.82

$$\frac{e^{\left(\frac{3(b \log(c)+a)}{bn}\right)} \log\_integral \left( \frac{e^{\left(-\frac{3(b \log(c)+a)}{bn}\right)}}{x^3} \right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(3\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-3\*(b\*log(c) + a)/(b\*n))/x^3)/(b\*n)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^4), x)

**maple** [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*ln(c\*x^n)+a),x)

[Out] int(1/x^4/(b\*ln(c\*x^n)+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4 (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4\*(a + b\*log(c\*x^n))),x)

[Out] int(1/(x^4\*(a + b\*log(c\*x^n))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(x\*\*4\*(a + b\*log(c\*x\*\*n))), x)

$$3.73 \quad \int \frac{x^3}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$\frac{4x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^4}{bn(a+b \log(cx^n))}$$

[Out]  $4x^4 \operatorname{Ei}(4(a+b \ln(cx^n))/bn)/b^2/\exp(4a/bn)/n^2/((cx^n)^{(4/n)}) - x^4/b/n/(a+b \ln(cx^n))$

**Rubi [A]** time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{4x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^4}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Log[c\*x^n])^2, x]

[Out]  $(4x^4 \operatorname{ExpIntegralEi}[(4(a + b \operatorname{Log}[c*x^n]))/(b*n)])/(b^2 * E^{((4*a)/(b*n))} * n^2 * (c*x^n)^{(4/n)}) - x^4/(b*n*(a + b \operatorname{Log}[c*x^n]))$

Rule 2178

Int[(F\_)^((g\_)\*(e\_)+(f\_)\*(x\_)))/((c\_)+(d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rule 2306

Int[((a\_)+Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*Log[c\*x^n])^(p+1))/(b\*d\*n\*(p+1)), x] - Dist[(m+1)/(b\*n\*(p+1)), Int[(d\*x)^m\*(a+b\*Log[c\*x^n])^(p+1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2310

Int[((a\_)+Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^((m+1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a + b \log(cx^n))^2} dx &= -\frac{x^4}{bn(a + b \log(cx^n))} + \frac{4 \int \frac{x^3}{a + b \log(cx^n)} dx}{bn} \\ &= -\frac{x^4}{bn(a + b \log(cx^n))} + \frac{(4x^4 (cx^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2} \\ &= \frac{4e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \text{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^4}{bn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 70, normalized size = 0.92

$$\frac{x^4 \left( 4e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \text{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Log[c\*x^n])^2,x]

[Out] (x^4\*((4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((4\*a)/(b\*n))\*(c\*x^n)^(4/n)) - (b\*n)/(a + b\*Log[c\*x^n])))/(b^2\*n^2)

**fricas [A]** time = 0.62, size = 101, normalized size = 1.33

$$\frac{\left( bnx^4 e^{\left(\frac{4(b \log(c)+a)}{bn}\right)} - 4(bn \log(x) + b \log(c) + a) \log\_integral \left( x^4 e^{\left(\frac{4(b \log(c)+a)}{bn}\right)} \right) \right) e^{\left(-\frac{4(b \log(c)+a)}{bn}\right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -(b\*n\*x^4\*e^(4\*(b\*log(c) + a)/(b\*n)) - 4\*(b\*n\*log(x) + b\*log(c) + a)\*log\_integral(x^4\*e^(4\*(b\*log(c) + a)/(b\*n))))\*e^(-4\*(b\*log(c) + a)/(b\*n))/(b^3\*n^3\*log(x) + b^3\*n^2\*log(c) + a\*b^2\*n^2)

**giac** [B] time = 0.50, size = 261, normalized size = 3.43

$$\frac{bnx^4}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2} + \frac{4bn \operatorname{Ei}\left(\frac{4 \log(c)}{n} + \frac{4a}{bn} + 4 \log(x)\right) e^{\left(-\frac{4a}{bn}\right)} \log(x)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right) c^{\frac{4}{n}}} + \frac{4b \operatorname{Ei}\left(\frac{4 \log(c)}{n} + \frac{4a}{bn} + 4 \log(x)\right)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right) c^{\frac{4}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$-b*n*x^4/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + 4*b*n*\operatorname{Ei}(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(x)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(4/n)}) + 4*b*\operatorname{Ei}(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(c)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(4/n)}) + 4*a*\operatorname{Ei}(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(4/n)})$$

**maple** [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b \ln(c x^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*ln(c\*x^n)+a)^2,x)

[Out] int(x^3/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^4}{b^2n \log(c) + b^2n \log(x^n) + abn} + 4 \int \frac{x^3}{b^2n \log(c) + b^2n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 
$$-x^4/(b^2*n*\log(c) + b^2*n*\log(x^n) + a*b*n) + 4*\operatorname{integrate}(x^3/(b^2*n*\log(c) + b^2*n*\log(x^n) + a*b*n), x)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(a + b \ln(c x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x^3/(a + b*log(c*x^n))^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(x**3/(a + b*log(c*x**n))**2, x)
```

$$3.74 \quad \int \frac{x^2}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$\frac{3x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^3}{bn(a+b \log(cx^n))}$$

[Out]  $3x^3 \operatorname{Ei}(3(a+b \ln(cx^n))/bn)/b^2/\exp(3a/bn)/n^2/((cx^n)^{(3/n)}) - x^3/b/n/(a+b \ln(cx^n))$

**Rubi [A]** time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{3x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^3}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Log[c\*x^n])^2, x]

[Out]  $(3x^3 \operatorname{ExpIntegralEi}[(3(a + b \operatorname{Log}[c*x^n]))/(b*n)])/(b^2 * E^{((3*a)/(b*n))} * n^2 * (c*x^n)^{(3/n)}) - x^3/(b*n*(a + b \operatorname{Log}[c*x^n]))$

Rule 2178

Int[(F\_)^((g\_)\*(e\_)+(f\_)\*(x\_)))/((c\_)+(d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2306

Int[((a\_)+Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*Log[c\*x^n])^(p+1))/(b\*d\*n\*(p+1)), x] - Dist[(m+1)/(b\*n\*(p+1)), Int[(d\*x)^m\*(a+b\*Log[c\*x^n])^(p+1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2310

Int[((a\_)+Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^((m+1)\*x)



/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a + b \log(cx^n))^2} dx &= -\frac{x^3}{bn(a + b \log(cx^n))} + \frac{3 \int \frac{x^2}{a + b \log(cx^n)} dx}{bn} \\ &= -\frac{x^3}{bn(a + b \log(cx^n))} + \frac{(3x^3 (cx^n)^{-3/n}) \text{Subst}\left(\int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2} \\ &= \frac{3e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^3}{bn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 70, normalized size = 0.92

$$\frac{x^3 \left( 3e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Log[c\*x^n])^2,x]

[Out] (x^3\*((3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)) - (b\*n)/(a + b\*Log[c\*x^n])))/(b^2\*n^2)

**fricas [A]** time = 0.64, size = 101, normalized size = 1.33

$$\frac{\left( bnx^3 e^{\left(\frac{3(b \log(c)+a)}{bn}\right)} - 3(bn \log(x) + b \log(c) + a) \log\_integral \left( x^3 e^{\left(\frac{3(b \log(c)+a)}{bn}\right)} \right) \right) e^{\left(-\frac{3(b \log(c)+a)}{bn}\right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -(b\*n\*x^3\*e^(3\*(b\*log(c) + a)/(b\*n)) - 3\*(b\*n\*log(x) + b\*log(c) + a)\*log\_integral(x^3\*e^(3\*(b\*log(c) + a)/(b\*n))))\*e^(-3\*(b\*log(c) + a)/(b\*n))/(b^3\*n^3\*log(x) + b^3\*n^2\*log(c) + a\*b^2\*n^2)

**giac** [B] time = 0.44, size = 261, normalized size = 3.43

$$\frac{bnx^3}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2} + \frac{3bn \operatorname{Ei}\left(\frac{3 \log(c)}{n} + \frac{3a}{bn} + 3 \log(x)\right) e^{\left(-\frac{3a}{bn}\right)} \log(x)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right) c^{\frac{3}{n}}} + \frac{3b \operatorname{Ei}\left(\frac{3 \log(c)}{n} + \frac{3a}{bn} + 3 \log(x)\right)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right) c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$-b*n*x^3/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + 3*b*n*\operatorname{Ei}(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(x)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(3/n)}) + 3*b*\operatorname{Ei}(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(c)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(3/n)}) + 3*a*\operatorname{Ei}(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(3/n)})$$

**maple** [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \ln(c x^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*ln(c\*x^n)+a)^2,x)

[Out] int(x^2/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^3}{b^2n \log(c) + b^2n \log(x^n) + abn} + 3 \int \frac{x^2}{b^2n \log(c) + b^2n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 
$$-x^3/(b^2*n*\log(c) + b^2*n*\log(x^n) + a*b*n) + 3*\operatorname{integrate}(x^2/(b^2*n*\log(c) + b^2*n*\log(x^n) + a*b*n), x)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a + b \ln(c x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*log(c*x^n))^2,x)`

[Out] `int(x^2/(a + b*log(c*x^n))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*ln(c*x**n))**2,x)`

[Out] `Integral(x**2/(a + b*log(c*x**n))**2, x)`

$$3.75 \quad \int \frac{x}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^2}{bn(a+b \log(cx^n))}$$

[Out]  $2*x^2*Ei(2*(a+b*\ln(c*x^n))/b/n)/b^2/\exp(2*a/b/n)/n^2/((c*x^n)^(2/n))-x^2/b/n/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2306, 2310, 2178}

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^2}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Log[c\*x^n])^2, x]

[Out]  $(2*x^2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(b^2*E^((2*a)/(b*n))*n^2*(c*x^n)^(2/n)) - x^2/(b*n*(a + b*Log[c*x^n]))$

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x}{(a + b \log(cx^n))^2} dx &= -\frac{x^2}{bn(a + b \log(cx^n))} + \frac{2 \int \frac{x}{a + b \log(cx^n)} dx}{bn} \\ &= -\frac{x^2}{bn(a + b \log(cx^n))} + \frac{(2x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int \frac{e^{\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2} \\ &= \frac{2e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \text{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^2}{bn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 70, normalized size = 0.92

$$\frac{x^2 \left( 2e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \text{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Log[c\*x^n])^2,x]

[Out] (x^2\*((2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)) - (b\*n)/(a + b\*Log[c\*x^n])))/(b^2\*n^2)

**fricas [A]** time = 0.42, size = 101, normalized size = 1.33

$$\frac{\left( bnx^2 e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} - 2(bn \log(x) + b \log(c) + a) \log\_integral \left( x^2 e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} \right) \right) e^{\left(-\frac{2(b \log(c)+a)}{bn}\right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -(b\*n\*x^2\*e^(2\*(b\*log(c) + a)/(b\*n)) - 2\*(b\*n\*log(x) + b\*log(c) + a)\*log\_integral(x^2\*e^(2\*(b\*log(c) + a)/(b\*n))))\*e^(-2\*(b\*log(c) + a)/(b\*n))/(b^3\*n^3\*log(x) + b^3\*n^2\*log(c) + a\*b^2\*n^2)

**giac** [B] time = 0.42, size = 261, normalized size = 3.43

$$\frac{bnx^2}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2} + \frac{2bn\text{Ei}\left(\frac{2\log(c)}{n} + \frac{2a}{bn} + 2\log(x)\right)e^{\left(-\frac{2a}{bn}\right)}\log(x)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right)c^{\frac{2}{n}}} + \frac{2b\text{Ei}\left(\frac{2\log(c)}{n} + \frac{2a}{bn} + 2\log(x)\right)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right)c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$-b*n*x^2/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + 2*b*n*\text{Ei}(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(x)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(2/n)}) + 2*b*\text{Ei}(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(2/n)}) + 2*a*\text{Ei}(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(2/n)})$$

**maple** [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \ln(c x^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*ln(c\*x^n)+a)^2,x)

[Out] int(x/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^2}{b^2n \log(c) + b^2n \log(x^n) + abn} + 2 \int \frac{x}{b^2n \log(c) + b^2n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 
$$-x^2/(b^2*n*\log(c) + b^2*n*\log(x^n) + a*b*n) + 2*\integrate(x/(b^2*n*\log(c) + b^2*n*\log(x^n) + a*b*n), x)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \ln(c x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*log(c*x^n))^2,x)`

[Out] `int(x/(a + b*log(c*x^n))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*ln(c*x**n))**2,x)`

[Out] `Integral(x/(a + b*log(c*x**n))**2, x)`

$$3.76 \quad \int \frac{1}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=70

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2} - \frac{x}{bn(a+b \log(cx^n))}$$

[Out] x\*Ei((a+b\*ln(c\*x^n))/b/n)/b^2/exp(a/b/n)/n^2/((c\*x^n)^(1/n))-x/b/n/(a+b\*ln(c\*x^n))

**Rubi [A]** time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2297, 2300, 2178}

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2} - \frac{x}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^(-2), x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b^2\*E^(a/(b\*n))\*n^2\*(c\*x^n)^n^(-1)) - x/(b\*n\*(a + b\*Log[c\*x^n]))

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] := Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]



Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \log(cx^n))^2} dx &= -\frac{x}{bn(a + b \log(cx^n))} + \frac{\int \frac{1}{a + b \log(cx^n)} dx}{bn} \\
&= -\frac{x}{bn(a + b \log(cx^n))} + \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a + bx} dx, x, \log(cx^n)\right)}{bn^2} \\
&= \frac{e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \text{Ei}\left(\frac{a + b \log(cx^n)}{bn}\right)}{b^2 n^2} - \frac{x}{bn(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 66, normalized size = 0.94

$$\frac{x \left( e^{-\frac{a}{bn}} (cx^n)^{-1/n} \text{Ei}\left(\frac{a + b \log(cx^n)}{bn}\right) - \frac{bn}{a + b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^(-2), x]

[Out] (x\*(ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)]/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)) - (b\*n)/(a + b\*Log[c\*x^n])))/(b^2\*n^2)

**fricas [A]** time = 0.40, size = 95, normalized size = 1.36

$$\frac{\left( b n x e^{\left( \frac{b \log(c) + a}{bn} \right)} - (bn \log(x) + b \log(c) + a) \log\_integral \left( x e^{\left( \frac{b \log(c) + a}{bn} \right)} \right) \right) e^{-\frac{b \log(c) + a}{bn}}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -(b\*n\*x\*e^((b\*log(c) + a)/(b\*n)) - (b\*n\*log(x) + b\*log(c) + a)\*log\_integral(x\*e^((b\*log(c) + a)/(b\*n))))\*e^(-(b\*log(c) + a)/(b\*n))/(b^3\*n^3\*log(x) + b^3\*n^2\*log(c) + a\*b^2\*n^2)

**giac [B]** time = 0.31, size = 238, normalized size = 3.40

$$\frac{bn \text{Ei}\left(\frac{\log(c)}{n} + \frac{a}{bn} + \log(x)\right) e^{-\frac{a}{bn}} \log(x)}{(b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2) c^{\left(\frac{1}{n}\right)}} - \frac{bnx}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2} + \frac{b \text{Ei}\left(\frac{\log(c)}{n} + \frac{a}{bn} + \log(x)\right) e^{-\frac{a}{bn}}}{(b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out]  $b^n \operatorname{Ei}(\log(c)/n + a/(b^n) + \log(x)) e^{-a/(b^n)} \log(x) / ((b^{3n^3} \log(x) + b^{3n^2} \log(c) + a b^{2n^2}) c^{1/n}) - b^n x / (b^{3n^3} \log(x) + b^{3n^2} \log(c) + a b^{2n^2}) + b^n \operatorname{Ei}(\log(c)/n + a/(b^n) + \log(x)) e^{-a/(b^n)} \log(c) / (b^{3n^3} \log(x) + b^{3n^2} \log(c) + a b^{2n^2}) c^{1/n} + a \operatorname{Ei}(\log(c)/n + a/(b^n) + \log(x)) e^{-a/(b^n)} / ((b^{3n^3} \log(x) + b^{3n^2} \log(c) + a b^{2n^2}) c^{1/n})$

maple [C] time = 0.49, size = 350, normalized size = 5.00

$$x c^{-\frac{1}{n}} (x^n)^{-\frac{1}{n}} \operatorname{Ei} \left( 1, -\ln(x) - \frac{-i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2 - i\pi b \operatorname{csgn}(ic x^n)^3 + 2b \ln}{2bn} \right)$$

$b^{2n^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*ln(c\*x^n)+a)^2,x)

[Out]  $-2/b/n*x/(2*a+2*b*\ln(c)+2*b*\ln(x^n)+I*\pi*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)-I*\pi*b*\operatorname{csgn}(I*c*x^n)^3+I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2)-1/b^2/n^2*x*(x^n)^{-1/n}*c^{-1/n}*\exp(-1/2*(-I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)+I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2+I*\pi*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-I*\pi*b*\operatorname{csgn}(I*c*x^n)^3+2*a)/b/n)*\operatorname{Ei}(1,-\ln(x)-1/2*(-I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)+I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2+I*\pi*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-I*\pi*b*\operatorname{csgn}(I*c*x^n)^3+2*b*\ln(c)+2*a+2*(-n*\ln(x)+\ln(x^n))*b)/b/n)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x}{b^2 n \log(c) + b^2 n \log(x^n) + a b n} + \int \frac{1}{b^2 n \log(c) + b^2 n \log(x^n) + a b n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out]  $-x/(b^{2n} \log(c) + b^{2n} \log(x^n) + a b^n) + \operatorname{integrate}(1/(b^{2n} \log(c) + b^{2n} \log(x^n) + a b^n), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \ln(c x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*log(c*x^n))^2,x)`

[Out] `int(1/(a + b*log(c*x^n))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*ln(c*x**n))**2,x)`

[Out] `Integral((a + b*log(c*x**n))**(-2), x)`

$$3.77 \quad \int \frac{1}{x(a+b \log(cx^n))^2} dx$$

Optimal. Leaf size=20

$$-\frac{1}{bn(a+b \log(cx^n))}$$

[Out] -1/b/n/(a+b\*ln(c\*x^n))

**Rubi [A]** time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$-\frac{1}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*Log[c\*x^n])^2), x]

[Out] -(1/(b\*n\*(a + b\*Log[c\*x^n])))

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+b \log(cx^n))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, a + b \log(cx^n)\right)}{bn} \\ &= -\frac{1}{bn(a+b \log(cx^n))} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{1}{bn(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*Log[c\*x^n])^2), x]

[Out] -(1/(b\*n\*(a + b\*Log[c\*x^n])))

**fricas** [A] time = 0.40, size = 25, normalized size = 1.25

$$\frac{1}{b^2 n^2 \log(x) + b^2 n \log(c) + abn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -1/(b^2\*n^2\*log(x) + b^2\*n\*log(c) + a\*b\*n)

**giac** [A] time = 0.25, size = 21, normalized size = 1.05

$$\frac{1}{(bn \log(x) + b \log(c) + a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] -1/((b\*n\*log(x) + b\*log(c) + a)\*b\*n)

**maple** [A] time = 0.02, size = 21, normalized size = 1.05

$$\frac{1}{(b \ln(cx^n) + a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*ln(c\*x^n)+a)^2,x)

[Out] -1/b/n/(b\*ln(c\*x^n)+a)

**maxima** [A] time = 0.50, size = 20, normalized size = 1.00

$$\frac{1}{(b \log(cx^n) + a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/((b\*log(c\*x^n) + a)\*b\*n)

mupad [B] time = 3.27, size = 20, normalized size = 1.00

$$-\frac{1}{n \ln(c x^n) b^2 + a n b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*log(c\*x^n))^2),x)

[Out] -1/(b^2\*n\*log(c\*x^n) + a\*b\*n)

sympy [A] time = 22.74, size = 70, normalized size = 3.50

$$\left\{ \begin{array}{ll} \frac{\infty \log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \tilde{\infty} n \log(x) & \text{for } a = -b(n \log(x) + \log(c)) \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b \log(c))^2} & \text{for } n = 0 \\ -\frac{1}{abn+b^2n^2 \log(x)+b^2n \log(c)} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Piecewise((zoo\*log(x)/log(c)\*\*2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (zoo\*n\*log(x), Eq(a, -b\*(n\*log(x) + log(c)))), (log(x)/a\*\*2, Eq(b, 0)), (log(x)/(a + b\*log(c))\*\*2, Eq(n, 0)), (-1/(a\*b\*n + b\*\*2\*n\*\*2\*log(x) + b\*\*2\*n\*log(c)), True))

$$3.78 \quad \int \frac{1}{x^2(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=73

$$-\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2 x} - \frac{1}{bnx(a+b \log(cx^n))}$$

[Out]  $-\exp(a/b/n)*(c*x^n)^{(1/n)}*Ei((-a-b*\ln(c*x^n))/b/n)/b^2/n^2/x-1/b/n/x/(a+b*1n(c*x^n))$

**Rubi [A]** time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$-\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2 x} - \frac{1}{bnx(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a + b*Log[c*x^n])^2), x]`

[Out]  $-\left(\left(E^{a/(b*n)}\right)*(c*x^n)^{n^{-1}}*\operatorname{ExpIntegralEi}\left[-\left(a + b*\operatorname{Log}[c*x^n]\right)/(b*n)\right]\right)/(b^2*n^2*x) - 1/(b*n*x*(a + b*\operatorname{Log}[c*x^n]))$

Rule 2178

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True`

Rule 2306

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

Rule 2310

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p*((d_.)*(x_))^(m_.), x_Symbol] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + b \log(cx^n))^2} dx &= -\frac{1}{bnx (a + b \log(cx^n))} - \frac{\int \frac{1}{x^2 (a + b \log(cx^n))} dx}{bn} \\
&= -\frac{1}{bnx (a + b \log(cx^n))} - \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{a + bx} dx, x, \log(cx^n)\right)}{bn^2 x} \\
&= -\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \text{Ei}\left(-\frac{a + b \log(cx^n)}{bn}\right)}{b^2 n^2 x} - \frac{1}{bnx (a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 76, normalized size = 1.04

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n)) \text{Ei}\left(-\frac{a + b \log(cx^n)}{bn}\right) + bn}{b^2 n^2 x (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*Log[c\*x^n])^2),x]

[Out] -((b\*n + E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*x\*(a + b\*Log[c\*x^n]))

**fricas [A]** time = 0.42, size = 88, normalized size = 1.21

$$\frac{(bnx \log(x) + bx \log(c) + ax) e^{\left(\frac{b \log(c) + a}{bn}\right)} \log\_integral\left(\frac{e^{\left(\frac{-b \log(c) + a}{bn}\right)}}{x}\right) + bn}{b^3 n^3 x \log(x) + b^3 n^2 x \log(c) + ab^2 n^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -((b\*n\*x\*log(x) + b\*x\*log(c) + a\*x)\*e^((b\*log(c) + a)/(b\*n))\*log\_integral(e^(-((b\*log(c) + a)/(b\*n))/x) + b\*n)/(b^3\*n^3\*x\*log(x) + b^3\*n^2\*x\*log(c) + a\*b^2\*n^2\*x))



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^2\*x^2), x)

**maple** [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*ln(c\*x^n)+a)^2,x)

[Out] int(1/x^2/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{b^2 n x \log(x^n) + (b^2 n \log(c) + a b n) x} - \int \frac{1}{b^2 n x^2 \log(x^n) + (b^2 n \log(c) + a b n) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/(b^2\*n\*x\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x) - integrate(1/(b^2\*n\*x^2\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b\*log(c\*x^n))^2),x)

[Out] int(1/(x^2\*(a + b\*log(c\*x^n))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*log(c\*x\*\*n))\*\*2), x)

$$3.79 \quad \int \frac{1}{x^3 (a + b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$-\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^2} - \frac{1}{bnx^2 (a + b \log(cx^n))}$$

[Out]  $-2*\exp(2*a/b/n)*(c*x^n)^{(2/n)}*Ei(-2*(a+b*\ln(c*x^n))/b/n)/b^2/n^2/x^2-1/b/n/x^2/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$-\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^2} - \frac{1}{bnx^2 (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*Log[c\*x^n])^2), x]

[Out]  $(-2*E^{((2*a)/(b*n))}*(c*x^n)^{(2/n)}*ExpIntegralEi[(-2*(a + b*Log[c*x^n]))/(b*n)])/(b^2*n^2*x^2) - 1/(b*n*x^2*(a + b*Log[c*x^n]))$

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + b \log(cx^n))^2} dx &= -\frac{1}{bnx^2 (a + b \log(cx^n))} - \frac{2 \int \frac{1}{x^3 (a + b \log(cx^n))} dx}{bn} \\ &= -\frac{1}{bnx^2 (a + b \log(cx^n))} - \frac{(2 (cx^n)^{2/n}) \text{Subst} \left( \int \frac{e^{-\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bn^2 x^2} \\ &= -\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right)}{b^2 n^2 x^2} - \frac{1}{bnx^2 (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 80, normalized size = 1.05

$$\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} (a + b \log(cx^n)) \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right) + bn}{b^2 n^2 x^2 (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*Log[c\*x^n])^2), x]

[Out] -((b\*n + 2\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))]/(b\*n)]\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*x^2\*(a + b\*Log[c\*x^n]))

**fricas [A]** time = 0.46, size = 102, normalized size = 1.34

$$\frac{2 (bnx^2 \log(x) + bx^2 \log(c) + ax^2) e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} \log\_integral \left( \frac{e^{\left(\frac{2(b \log(c)+a)}{bn}\right)}}{x^2} \right) + bn}{b^3 n^3 x^2 \log(x) + b^3 n^2 x^2 \log(c) + ab^2 n^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out]  $-(2*(b*n*x^2*\log(x) + b*x^2*\log(c) + a*x^2)*e^{(2*(b*\log(c) + a)/(b*n))}*\log\_integral(e^{(-2*(b*\log(c) + a)/(b*n))}/x^2) + b*n)/(b^3*n^3*x^2*\log(x) + b^3*n^2*x^2*\log(c) + a*b^2*n^2*x^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="giac")`

[Out] `integrate(1/((b*log(c*x^n) + a)^2*x^3), x)`

**maple** [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*ln(c*x^n)+a)^2,x)`

[Out] `int(1/x^3/(b*ln(c*x^n)+a)^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{b^2 n x^2 \log(x^n) + (b^2 n \log(c) + a b n) x^2} - 2 \int \frac{1}{b^2 n x^3 \log(x^n) + (b^2 n \log(c) + a b n) x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $-1/(b^2*n*x^2*\log(x^n) + (b^2*n*\log(c) + a*b*n)*x^2) - 2*integrate(1/(b^2*n*x^3*\log(x^n) + (b^2*n*\log(c) + a*b*n)*x^3), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a + b*log(c*x^n))^2),x)`

[Out] `int(1/(x^3*(a + b*log(c*x^n))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a+b*ln(c*x**n))**2,x)`

[Out] `Integral(1/(x**3*(a + b*log(c*x**n))**2), x)`

$$3.80 \quad \int \frac{1}{x^4(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$-\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^3} - \frac{1}{bnx^3(a+b \log(cx^n))}$$

[Out]  $-3*\exp(3*a/b/n)*(c*x^n)^{(3/n)}*Ei(-3*(a+b*\ln(c*x^n))/b/n)/b^2/n^2/x^3-1/b/n/x^3/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$-\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^3} - \frac{1}{bnx^3(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*Log[c\*x^n])^2), x]

[Out]  $(-3*E^{((3*a)/(b*n))}*(c*x^n)^{(3/n)}*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n)])/(b^2*n^2*x^3) - 1/(b*n*x^3*(a + b*Log[c*x^n]))$

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 (a + b \log(cx^n))^2} dx &= -\frac{1}{bnx^3 (a + b \log(cx^n))} - \frac{3 \int \frac{1}{x^4 (a + b \log(cx^n))} dx}{bn} \\ &= -\frac{1}{bnx^3 (a + b \log(cx^n))} - \frac{(3 (cx^n)^{3/n}) \text{Subst} \left( \int \frac{e^{-\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bn^2 x^3} \\ &= -\frac{3e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right)}{b^2 n^2 x^3} - \frac{1}{bnx^3 (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 80, normalized size = 1.05

$$\frac{3e^{\frac{3a}{bn}} (cx^n)^{3/n} (a + b \log(cx^n)) \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right) + bn}{b^2 n^2 x^3 (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*Log[c\*x^n])^2), x]

[Out] -((b\*n + 3\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))]/(b\*n)]\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*x^3\*(a + b\*Log[c\*x^n]))

**fricas [A]** time = 0.42, size = 102, normalized size = 1.34

$$\frac{3 (bnx^3 \log(x) + bx^3 \log(c) + ax^3) e^{\left( \frac{3(b \log(c)+a)}{bn} \right)} \log\_integral \left( \frac{e^{\left( -\frac{3(b \log(c)+a)}{bn} \right)}}{x^3} \right) + bn}{b^3 n^3 x^3 \log(x) + b^3 n^2 x^3 \log(c) + ab^2 n^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")



[Out]  $-(3*(b*n*x^3*\log(x) + b*x^3*\log(c) + a*x^3)*e^{(3*(b*\log(c) + a)/(b*n))}*\log\_integral(e^{(-3*(b*\log(c) + a)/(b*n))}/x^3) + b*n)/(b^3*n^3*x^3*\log(x) + b^3*n^2*x^3*\log(c) + a*b^2*n^2*x^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*log(c*x^n))^2,x, algorithm="giac")`

[Out] `integrate(1/((b*log(c*x^n) + a)^2*x^4), x)`

**maple** [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*ln(c*x^n)+a)^2,x)`

[Out] `int(1/x^4/(b*ln(c*x^n)+a)^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{b^2 n x^3 \log(x^n) + (b^2 n \log(c) + a b n) x^3} - 3 \int \frac{1}{b^2 n x^4 \log(x^n) + (b^2 n \log(c) + a b n) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $-1/(b^2*n*x^3*\log(x^n) + (b^2*n*\log(c) + a*b*n)*x^3) - 3*integrate(1/(b^2*n*x^4*\log(x^n) + (b^2*n*\log(c) + a*b*n)*x^4), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a + b*log(c*x^n))^2),x)`

[Out] `int(1/(x^4*(a + b*log(c*x^n))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(a+b*ln(c*x**n))**2,x)`

[Out] `Integral(1/(x**4*(a + b*log(c*x**n))**2), x)`

$$3.81 \quad \int \frac{x^3}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=101

$$\frac{8x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{2x^4}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^4}{2bn (a+b \log(cx^n))^2}$$

[Out]  $8*x^4*Ei(4*(a+b*\ln(c*x^n))/b/n)/b^3/\exp(4*a/b/n)/n^3/((c*x^n)^(4/n))-1/2*x^4/b/n/(a+b*\ln(c*x^n))^{2-2*x^4/b^2/n^2/(a+b*\ln(c*x^n))}$

**Rubi [A]** time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.188, Rules used = {2306, 2310, 2178}

$$\frac{8x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{2x^4}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^4}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Log[c\*x^n])^3, x]

[Out]  $(8*x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b^3*E^((4*a)/(b*n))*n^3*(c*x^n)^(4/n)) - x^4/(2*b*n*(a + b*Log[c*x^n])^2) - (2*x^4)/(b^2*n^2*(a + b*Log[c*x^n]))$

Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol
] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a + b \log(cx^n))^3} dx &= -\frac{x^4}{2bn(a + b \log(cx^n))^2} + \frac{2 \int \frac{x^3}{(a + b \log(cx^n))^2} dx}{bn} \\ &= -\frac{x^4}{2bn(a + b \log(cx^n))^2} - \frac{2x^4}{b^2n^2(a + b \log(cx^n))} + \frac{8 \int \frac{x^3}{a + b \log(cx^n)} dx}{b^2n^2} \\ &= -\frac{x^4}{2bn(a + b \log(cx^n))^2} - \frac{2x^4}{b^2n^2(a + b \log(cx^n))} + \frac{(8x^4 (cx^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{a+bx}} dx, x}{b^2n^3}\right)}{b^2n^3} \\ &= \frac{8e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \text{Ei}\left(\frac{4(a + b \log(cx^n))}{bn}\right)}{b^3n^3} - \frac{x^4}{2bn(a + b \log(cx^n))^2} - \frac{2x^4}{b^2n^2(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 89, normalized size = 0.88

$$\frac{x^4 \left( 16e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \text{Ei}\left(\frac{4(a + b \log(cx^n))}{bn}\right) - \frac{bn(4a + 4b \log(cx^n) + bn)}{(a + b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^4*((16*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(E^((4*a)/(b*n)))*(c*
x^n)^(4/n)) - (b*n*(4*a + b*n + 4*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*
b^3*n^3)
```

**fricas [B]** time = 0.44, size = 211, normalized size = 2.09

$$\frac{\left( (4b^2n^2x^4 \log(x) + 4b^2nx^4 \log(c) + (b^2n^2 + 4abn)x^4 \right) e^{\left(\frac{4(b \log(c) + a)}{bn}\right)} - 16(b^2n^2 \log(x)^2 + b^2 \log(c)^2 + 2ab \log(c))}{2(b^5n^5 \log(x)^2 + b^5n^3 \log(c)^2 + 2ab^4n^3 \log(c) + a^2b^3n^3 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 
$$-1/2*((4*b^2*n^2*x^4*\log(x) + 4*b^2*n*x^4*\log(c) + (b^2*n^2 + 4*a*b*n)*x^4) * e^{(4*(b*\log(c) + a)/(b*n))} - 16*(b^2*n^2*\log(x)^2 + b^2*\log(c)^2 + 2*a*b*\log(c) + a^2 + 2*(b^2*n*\log(c) + a*b*n)*\log(x))*\log\_integral(x^4*e^{(4*(b*\log(c) + a)/(b*n))}) * e^{(-4*(b*\log(c) + a)/(b*n))}/(b^5*n^5*\log(x)^2 + b^5*n^3*\log(c)^2 + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*\log(c) + a*b^4*n^4)*\log(x)))$$

**giac** [B] time = 0.55, size = 1029, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2*b^2*n^2*x^4*\log(x)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - 1/2*b^2*n^2*x^4/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - 2*b^2*n*x^4*\log(c)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - 2*a*b*n*x^4/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 8*b^2*n^2*Ei(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(x)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 16*b^2*n*Ei(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(c)*\log(x)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 8*b^2*Ei(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(c)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 16*a*b*n*Ei(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(c)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 16*a*b*Ei(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(c)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 8*a^2*Ei(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) \end{aligned}$$

**maple** [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b \ln(c x^n) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*ln(c*x^n)+a)^3,x)`

[Out] `int(x^3/(b*ln(c*x^n)+a)^3,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4bx^4 \log(x^n) + (b(n + 4 \log(c)) + 4a)x^4}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + 8 \int \frac{1}{b^3n^2 \log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+b*log(c*x^n))^3,x, algorithm="maxima")`

[Out] `-1/2*(4*b*x^4*log(x^n) + (b*(n + 4*log(c)) + 4*a)*x^4)/(b^4*n^2*log(c)^2 + b^4*n^2*log(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) + a*b^3*n^2)*log(x^n)) + 8*integrate(x^3/(b^3*n^2*log(c) + b^3*n^2*log(x^n) + a*b^2*n^2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b*log(c*x^n))^3,x)`

[Out] `int(x^3/(a + b*log(c*x^n))^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a+b*ln(c*x**n))**3,x)`

[Out] `Integral(x**3/(a + b*log(c*x**n))**3, x)`

$$3.82 \quad \int \frac{x^2}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=105

$$\frac{9x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3} - \frac{3x^3}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x^3}{2bn (a+b \log(cx^n))^2}$$

[Out]  $9/2*x^3*Ei(3*(a+b*\ln(c*x^n))/b/n)/b^3/\exp(3*a/b/n)/n^3/((c*x^n)^(3/n))-1/2*x^3/b/n/(a+b*\ln(c*x^n))^2-3/2*x^3/b^2/n^2/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{9x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3} - \frac{3x^3}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x^3}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Log[c\*x^n])^3, x]

[Out]  $(9*x^3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(2*b^3*E^((3*a)/(b*n))*n^3*(c*x^n)^(3/n)) - x^3/(2*b*n*(a + b*Log[c*x^n])^2) - (3*x^3)/(2*b^2*n^2*(a + b*Log[c*x^n]))$

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a + b \log(cx^n))^3} dx &= -\frac{x^3}{2bn(a + b \log(cx^n))^2} + \frac{3 \int \frac{x^2}{(a + b \log(cx^n))^2} dx}{2bn} \\ &= -\frac{x^3}{2bn(a + b \log(cx^n))^2} - \frac{3x^3}{2b^2n^2(a + b \log(cx^n))} + \frac{9 \int \frac{x^2}{a + b \log(cx^n)} dx}{2b^2n^2} \\ &= -\frac{x^3}{2bn(a + b \log(cx^n))^2} - \frac{3x^3}{2b^2n^2(a + b \log(cx^n))} + \frac{(9x^3 (cx^n)^{-3/n}) \text{Subst}\left(\int \frac{e^{\frac{3x}{a+bx}} dx, x}{2b^2n^3}\right)}{2b^2n^3} \\ &= \frac{9e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3n^3} - \frac{x^3}{2bn(a + b \log(cx^n))^2} - \frac{3x^3}{2b^2n^2(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 89, normalized size = 0.85

$$\frac{x^3 \left( 9e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right) - \frac{bn(3a+3b \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^3*((9*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(E^((3*a)/(b*n))*(c*x
^n)^(3/n)) - (b*n*(3*a + b*n + 3*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*b
^3*n^3)
```

**fricas [B]** time = 0.44, size = 211, normalized size = 2.01

$$\frac{\left( (3b^2n^2x^3 \log(x) + 3b^2nx^3 \log(c) + (b^2n^2 + 3abn)x^3 \right) e^{\left(\frac{3(b \log(c)+a)}{bn}\right)} - 9(b^2n^2 \log(x)^2 + b^2 \log(c)^2 + 2ab \log(c)) \right)}{2(b^5n^5 \log(x)^2 + b^5n^3 \log(c)^2 + 2ab^4n^3 \log(c) + a^2b^3n^3 + 2ab^2n^2 \log(x) + a^2bn^2 \log(c) + a^2n^2)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 
$$-1/2*((3*b^2*n^2*x^3*\log(x) + 3*b^2*n*x^3*\log(c) + (b^2*n^2 + 3*a*b*n)*x^3) * e^{(3*(b*\log(c) + a)/(b*n))} - 9*(b^2*n^2*\log(x)^2 + b^2*\log(c)^2 + 2*a*b*\log(c) + a^2 + 2*(b^2*n*\log(c) + a*b*n)*\log(x))*\log\_integral(x^3*e^{(3*(b*\log(c) + a)/(b*n))}) * e^{(-3*(b*\log(c) + a)/(b*n))}/(b^5*n^5*\log(x)^2 + b^5*n^3*\log(c)^2 + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*\log(c) + a*b^4*n^4)*\log(x)))$$

**giac** [B] time = 0.56, size = 1029, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -3/2*b^2*n^2*x^3*\log(x)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - 1/2* \\ & b^2*n^2*x^3/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - 3/2*b^2*n*x^3*\log(c)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 9/2*b^2*n^2*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(x)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(3/n)}) - 3/2*a*b*n*x^3/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 9*b^2*n*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(c)*\log(x)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(3/n)}) + 9/2*b^2*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(c)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(3/n)}) + 9*a*b*n*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(c)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(3/n)}) + 9/2*a^2*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(3/n)}) + 9/2*a^2*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(3/n)}) \end{aligned}$$

**maple** [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \ln(c x^n) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*ln(c*x^n)+a)^3,x)`

[Out] `int(x^2/(b*ln(c*x^n)+a)^3,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3bx^3 \log(x^n) + (b(n + 3 \log(c)) + 3a)x^3}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + 9 \int \frac{1}{2(b^3n^2 \log(x^n) + a^2b^2n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+b*log(c*x^n))^3,x, algorithm="maxima")`

[Out] `-1/2*(3*b*x^3*log(x^n) + (b*(n + 3*log(c)) + 3*a)*x^3)/(b^4*n^2*log(c)^2 + b^4*n^2*log(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) + a*b^3*n^2)*log(x^n)) + 9*integrate(1/2*x^2/(b^3*n^2*log(c) + b^3*n^2*log(x^n) + a*b^2*n^2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*log(c*x^n))^3,x)`

[Out] `int(x^2/(a + b*log(c*x^n))^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*ln(c*x**n))**3,x)`

[Out] `Integral(x**2/(a + b*log(c*x**n))**3, x)`

$$3.83 \quad \int \frac{x}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=101

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{x^2}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^2}{2bn (a+b \log(cx^n))^2}$$

[Out]  $2*x^2*Ei(2*(a+b*\ln(c*x^n))/b/n)/b^3/\exp(2*a/b/n)/n^3/((c*x^n)^(2/n))-1/2*x^2/b/n/(a+b*\ln(c*x^n))^{-2}-x^2/b^2/n^2/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2306, 2310, 2178}

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{x^2}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^2}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Log[c\*x^n])^3,x]

[Out]  $(2*x^2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)]/(b^3*E^((2*a)/(b*n))*n^3*(c*x^n)^(2/n)) - x^2/(2*b*n*(a + b*Log[c*x^n])^2) - x^2/(b^2*n^2*(a + b*Log[c*x^n]))$

**Rule 2178**

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

**Rule 2306**

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

**Rule 2310**

Int[(a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{(a + b \log(cx^n))^3} dx &= -\frac{x^2}{2bn(a + b \log(cx^n))^2} + \frac{\int \frac{x}{(a+b \log(cx^n))^2} dx}{bn} \\
 &= -\frac{x^2}{2bn(a + b \log(cx^n))^2} - \frac{x^2}{b^2 n^2 (a + b \log(cx^n))} + \frac{2 \int \frac{x}{a+b \log(cx^n)} dx}{b^2 n^2} \\
 &= -\frac{x^2}{2bn(a + b \log(cx^n))^2} - \frac{x^2}{b^2 n^2 (a + b \log(cx^n))} + \frac{(2x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int \frac{e^{\frac{2x}{a+bx}}}{a+bx} dx, x, \right)}{b^2 n^3} \\
 &= \frac{2e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \text{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{x^2}{2bn(a + b \log(cx^n))^2} - \frac{x^2}{b^2 n^2 (a + b \log(cx^n))}
 \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 89, normalized size = 0.88

$$\frac{x^2 \left( 4e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \text{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right) - \frac{bn(2a+2b \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3 n^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Log[c\*x^n])^3, x]

[Out] (x^2\*((4\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)) - (b\*n\*(2\*a + b\*n + 2\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2))/(2\*b^3\*n^3)

**fricas [B]** time = 0.46, size = 211, normalized size = 2.09

$$\frac{\left( (2b^2 n^2 x^2 \log(x) + 2b^2 n x^2 \log(c) + (b^2 n^2 + 2abn)x^2 \right) e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} - 4(b^2 n^2 \log(x)^2 + b^2 \log(c)^2 + 2ab \log(c))}{2(b^5 n^5 \log(x)^2 + b^5 n^3 \log(c)^2 + 2ab^4 n^3 \log(c) + a^2 b^3 n^3 + 2ab^2 n^2 \log(x) + 2ab^2 n^2 \log(c) + 2ab^2 n^2 \log(c) \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 
$$-1/2*((2*b^2*n^2*x^2*\log(x) + 2*b^2*n*x^2*\log(c) + (b^2*n^2 + 2*a*b*n)*x^2) * e^{(2*(b*\log(c) + a)/(b*n))} - 4*(b^2*n^2*\log(x)^2 + b^2*\log(c)^2 + 2*a*b*\log(c) + a^2 + 2*(b^2*n*\log(c) + a*b*n)*\log(x))*\log\_integral(x^2*e^{(2*(b*\log(c) + a)/(b*n))}) * e^{-2*(b*\log(c) + a)/(b*n)})/(b^5*n^5*\log(x)^2 + b^5*n^3*\log(c)^2 + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*\log(c) + a*b^4*n^4)*\log(x))$$

**giac** [B] time = 0.56, size = 1029, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 
$$-b^2*n^2*x^2*\log(x)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 2*b^2*n^2*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(x)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) - 1/2*b^2*n^2*x^2/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - b^2*n*x^2*\log(c)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 4*b^2*n*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)*\log(x)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) - a*b*n*x^2/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 2*b^2*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) + 4*a*b*n*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) + 2*a^2*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) + 4*a*b*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) + 2*a^2*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)})$$

**maple** [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \ln(c x^n) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b*ln(c*x^n)+a)^3,x)`

[Out] `int(x/(b*ln(c*x^n)+a)^3,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2bx^2 \log(x^n) + (b(n + 2 \log(c)) + 2a)x^2}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + 2 \int \frac{1}{b^3n^2 \log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*log(c*x^n))^3,x, algorithm="maxima")`

[Out] `-1/2*(2*b*x^2*log(x^n) + (b*(n + 2*log(c)) + 2*a)*x^2)/(b^4*n^2*log(c)^2 + b^4*n^2*log(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) + a*b^3*n^2)*log(x^n)) + 2*integrate(x/(b^3*n^2*log(c) + b^3*n^2*log(x^n) + a*b^2*n^2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*log(c*x^n))^3,x)`

[Out] `int(x/(a + b*log(c*x^n))^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*ln(c*x**n))**3,x)`

[Out] `Integral(x/(a + b*log(c*x**n))**3, x)`

$$3.84 \quad \int \frac{1}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=98

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3} - \frac{x}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x}{2bn (a+b \log(cx^n))^2}$$

[Out]  $1/2*x*Ei((a+b*\ln(c*x^n))/b/n)/b^3/\exp(a/b/n)/n^3/((c*x^n)^{(1/n)})^{-1/2*x/b/n/(a+b*\ln(c*x^n))^{-2-1/2*x/b^2/n^2/(a+b*\ln(c*x^n))}$

**Rubi [A]** time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2297, 2300, 2178}

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3} - \frac{x}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Log}[c*x^n])^{-3}, x]$

[Out]  $(x*\operatorname{ExpIntegralEi}[(a + b*\operatorname{Log}[c*x^n])/(b*n)])/(2*b^3*E^{(a/(b*n))*n^3*(c*x^n)^{-1}}) - x/(2*b*n*(a + b*\operatorname{Log}[c*x^n])^2) - x/(2*b^2*n^2*(a + b*\operatorname{Log}[c*x^n]))$

Rule 2178

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d)})*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \text{!}\$UseGamma == True$

Rule 2297

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(x*(a + b*\operatorname{Log}[c*x^n])^{(p+1)})/(b*n*(p+1)), x] - \operatorname{Dist}[1/(b*n*(p+1)), \operatorname{Int}[(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, n\}, x] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{IntegerQ}[2*p]$

Rule 2300

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[x/(n*(c*x^n)^{(1/n})], \operatorname{Subst}[\operatorname{Int}[E^{(x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /; \operatorname{FreeQ}[\dots]$

{a, b, c, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + b \log(cx^n))^3} dx &= -\frac{x}{2bn(a + b \log(cx^n))^2} + \frac{\int \frac{1}{(a + b \log(cx^n))^2} dx}{2bn} \\
 &= -\frac{x}{2bn(a + b \log(cx^n))^2} - \frac{x}{2b^2n^2(a + b \log(cx^n))} + \frac{\int \frac{1}{a + b \log(cx^n)} dx}{2b^2n^2} \\
 &= -\frac{x}{2bn(a + b \log(cx^n))^2} - \frac{x}{2b^2n^2(a + b \log(cx^n))} + \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, 1\right)}{2b^2n^3} \\
 &= \frac{e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{2b^3n^3} - \frac{x}{2bn(a + b \log(cx^n))^2} - \frac{x}{2b^2n^2(a + b \log(cx^n))}
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 82, normalized size = 0.84

$$\frac{x \left( e^{-\frac{a}{bn}} (cx^n)^{-1/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right) - \frac{bn(a+b \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^(-3),x]

[Out] (x\*(ExpIntegralEi[(a + b\*Log[c\*x^n])]/(b\*n)]/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)) - (b\*n\*(a + b\*n + b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2)/(2\*b^3\*n^3)

**fricas [B]** time = 0.44, size = 198, normalized size = 2.02

$$\frac{\left( (b^2n^2x \log(x) + b^2nx \log(c) + (b^2n^2 + abn)x \right) e^{\left(\frac{b \log(c)+a}{bn}\right)} - (b^2n^2 \log(x)^2 + b^2 \log(c)^2 + 2ab \log(c) + a^2 + 2(b^2n^2x \log(x) + b^2nx \log(c) + (b^2n^2 + abn)x))}{2(b^5n^5 \log(x)^2 + b^5n^3 \log(c)^2 + 2ab^4n^3 \log(c) + a^2b^3n^3 + 2(b^5n^4 \log(x) + b^5n^2 \log(c) + ab^4n^2 \log(c) + a^2b^2n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")



[Out]  $-1/2*((b^2*n^2*x*\log(x) + b^2*n*x*\log(c) + (b^2*n^2 + a*b*n)*x)*e^{((b*\log(c) + a)/(b*n))} - (b^2*n^2*\log(x)^2 + b^2*\log(c)^2 + 2*a*b*\log(c) + a^2 + 2*(b^2*n*\log(c) + a*b*n)*\log(x))*\log\_integral(x*e^{((b*\log(c) + a)/(b*n))}))*e^{(-(b*\log(c) + a)/(b*n))}/(b^5*n^5*\log(x)^2 + b^5*n^3*\log(c)^2 + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*\log(c) + a*b^4*n^4)*\log(x))$

**giac [B]** time = 0.36, size = 982, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*log(c*x^n))^3,x, algorithm="giac")`

[Out]  $1/2*b^2*n^2*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(x)^2/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)}) - 1/2*b^2*n^2*x*\log(x)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + b^2*n*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(c)*\log(x)/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)}) - 1/2*b^2*n^2*x/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - 1/2*b^2*n*x*\log(c)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 1/2*b^2*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(c)^2/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)}) + a*b*n*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(x)/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)}) - 1/2*a*b*n*x/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + a*b*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(c)/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)}) + 1/2*a^2*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)})$

**maple [C]** time = 0.50, size = 459, normalized size = 4.68

$$x c^{-\frac{1}{n}} (x^n)^{-\frac{1}{n}} Ei\left(1, -\ln(x) - \frac{-i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2 - i\pi b \operatorname{csgn}(ic x^n)^3 + 2b}{2bn}\right)$$

$2b^3n^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*ln(c\*x^n)+a)^3,x)

[Out] 
$$-(2*b*n*x+I*\pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)^2-I*\pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*\pi*b*x*csgn(I*c*x^n)^3+I*\pi*b*x*csgn(I*c*x^n)^2*csgn(I*c)+2*\ln(c)*b*x+2*b*x*\ln(x^n)+2*a*x)/(-I*\pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*\pi*b*csgn(I*c)*csgn(I*c*x^n)^2+I*\pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*\pi*b*csgn(I*c*x^n)^3+2*b*\ln(c)+2*b*\ln(x^n)+2*a)^2/b^2/n^2-1/2/b^3/n^3*x*c^{(-1/n)}*(x^n)^{(-1/n)}*\exp(-1/2*(-I*\pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*\pi*b*csgn(I*c)*csgn(I*c*x^n)^2+I*\pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*\pi*b*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-\ln(x)-1/2*(-I*\pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*\pi*b*csgn(I*c)*csgn(I*c*x^n)^2+I*\pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2-I*\pi*b*csgn(I*c*x^n)^3+2*b*\ln(c)+2*a+2*(-n*\ln(x)+\ln(x^n))*b)/b/n)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx \log(x^n) + (b(n + \log(c)) + a)x}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + \int \frac{1}{2(b^3n^2 \log(c) + b^3n^2 \log(x^n) + a*b^2n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 
$$-1/2*(b*x*\log(x^n) + (b*(n + \log(c)) + a)*x)/(b^4*n^2*\log(c)^2 + b^4*n^2*\log(x^n)^2 + 2*a*b^3*n^2*\log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*\log(c) + a*b^3*n^2)*\log(x^n)) + \text{integrate}(1/2/(b^3*n^2*\log(c) + b^3*n^2*\log(x^n) + a*b^2*n^2), x)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*log(c\*x^n))^3,x)

[Out] int(1/(a + b\*log(c\*x^n))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*(-3), x)

$$3.85 \quad \int \frac{1}{x(a+b \log(cx^n))^3} dx$$

Optimal. Leaf size=22

$$-\frac{1}{2bn(a+b \log(cx^n))^2}$$

[Out] -1/2/b/n/(a+b\*ln(c\*x^n))^2

**Rubi [A]** time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$-\frac{1}{2bn(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*Log[c\*x^n])^3),x]

[Out] -1/(2\*b\*n\*(a + b\*Log[c\*x^n])^2)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+b \log(cx^n))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, a+b \log(cx^n)\right)}{bn} \\ &= -\frac{1}{2bn(a+b \log(cx^n))^2} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{2bn \left(a + b \log(cx^n)\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*Log[c\*x^n])^3), x]

[Out] -1/2\*1/(b\*n\*(a + b\*Log[c\*x^n])^2)

**fricas [B]** time = 0.43, size = 62, normalized size = 2.82

$$-\frac{1}{2 \left(b^3 n^3 \log(x)^2 + b^3 n \log(c)^2 + 2 ab^2 n \log(c) + a^2 bn + 2 \left(b^3 n^2 \log(c) + ab^2 n^2\right) \log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] -1/2/(b^3\*n^3\*log(x)^2 + b^3\*n\*log(c)^2 + 2\*a\*b^2\*n\*log(c) + a^2\*b\*n + 2\*(b^3\*n^2\*log(c) + a\*b^2\*n^2)\*log(x))

**giac [A]** time = 0.33, size = 21, normalized size = 0.95

$$-\frac{1}{2 \left(bn \log(x) + b \log(c) + a\right)^2 bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] -1/2/((b\*n\*log(x) + b\*log(c) + a)^2\*b\*n)

**maple [A]** time = 0.02, size = 21, normalized size = 0.95

$$-\frac{1}{2 \left(b \ln(cx^n) + a\right)^2 bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*ln(c\*x^n)+a)^3,x)

[Out] -1/2/b/n/(b\*ln(c\*x^n)+a)^2

**maxima [A]** time = 0.58, size = 20, normalized size = 0.91

$$\frac{1}{2(b \log(cx^n) + a)^2 bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2/((b\*log(c\*x^n) + a)^2\*b\*n)

**mupad [B]** time = 3.53, size = 39, normalized size = 1.77

$$\frac{1}{2na^2b + 4nab^2 \ln(cx^n) + 2nb^3 \ln(cx^n)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*log(c\*x^n))^3),x)

[Out] -1/(2\*b^3\*n\*log(c\*x^n)^2 + 2\*a^2\*b\*n + 4\*a\*b^2\*n\*log(c\*x^n))

**sympy [A]** time = 93.79, size = 121, normalized size = 5.50

$$\left\{ \begin{array}{ll} \frac{\infty \log(x)}{\log(c)^3} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \infty n \log(x) & \text{for } a = -b(n \log(x) + \log(c)) \\ \frac{\log(x)}{a^3} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b \log(c))^3} & \text{for } n = 0 \\ \frac{1}{2a^2bn+4ab^2n^2 \log(x)+4ab^2n \log(c)+2b^3n^3 \log(x)^2+4b^3n^2 \log(c) \log(x)+2b^3n \log(c)^2} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Piecewise((zoo\*log(x)/log(c)\*\*3, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (zoo\*n\*log(x), Eq(a, -b\*(n\*log(x) + log(c)))), (log(x)/a\*\*3, Eq(b, 0)), (log(x)/(a + b\*log(c))\*\*3, Eq(n, 0)), (-1/(2\*a\*\*2\*b\*n + 4\*a\*b\*\*2\*n\*\*2\*log(x) + 4\*a\*b\*\*2\*n\*log(c) + 2\*b\*\*3\*n\*\*3\*log(x)\*\*2 + 4\*b\*\*3\*n\*\*2\*log(c)\*log(x) + 2\*b\*\*3\*n\*log(c)\*\*2), True))

$$3.86 \quad \int \frac{1}{x^2(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=102

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3 x} + \frac{1}{2b^2 n^2 x (a+b \log(cx^n))} - \frac{1}{2bnx (a+b \log(cx^n))^2}$$

[Out]  $1/2*\exp(a/b/n)*(c*x^n)^{(1/n)}*Ei((-a-b*\ln(c*x^n))/b/n)/b^3/n^3/x-1/2/b/n/x/(a+b*\ln(c*x^n))^2+1/2/b^2/n^2/x/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3 x} + \frac{1}{2b^2 n^2 x (a+b \log(cx^n))} - \frac{1}{2bnx (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a + b*Log[c*x^n])^3), x]`

[Out]  $(E^{(a/(b*n))}*(c*x^n)^n*(-1)*\operatorname{ExpIntegralEi}[-((a + b*\operatorname{Log}[c*x^n])/(b*n))])/(2*b^3*n^3*x) - 1/(2*b*n*x*(a + b*\operatorname{Log}[c*x^n])^2) + 1/(2*b^2*n^2*x*(a + b*\operatorname{Log}[c*x^n]))$

#### Rule 2178

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True`

#### Rule 2306

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[(((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

#### Rule 2310

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)`

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 (a + b \log(cx^n))^3} dx &= -\frac{1}{2bnx (a + b \log(cx^n))^2} - \frac{\int \frac{1}{x^2 (a + b \log(cx^n))^2} dx}{2bn} \\
 &= -\frac{1}{2bnx (a + b \log(cx^n))^2} + \frac{1}{2b^2 n^2 x (a + b \log(cx^n))} + \frac{\int \frac{1}{x^2 (a + b \log(cx^n))} dx}{2b^2 n^2} \\
 &= -\frac{1}{2bnx (a + b \log(cx^n))^2} + \frac{1}{2b^2 n^2 x (a + b \log(cx^n))} + \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int \frac{e^{-\frac{x}{a+bx}}}{a+bx} dx, x, (cx^n)^{\frac{1}{n}}\right)}{2b^2 n^3 x} \\
 &= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \text{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3 x} - \frac{1}{2bnx (a + b \log(cx^n))^2} + \frac{1}{2b^2 n^2 x (a + b \log(cx^n))}
 \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 94, normalized size = 0.92

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^2 \text{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right) + bn (a + b \log(cx^n) - bn)}{2b^3 n^3 x (a + b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*Log[c\*x^n])^3), x]

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))]\*(a + b\*Log[c\*x^n])^2 + b\*n\*(a - b\*n + b\*Log[c\*x^n]))/(2\*b^3\*n^3\*x\*(a + b\*Log[c\*x^n])^2)

**fricas [B]** time = 0.44, size = 192, normalized size = 1.88

$$\frac{b^2 n^2 \log(x) - b^2 n^2 + b^2 n \log(c) + abn + (b^2 n^2 x \log(x)^2 + b^2 x \log(c)^2 + 2 abx \log(c) + a^2 x + 2 (b^2 n x \log(c) + abx))}{2 (b^5 n^5 x \log(x)^2 + b^5 n^3 x \log(c)^2 + 2 ab^4 n^3 x \log(c) + a^2 b^3 n^3 x + 2 (b^5 n^4 x \log(c) + ab^2 n^2 x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(b^2\*n^2\*log(x) - b^2\*n^2 + b^2\*n\*log(c) + a\*b\*n + (b^2\*n^2\*x\*log(x))^2 + b^2\*x\*log(c)^2 + 2\*a\*b\*x\*log(c) + a^2\*x + 2\*(b^2\*n\*x\*log(c) + a\*b\*n\*x)\*log(x))\*e^((b\*log(c) + a)/(b\*n))\*log\_integral(e^(-(b\*log(c) + a)/(b\*n))/x))/(b^5\*n^5\*x\*log(x)^2 + b^5\*n^3\*x\*log(c)^2 + 2\*a\*b^4\*n^3\*x\*log(c) + a^2\*b^3\*n^3\*x + 2\*(b^5\*n^4\*x\*log(c) + a\*b^4\*n^4\*x)\*log(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^3\*x^2), x)

**maple** [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*ln(c\*x^n)+a)^3,x)

[Out] int(1/x^2/(b\*ln(c\*x^n)+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b(n - \log(c)) - b \log(x^n) - a}{2(b^4 n^2 x \log(x^n))^2 + 2(b^4 n^2 \log(c) + ab^3 n^2)x \log(x^n) + (b^4 n^2 \log(c)^2 + 2ab^3 n^2 \log(c) + a^2 b^2 n^2)x} + \int \frac{1}{2(b^3 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(b\*(n - log(c)) - b\*log(x^n) - a)/(b^4\*n^2\*x\*log(x^n)^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*x\*log(x^n) + (b^4\*n^2\*log(c)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2)\*x) + integrate(1/2/(b^3\*n^2\*x^2\*log(x^n) + (b^3\*n^2\*log(c) + a\*b^2\*n^2)\*x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 (a + b \ln(cx^n))^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*log(c*x^n))^3), x)`

[Out] `int(1/(x^2*(a + b*log(c*x^n))^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(a+b*ln(c*x**n))**3, x)`

[Out] `Integral(1/(x**2*(a + b*log(c*x**n))**3), x)`

$$3.87 \quad \int \frac{1}{x^3(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=100

$$\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3 x^2} + \frac{1}{b^2 n^2 x^2 (a+b \log(cx^n))} - \frac{1}{2bnx^2 (a+b \log(cx^n))^2}$$

[Out]  $2*\exp(2*a/b/n)*(c*x^n)^{(2/n)}*Ei(-2*(a+b*\ln(c*x^n))/b/n)/b^3/n^3/x^2-1/2/b/n/x^2/(a+b*\ln(c*x^n))^{2+1/b^2/n^2/x^2/(a+b*\ln(c*x^n))}$

**Rubi [A]** time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3 x^2} + \frac{1}{b^2 n^2 x^2 (a+b \log(cx^n))} - \frac{1}{2bnx^2 (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*Log[c\*x^n])^3), x]

[Out]  $(2*E^{((2*a)/(b*n))}*(c*x^n)^{(2/n)}*\operatorname{ExpIntegralEi}[(-2*(a + b*\operatorname{Log}[c*x^n]))/(b*n)])/((b^3*n^3*x^2) - 1/(2*b*n*x^2*(a + b*\operatorname{Log}[c*x^n])^2) + 1/(b^2*n^2*x^2*(a + b*\operatorname{Log}[c*x^n])))$

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[(((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a + b \log(cx^n))^3} dx &= -\frac{1}{2bnx^2 (a + b \log(cx^n))^2} - \frac{\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx}{bn} \\
 &= -\frac{1}{2bnx^2 (a + b \log(cx^n))^2} + \frac{1}{b^2 n^2 x^2 (a + b \log(cx^n))} + \frac{2 \int \frac{1}{x^3 (a + b \log(cx^n))} dx}{b^2 n^2} \\
 &= -\frac{1}{2bnx^2 (a + b \log(cx^n))^2} + \frac{1}{b^2 n^2 x^2 (a + b \log(cx^n))} + \frac{(2 (cx^n)^{2/n}) \text{Subst} \left( \int \frac{e^{-\frac{2x}{n}}}{a+bx} \right)}{b^2 n^3 x^2} \\
 &= \frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right)}{b^3 n^3 x^2} - \frac{1}{2bnx^2 (a + b \log(cx^n))^2} + \frac{1}{b^2 n^2 x^2 (a + b \log(cx^n))}
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 89, normalized size = 0.89

$$\frac{4e^{\frac{2a}{bn}} (cx^n)^{2/n} \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right) + \frac{bn(2a+2b \log(cx^n)-bn)}{(a+b \log(cx^n))^2}}{2b^3 n^3 x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*Log[c\*x^n])^3), x]

[Out] (4\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))]/(b\*n)) + (b\*n\*(2\*a - b\*n + 2\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2)/(2\*b^3\*n^3\*x^2)

**fricas [B]** time = 0.46, size = 221, normalized size = 2.21

$$2b^2n^2 \log(x) - b^2n^2 + 2b^2n \log(c) + 2abn + 4(b^2n^2x^2 \log(x)^2 + b^2x^2 \log(c)^2 + 2abx^2 \log(c) + a^2x^2 + 2(b^2nx^2$$

---


$$2(b^5n^5x^2 \log(x)^2 + b^5n^3x^2 \log(c)^2 + 2ab^4n^3x^2 \log(c) + a^2b^3n^3x^2 + 2(b^5n^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(2\*b^2\*n^2\*log(x) - b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*a\*b\*n + 4\*(b^2\*n^2\*x^2\*log(x)^2 + b^2\*x^2\*log(c)^2 + 2\*a\*b\*x^2\*log(c) + a^2\*x^2 + 2\*(b^2\*n\*x^2\*log(c) + a\*b\*n\*x^2)\*log(x))\*e^(2\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-2\*(b\*log(c) + a)/(b\*n))/x^2))/(b^5\*n^5\*x^2\*log(x)^2 + b^5\*n^3\*x^2\*log(c)^2 + 2\*a\*b^4\*n^3\*x^2\*log(c) + a^2\*b^3\*n^3\*x^2 + 2\*(b^5\*n^4\*x^2\*log(c) + a\*b^4\*n^4\*x^2)\*log(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^3\*x^3), x)

**maple** [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b\*ln(c\*x^n)+a)^3,x)

[Out] int(1/x^3/(b\*ln(c\*x^n)+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b(n - 2 \log(c)) - 2b \log(x^n) - 2a}{2(b^4 n^2 x^2 \log(x^n)^2 + 2(b^4 n^2 \log(c) + ab^3 n^2)x^2 \log(x^n) + (b^4 n^2 \log(c)^2 + 2ab^3 n^2 \log(c) + a^2 b^2 n^2)x^2)} + 2 \int \frac{1}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(b\*(n - 2\*log(c)) - 2\*b\*log(x^n) - 2\*a)/(b^4\*n^2\*x^2\*log(x^n)^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*x^2\*log(x^n) + (b^4\*n^2\*log(c)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2)\*x^2) + 2\*integrate(1/(b^3\*n^2\*x^3\*log(x^n) + (b^3\*n^2\*log(c) + a\*b^2\*n^2)\*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 (a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a + b*log(c*x^n))^3), x)`

[Out] `int(1/(x^3*(a + b*log(c*x^n))^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a+b*ln(c*x**n))**3, x)`

[Out] `Integral(1/(x**3*(a + b*log(c*x**n))**3), x)`

$$3.88 \quad \int \frac{1}{x^4(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=105

$$\frac{9e^{\frac{3a}{bn}}(cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3n^3x^3} + \frac{3}{2b^2n^2x^3(a+b \log(cx^n))} - \frac{1}{2bnx^3(a+b \log(cx^n))^2}$$

[Out]  $9/2*\exp(3*a/b/n)*(c*x^n)^{(3/n)}*Ei(-3*(a+b*\ln(c*x^n))/b/n)/b^3/n^3/x^3-1/2/b/n/x^3/(a+b*\ln(c*x^n))^2+3/2/b^2/n^2/x^3/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{9e^{\frac{3a}{bn}}(cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3n^3x^3} + \frac{3}{2b^2n^2x^3(a+b \log(cx^n))} - \frac{1}{2bnx^3(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*Log[c\*x^n])^3), x]

[Out]  $(9*E^{((3*a)/(b*n))}*(c*x^n)^{(3/n)}*\operatorname{ExpIntegralEi}[(-3*(a + b*\operatorname{Log}[c*x^n]))/(b*n)])/((2*b^3*n^3*x^3) - 1/(2*b*n*x^3*(a + b*\operatorname{Log}[c*x^n])^2) + 3/(2*b^2*n^2*x^3*(a + b*\operatorname{Log}[c*x^n])))$

**Rule 2178**

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

**Rule 2306**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

**Rule 2310**

Int[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 (a + b \log(cx^n))^3} dx &= -\frac{1}{2bnx^3 (a + b \log(cx^n))^2} - \frac{3 \int \frac{1}{x^4 (a + b \log(cx^n))^2} dx}{2bn} \\
 &= -\frac{1}{2bnx^3 (a + b \log(cx^n))^2} + \frac{3}{2b^2 n^2 x^3 (a + b \log(cx^n))} + \frac{9 \int \frac{1}{x^4 (a + b \log(cx^n))} dx}{2b^2 n^2} \\
 &= -\frac{1}{2bnx^3 (a + b \log(cx^n))^2} + \frac{3}{2b^2 n^2 x^3 (a + b \log(cx^n))} + \frac{(9 (cx^n)^{3/n}) \text{Subst}\left(\int \frac{e^{-\frac{3x}{a+b \log(cx^n)}}}{a+b \log(cx^n)} dx\right)}{2b^2 n^3 x^3} \\
 &= \frac{9e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3 x^3} - \frac{1}{2bnx^3 (a + b \log(cx^n))^2} + \frac{3}{2b^2 n^2 x^3 (a + b \log(cx^n))}
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 89, normalized size = 0.85

$$\frac{9e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right) + \frac{bn(3a+3b \log(cx^n)-bn)}{(a+b \log(cx^n))^2}}{2b^3 n^3 x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*Log[c\*x^n])^3), x]

[Out] (9\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))]/(b\*n)) + (b\*n\*(3\*a - b\*n + 3\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2)/(2\*b^3\*n^3\*x^3)

**fricas [B]** time = 0.47, size = 221, normalized size = 2.10

$3b^2n^2 \log(x) - b^2n^2 + 3b^2n \log(c) + 3abn + 9(b^2n^2x^3 \log(x)^2 + b^2x^3 \log(c)^2 + 2abx^3 \log(c) + a^2x^3 + 2(b^2nx^3$

---

$2(b^5n^5x^3 \log(x)^2 + b^5n^3x^3 \log(c)^2 + 2ab^4n^3x^3 \log(c) + a^2b^3n^3x^3 + 2(b^5n^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(3\*b^2\*n^2\*log(x) - b^2\*n^2 + 3\*b^2\*n\*log(c) + 3\*a\*b\*n + 9\*(b^2\*n^2\*x^3\*log(x)^2 + b^2\*x^3\*log(c)^2 + 2\*a\*b\*x^3\*log(c) + a^2\*x^3 + 2\*(b^2\*n\*x^3\*log(c) + a\*b\*n\*x^3)\*log(x))\*e^(3\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-3\*(b\*log(c) + a)/(b\*n))/x^3))/(b^5\*n^5\*x^3\*log(x)^2 + b^5\*n^3\*x^3\*log(c)^2 + 2\*a\*b^4\*n^3\*x^3\*log(c) + a^2\*b^3\*n^3\*x^3 + 2\*(b^5\*n^4\*x^3\*log(c) + a\*b^4\*n^4\*x^3)\*log(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \log(cx^n) + a)^3 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^3\*x^4), x)

**maple** [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \ln(cx^n) + a)^3 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*ln(c\*x^n)+a)^3,x)

[Out] int(1/x^4/(b\*ln(c\*x^n)+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b(n - 3 \log(c)) - 3b \log(x^n) - 3a}{2(b^4 n^2 x^3 \log(x^n)^2 + 2(b^4 n^2 \log(c) + ab^3 n^2)x^3 \log(x^n) + (b^4 n^2 \log(c)^2 + 2ab^3 n^2 \log(c) + a^2 b^2 n^2)x^3)} + 9 \int \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(b\*(n - 3\*log(c)) - 3\*b\*log(x^n) - 3\*a)/(b^4\*n^2\*x^3\*log(x^n)^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*x^3\*log(x^n) + (b^4\*n^2\*log(c)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2)\*x^3) + 9\*integrate(1/2/(b^3\*n^2\*x^4\*log(x^n) + (b^3\*n^2\*log(c) + a\*b^2\*n^2)\*x^4), x)



mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 (a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4\*(a + b\*log(c\*x^n))^3), x)

[Out] int(1/(x^4\*(a + b\*log(c\*x^n))^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*ln(c\*x\*\*n))\*\*3, x)

[Out] Integral(1/(x\*\*4\*(a + b\*log(c\*x\*\*n))\*\*3), x)

$$3.89 \quad \int (dx)^{5/2} \left( a + b \log(cx^n) \right) dx$$

Optimal. Leaf size=41

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))}{7d} - \frac{4bn(dx)^{7/2}}{49d}$$

[Out]  $-4/49*b*n*(d*x)^{(7/2)}/d+2/7*(d*x)^{(7/2)}*(a+b*\ln(c*x^n))/d$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))}{7d} - \frac{4bn(dx)^{7/2}}{49d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n]), x]

[Out]  $(-4*b*n*(d*x)^{(7/2)})/(49*d) + (2*(d*x)^{(7/2)}*(a + b*Log[c*x^n]))/(7*d)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int (dx)^{5/2} (a + b \log(cx^n)) dx = -\frac{4bn(dx)^{7/2}}{49d} + \frac{2(dx)^{7/2} (a + b \log(cx^n))}{7d}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.71

$$\frac{2}{49}x(dx)^{5/2} (7a + 7b \log(cx^n) - 2bn)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n]), x]

[Out]  $(2*x*(d*x)^{(5/2)}*(7*a - 2*b*n + 7*b*Log[c*x^n]))/49$

**fricas** [A] time = 0.48, size = 50, normalized size = 1.22

$$\frac{2}{49} (7bd^2nx^3 \log(x) + 7bd^2x^3 \log(c) - (2bd^2n - 7ad^2)x^3) \sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] 2/49\*(7\*b\*d^2\*n\*x^3\*log(x) + 7\*b\*d^2\*x^3\*log(c) - (2\*b\*d^2\*n - 7\*a\*d^2)\*x^3)\*sqrt(d\*x)

**giac** [C] time = 0.52, size = 117, normalized size = 2.85

$$\left(\frac{1}{7}i + \frac{1}{7}\right) \sqrt{2}bd^2nx^{\frac{7}{2}}\sqrt{|d|} \cos\left(\frac{1}{4}\pi\text{sgn}(d)\right) \log(x) - \left(\frac{1}{7}i - \frac{1}{7}\right) \sqrt{2}bd^2nx^{\frac{7}{2}}\sqrt{|d|} \log(x) \sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{2}{49}i + \frac{2}{49}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] (1/7\*I + 1/7)\*sqrt(2)\*b\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) - (1/7\*I - 1/7)\*sqrt(2)\*b\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) - (2/49\*I + 2/49)\*sqrt(2)\*b\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d)) + (2/49\*I - 2/49)\*sqrt(2)\*b\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*sin(1/4\*pi\*sgn(d)) + 2/7\*b\*d^(5/2)\*x^(7/2)\*log(c) + 2/7\*a\*d^(5/2)\*x^(7/2)

**maple** [C] time = 0.15, size = 128, normalized size = 3.12

$$\frac{2bd^3x^4 \ln(x^n)}{7\sqrt{dx}} + \frac{(-7i\pi b \text{csgn}(ic) \text{csgn}(ix^n) \text{csgn}(icx^n) + 7i\pi b \text{csgn}(ic) \text{csgn}(icx^n)^2 + 7i\pi b \text{csgn}(ix^n) \text{csgn}(icx^n))}{49\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)\*(b\*ln(c\*x^n)+a),x)

[Out] 2/7\*d^3\*b\*x^4/(d\*x)^(1/2)\*ln(x^n)+1/49\*d^3\*(7\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-7\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-7\*I\*b\*Pi\*csgn(I\*c\*x^n)^3+7\*I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+14\*b\*ln(c)-4\*b\*n+14\*a)\*x^4/(d\*x)^(1/2)

**maxima** [A] time = 0.53, size = 41, normalized size = 1.00

$$-\frac{4(dx)^{\frac{7}{2}}bn}{49d} + \frac{2(dx)^{\frac{7}{2}}b \log(cx^n)}{7d} + \frac{2(dx)^{\frac{7}{2}}a}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out]  $-4/49*(d*x)^{(7/2)}*b*n/d + 2/7*(d*x)^{(7/2)}*b*\log(c*x^n)/d + 2/7*(d*x)^{(7/2)}*a/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (dx)^{5/2} (a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)\*(a + b\*log(c\*x^n)),x)

[Out] int((d\*x)^(5/2)\*(a + b\*log(c\*x^n)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)\*(a+b\*ln(c\*x\*\*n)),x)

[Out] Timed out

### 3.90 $\int (dx)^{3/2} (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=41

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))}{5d} - \frac{4bn(dx)^{5/2}}{25d}$$

[Out]  $-4/25*b*n*(d*x)^{(5/2)}/d+2/5*(d*x)^{(5/2)}*(a+b*\ln(c*x^n))/d$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))}{5d} - \frac{4bn(dx)^{5/2}}{25d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]), x]

[Out]  $(-4*b*n*(d*x)^{(5/2)})/(25*d) + (2*(d*x)^{(5/2)}*(a + b*Log[c*x^n]))/(5*d)$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int (dx)^{3/2} (a + b \log(cx^n)) dx = -\frac{4bn(dx)^{5/2}}{25d} + \frac{2(dx)^{5/2} (a + b \log(cx^n))}{5d}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.71

$$\frac{2}{25}x(dx)^{3/2} (5a + 5b \log(cx^n) - 2bn)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]), x]

[Out]  $(2*x*(d*x)^{(3/2)}*(5*a - 2*b*n + 5*b*Log[c*x^n]))/25$

**fricas** [A] time = 0.45, size = 42, normalized size = 1.02

$$\frac{2}{25} \left( 5 b d n x^2 \log(x) + 5 b d x^2 \log(c) - (2 b d n - 5 a d) x^2 \right) \sqrt{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] 2/25\*(5\*b\*d\*n\*x^2\*log(x) + 5\*b\*d\*x^2\*log(c) - (2\*b\*d\*n - 5\*a\*d)\*x^2)\*sqrt(d\*x)

**giac** [C] time = 0.53, size = 108, normalized size = 2.63

$$-\frac{1}{25} \left( -(5i + 5) \sqrt{2} b n x^{\frac{5}{2}} \sqrt{|d|} \cos\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) \log(x) + (5i - 5) \sqrt{2} b n x^{\frac{5}{2}} \sqrt{|d|} \log(x) \sin\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) + (2i + 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] -1/25\*(-(5\*I + 5)\*sqrt(2)\*b\*n\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) + (5\*I - 5)\*sqrt(2)\*b\*n\*x^(5/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) + (2\*I + 2)\*sqrt(2)\*b\*n\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d)) - (2\*I - 2)\*sqrt(2)\*b\*n\*x^(5/2)\*sqrt(abs(d))\*sin(1/4\*pi\*sgn(d)) - 10\*b\*sqrt(d)\*x^(5/2)\*log(c) - 10\*a\*sqrt(d)\*x^(5/2))\*d

**maple** [C] time = 0.12, size = 128, normalized size = 3.12

$$\frac{2b d^2 x^3 \ln(x^n)}{5\sqrt{dx}} + \frac{(-5i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 5i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 5i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)}{25\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*(b\*ln(c\*x^n)+a),x)

[Out] 2/5\*d^2\*b\*x^3/(d\*x)^(1/2)\*ln(x^n)+1/25\*d^2\*(5\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-5\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-5\*I\*b\*Pi\*csgn(I\*c\*x^n)^3+5\*I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+10\*b\*ln(c)-4\*b\*n+10\*a)\*x^3/(d\*x)^(1/2)

**maxima** [A] time = 0.56, size = 41, normalized size = 1.00

$$-\frac{4 (dx)^{\frac{5}{2}} b n}{25 d} + \frac{2 (dx)^{\frac{5}{2}} b \log(cx^n)}{5 d} + \frac{2 (dx)^{\frac{5}{2}} a}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out]  $-4/25*(d*x)^{(5/2)}*b*n/d + 2/5*(d*x)^{(5/2)}*b*\log(c*x^n)/d + 2/5*(d*x)^{(5/2)}*a/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (dx)^{3/2} (a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*(a + b\*log(c\*x^n)),x)

[Out] int((d\*x)^(3/2)\*(a + b\*log(c\*x^n)), x)

**sympy** [A] time = 25.45, size = 70, normalized size = 1.71

$$\frac{2ad^{\frac{3}{2}}x^{\frac{5}{2}}}{5} + \frac{2bd^{\frac{3}{2}}nx^{\frac{5}{2}}\log(x)}{5} - \frac{4bd^{\frac{3}{2}}nx^{\frac{5}{2}}}{25} + \frac{2bd^{\frac{3}{2}}x^{\frac{5}{2}}\log(c)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*(a+b\*ln(c\*x\*\*n)),x)

[Out]  $2*a*d^{3/2}*x^{5/2}/5 + 2*b*d^{3/2}*n*x^{5/2}*\log(x)/5 - 4*b*d^{3/2}*n*x^{5/2}/25 + 2*b*d^{3/2}*x^{5/2}*\log(c)/5$

### 3.91 $\int \sqrt{dx} (a + b \log(cx^n)) dx$

Optimal. Leaf size=41

$$\frac{2(dx)^{3/2} (a + b \log(cx^n))}{3d} - \frac{4bn(dx)^{3/2}}{9d}$$

[Out]  $-4/9*b*n*(d*x)^{(3/2)}/d+2/3*(d*x)^{(3/2)}*(a+b*\ln(c*x^n))/d$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2(dx)^{3/2} (a + b \log(cx^n))}{3d} - \frac{4bn(dx)^{3/2}}{9d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*(a + b\*Log[c\*x^n]),x]

[Out]  $(-4*b*n*(d*x)^{(3/2)})/(9*d) + (2*(d*x)^{(3/2)}*(a + b*Log[c*x^n]))/(3*d)$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \sqrt{dx} (a + b \log(cx^n)) dx = -\frac{4bn(dx)^{3/2}}{9d} + \frac{2(dx)^{3/2} (a + b \log(cx^n))}{3d}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.71

$$\frac{2}{9}x\sqrt{dx} (3a + 3b \log(cx^n) - 2bn)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*Log[c\*x^n]),x]

[Out]  $(2*x*Sqrt[d*x]*(3*a - 2*b*n + 3*b*Log[c*x^n]))/9$



**fricas** [A] time = 0.47, size = 32, normalized size = 0.78

$$\frac{2}{9} (3 b n x \log(x) + 3 b x \log(c) - (2 b n - 3 a) x) \sqrt{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] 2/9\*(3\*b\*n\*x\*log(x) + 3\*b\*x\*log(c) - (2\*b\*n - 3\*a)\*x)\*sqrt(d\*x)

**giac** [C] time = 0.54, size = 105, normalized size = 2.56

$$\left(\frac{1}{3}i + \frac{1}{3}\right) \sqrt{2} b n x^{\frac{3}{2}} \sqrt{|d|} \cos\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) \log(x) - \left(\frac{1}{3}i - \frac{1}{3}\right) \sqrt{2} b n x^{\frac{3}{2}} \sqrt{|d|} \log(x) \sin\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) - \left(\frac{2}{9}i + \frac{2}{9}\right) \sqrt{2} b x^{\frac{3}{2}} \sqrt{|d|} \log(x) \sin\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) - \left(\frac{2}{9}i - \frac{2}{9}\right) \sqrt{2} b x^{\frac{3}{2}} \sqrt{|d|} \log(x) \cos\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) + \frac{2}{3} a \sqrt{d} x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] (1/3\*I + 1/3)\*sqrt(2)\*b\*n\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) - (1/3\*I - 1/3)\*sqrt(2)\*b\*n\*x^(3/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) - (2/9\*I + 2/9)\*sqrt(2)\*b\*n\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d)) + (2/9\*I - 2/9)\*sqrt(2)\*b\*n\*x^(3/2)\*sqrt(abs(d))\*sin(1/4\*pi\*sgn(d)) + 2/3\*b\*sqrt(d)\*x^(3/2)\*log(c) + 2/3\*a\*sqrt(d)\*x^(3/2)

**maple** [C] time = 0.12, size = 124, normalized size = 3.02

$$\frac{2 b d x^2 \ln(x^n)}{3 \sqrt{d x}} + \frac{(-3 i \pi b \operatorname{csgn}(i c) \operatorname{csgn}(i x^n) \operatorname{csgn}(i c x^n) + 3 i \pi b \operatorname{csgn}(i c) \operatorname{csgn}(i c x^n)^2 + 3 i \pi b \operatorname{csgn}(i x^n) \operatorname{csgn}(i c x^n) - 3 i \pi b \operatorname{csgn}(i c) \operatorname{csgn}(i c x^n)^2 - 3 i \pi b \operatorname{csgn}(i x^n) \operatorname{csgn}(i c x^n)^2) \sqrt{d x}}{9 \sqrt{d x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(b\*ln(c\*x^n)+a),x)

[Out] 2/3\*d\*b\*x^2/(d\*x)^(1/2)\*ln(x^n)+1/9\*d\*(3\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2 - 3\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c) - 3\*I\*b\*Pi\*csgn(I\*c\*x^n)^3 + 3\*I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c) + 6\*b\*ln(c) - 4\*b\*n + 6\*a)\*x^2/(d\*x)^(1/2)

**maxima** [A] time = 0.53, size = 41, normalized size = 1.00

$$-\frac{4 (d x)^{\frac{3}{2}} b n}{9 d} + \frac{2 (d x)^{\frac{3}{2}} b \log(c x^n)}{3 d} + \frac{2 (d x)^{\frac{3}{2}} a}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out]  $-4/9*(d*x)^{(3/2)}*b*n/d + 2/3*(d*x)^{(3/2)}*b*\log(c*x^n)/d + 2/3*(d*x)^{(3/2)}*a/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{d} x (a + b \ln(c x^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(1/2)*(a + b*log(c*x^n)),x)`

[Out] `int((d*x)^(1/2)*(a + b*log(c*x^n)), x)`

sympy [A] time = 1.06, size = 70, normalized size = 1.71

$$\frac{2a\sqrt{d}x^{\frac{3}{2}}}{3} + \frac{2b\sqrt{d}nx^{\frac{3}{2}}\log(x)}{3} - \frac{4b\sqrt{d}nx^{\frac{3}{2}}}{9} + \frac{2b\sqrt{d}x^{\frac{3}{2}}\log(c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(1/2)*(a+b*ln(c*x**n)),x)`

[Out] `2*a*sqrt(d)*x**(3/2)/3 + 2*b*sqrt(d)*n*x**(3/2)*log(x)/3 - 4*b*sqrt(d)*n*x**  
*(3/2)/9 + 2*b*sqrt(d)*x**(3/2)*log(c)/3`

$$3.92 \quad \int \frac{a+b \log(cx^n)}{\sqrt{dx}} dx$$

Optimal. Leaf size=37

$$\frac{2\sqrt{dx} (a + b \log(cx^n))}{d} - \frac{4bn\sqrt{dx}}{d}$$

[Out]  $-4*b*n*(d*x)^{(1/2)}/d+2*(a+b*\ln(c*x^n))*(d*x)^{(1/2)}/d$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2\sqrt{dx} (a + b \log(cx^n))}{d} - \frac{4bn\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/Sqrt[d\*x], x]

[Out]  $(-4*b*n*Sqrt[d*x])/d + (2*Sqrt[d*x]*(a + b*Log[c*x^n]))/d$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{a + b \log(cx^n)}{\sqrt{dx}} dx = -\frac{4bn\sqrt{dx}}{d} + \frac{2\sqrt{dx} (a + b \log(cx^n))}{d}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{2x (a + b \log(cx^n) - 2bn)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/Sqrt[d\*x], x]

[Out]  $(2*x*(a - 2*b*n + b*\text{Log}[c*x^n]))/\text{Sqrt}[d*x]$

**fricas** [A] time = 0.47, size = 25, normalized size = 0.68

$$\frac{2(bn \log(x) - 2bn + b \log(c) + a)\sqrt{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="fricas")`

[Out]  $2*(b*n*\log(x) - 2*b*n + b*\log(c) + a)*\text{sqrt}(d*x)/d$

**giac** [A] time = 0.29, size = 41, normalized size = 1.11

$$\frac{2((\sqrt{dx} \log(x) - 2\sqrt{dx})bn + \sqrt{dx} b \log(c) + \sqrt{dx} a)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="giac")`

[Out]  $2*((\text{sqrt}(d*x)*\log(x) - 2*\text{sqrt}(d*x))*b*n + \text{sqrt}(d*x)*b*\log(c) + \text{sqrt}(d*x)*a)/d$

**maple** [A] time = 0.04, size = 42, normalized size = 1.14

$$-\frac{4\sqrt{dx} bn}{d} + \frac{2\sqrt{dx} b \ln(cx^n)}{d} + \frac{2\sqrt{dx} a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^n)+a)/(d*x)^(1/2),x)`

[Out]  $2/d*(d*x)^(1/2)*b*\ln(c*x^n)-4*b*n*(d*x)^(1/2)/d+2/d*(d*x)^(1/2)*a$

**maxima** [A] time = 0.56, size = 41, normalized size = 1.11

$$-\frac{4\sqrt{dx} bn}{d} + \frac{2\sqrt{dx} b \log(cx^n)}{d} + \frac{2\sqrt{dx} a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="maxima")`

[Out]  $-4*\text{sqrt}(d*x)*b*n/d + 2*\text{sqrt}(d*x)*b*\log(c*x^n)/d + 2*\text{sqrt}(d*x)*a/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \ln(cx^n)}{\sqrt{d}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))/(d\*x)^(1/2), x)

[Out] int((a + b\*log(c\*x^n))/(d\*x)^(1/2), x)

**sympy** [A] time = 0.77, size = 63, normalized size = 1.70

$$\frac{2a\sqrt{x}}{\sqrt{d}} + \frac{2bn\sqrt{x} \log(x)}{\sqrt{d}} - \frac{4bn\sqrt{x}}{\sqrt{d}} + \frac{2b\sqrt{x} \log(c)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))/(d\*x)\*\*(1/2), x)

[Out] 2\*a\*sqrt(x)/sqrt(d) + 2\*b\*n\*sqrt(x)\*log(x)/sqrt(d) - 4\*b\*n\*sqrt(x)/sqrt(d)  
+ 2\*b\*sqrt(x)\*log(c)/sqrt(d)

$$3.93 \quad \int \frac{a+b \log(cx^n)}{(dx)^{3/2}} dx$$

Optimal. Leaf size=37

$$-\frac{2(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{4bn}{d\sqrt{dx}}$$

[Out]  $-4*b*n/d/(d*x)^{(1/2)}-2*(a+b*\ln(c*x^n))/d/(d*x)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$-\frac{2(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{4bn}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/(d\*x)^(3/2), x]

[Out]  $(-4*b*n)/(d*\text{Sqrt}[d*x]) - (2*(a + b*\text{Log}[c*x^n]))/(d*\text{Sqrt}[d*x])$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{a+b \log(cx^n)}{(dx)^{3/2}} dx = -\frac{4bn}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))}{d\sqrt{dx}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{2x(a+b \log(cx^n) + 2bn)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/(d\*x)^(3/2), x]

[Out]  $(-2*x*(a + 2*b*n + b*\text{Log}[c*x^n]))/(d*x)^{(3/2)}$

**fricas** [A] time = 0.47, size = 28, normalized size = 0.76

$$\frac{2(bn \log(x) + 2bn + b \log(c) + a)\sqrt{dx}}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(3/2),x, algorithm="fricas")`

[Out]  $-2*(b*n*\log(x) + 2*b*n + b*\log(c) + a)*\text{sqrt}(d*x)/(d^2*x)$

**giac** [A] time = 0.38, size = 43, normalized size = 1.16

$$\frac{2\left(\frac{bn \log(dx)}{\sqrt{dx}} - \frac{bn \log(d) - 2bn - b \log(c) - a}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(3/2),x, algorithm="giac")`

[Out]  $-2*(b*n*\log(d*x)/\text{sqrt}(d*x) - (b*n*\log(d) - 2*b*n - b*\log(c) - a)/\text{sqrt}(d*x))/d$

**maple** [C] time = 0.13, size = 122, normalized size = 3.30

$$\frac{2b \ln(x^n)}{\sqrt{dx} d} - \frac{-i\pi b \text{csgn}(ic) \text{csgn}(ix^n) \text{csgn}(ic x^n) + i\pi b \text{csgn}(ic) \text{csgn}(ic x^n)^2 + i\pi b \text{csgn}(ix^n) \text{csgn}(ic x^n)^2 - i\pi b \text{csgn}(ic) \text{csgn}(ix^n) \text{csgn}(ic x^n)^2}{\sqrt{dx} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^n)+a)/(d*x)^(3/2),x)`

[Out]  $-2/d*b/(d*x)^{(1/2)}*\ln(x^n) - 1/d*(I*\text{Pi}*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2 - I*\text{Pi}*b*\text{csgn}(I*c)*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n) - I*\text{Pi}*b*\text{csgn}(I*c*x^n)^3 + I*\text{Pi}*b*\text{csgn}(I*c)*\text{csgn}(I*c*x^n)^2 + 2*b*\ln(c) + 4*b*n + 2*a)/(d*x)^{(1/2)}$

**maxima** [A] time = 0.55, size = 41, normalized size = 1.11

$$-\frac{4bn}{\sqrt{dx} d} - \frac{2b \log(cx^n)}{\sqrt{dx} d} - \frac{2a}{\sqrt{dx} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(3/2),x, algorithm="maxima")`

[Out]  $-4*b*n/(sqrt(d*x)*d) - 2*b*log(c*x^n)/(sqrt(d*x)*d) - 2*a/(sqrt(d*x)*d)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \ln(cx^n)}{(dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))/(d*x)^(3/2), x)`

[Out] `int((a + b*log(c*x^n))/(d*x)^(3/2), x)`

**sympy** [A] time = 2.86, size = 65, normalized size = 1.76

$$-\frac{2a}{d^{\frac{3}{2}}\sqrt{x}} - \frac{2bn \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4bn}{d^{\frac{3}{2}}\sqrt{x}} - \frac{2b \log(c)}{d^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/(d*x)**(3/2), x)`

[Out] `-2*a/(d**(3/2)*sqrt(x)) - 2*b*n*log(x)/(d**(3/2)*sqrt(x)) - 4*b*n/(d**(3/2)*sqrt(x)) - 2*b*log(c)/(d**(3/2)*sqrt(x))`



$$3.94 \quad \int \frac{a+b \log(cx^n)}{(dx)^{5/2}} dx$$

Optimal. Leaf size=41

$$-\frac{2(a+b \log(cx^n))}{3d(dx)^{3/2}} - \frac{4bn}{9d(dx)^{3/2}}$$

[Out]  $-4/9*b*n/d/(d*x)^{(3/2)}-2/3*(a+b*\ln(c*x^n))/d/(d*x)^{(3/2)}$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$-\frac{2(a+b \log(cx^n))}{3d(dx)^{3/2}} - \frac{4bn}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/(d\*x)^(5/2), x]

[Out]  $(-4*b*n)/(9*d*(d*x)^{(3/2)}) - (2*(a + b*Log[c*x^n]))/(3*d*(d*x)^{(3/2)})$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{a+b \log(cx^n)}{(dx)^{5/2}} dx = -\frac{4bn}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))}{3d(dx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.71

$$-\frac{2x(3a + 3b \log(cx^n) + 2bn)}{9(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/(d\*x)^(5/2), x]

[Out]  $(-2*x*(3*a + 2*b*n + 3*b*Log[c*x^n]))/(9*(d*x)^{(5/2)})$

**fricas** [A] time = 0.48, size = 32, normalized size = 0.78

$$\frac{2(3bn \log(x) + 2bn + 3b \log(c) + 3a)\sqrt{dx}}{9d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/(d\*x)^(5/2),x, algorithm="fricas")

[Out] -2/9\*(3\*b\*n\*log(x) + 2\*b\*n + 3\*b\*log(c) + 3\*a)\*sqrt(d\*x)/(d^3\*x^2)

**giac** [B] time = 0.35, size = 67, normalized size = 1.63

$$-\frac{2\left(\frac{3bdn \log(dx)}{\sqrt{dx}x} - \frac{3bd^2n \log(d) - 2bd^2n - 3bd^2 \log(c) - 3ad^2}{\sqrt{dx}dx}\right)}{9d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/(d\*x)^(5/2),x, algorithm="giac")

[Out] -2/9\*(3\*b\*d\*n\*log(d\*x)/(sqrt(d\*x)\*x) - (3\*b\*d^2\*n\*log(d) - 2\*b\*d^2\*n - 3\*b\*d^2\*log(c) - 3\*a\*d^2)/(sqrt(d\*x)\*d\*x))/d^3

**maple** [C] time = 0.13, size = 128, normalized size = 3.12

$$\frac{2b \ln(x^n)}{3\sqrt{dx} d^2x} - \frac{3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 3i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2}{9\sqrt{dx} d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^n)+a)/(d\*x)^(5/2),x)

[Out] -2/3/d^2\*b/x/(d\*x)^(1/2)\*ln(x^n)-1/9/d^2\*(3\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-3\*I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-3\*I\*b\*Pi\*csgn(I\*c\*x^n)^3+3\*I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+6\*b\*ln(c)+4\*b\*n+6\*a)/x/(d\*x)^(1/2)

**maxima** [A] time = 0.63, size = 41, normalized size = 1.00

$$-\frac{4bn}{9(dx)^{\frac{3}{2}}d} - \frac{2b \log(cx^n)}{3(dx)^{\frac{3}{2}}d} - \frac{2a}{3(dx)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/(d\*x)^(5/2),x, algorithm="maxima")

[Out] -4/9\*b\*n/((d\*x)^(3/2)\*d) - 2/3\*b\*log(c\*x^n)/((d\*x)^(3/2)\*d) - 2/3\*a/((d\*x)^(3/2)\*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \ln(cx^n)}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))/(d\*x)^(5/2), x)

[Out] int((a + b\*log(c\*x^n))/(d\*x)^(5/2), x)

sympy [A] time = 28.21, size = 71, normalized size = 1.73

$$-\frac{2a}{3d^{5/2}x^{3/2}} - \frac{2bn \log(x)}{3d^{5/2}x^{3/2}} - \frac{4bn}{9d^{5/2}x^{3/2}} - \frac{2b \log(c)}{3d^{5/2}x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))/(d\*x)\*\*(5/2), x)

[Out] -2\*a/(3\*d\*\*(5/2)\*x\*\*(3/2)) - 2\*b\*n\*log(x)/(3\*d\*\*(5/2)\*x\*\*(3/2)) - 4\*b\*n/(9\*d\*\*(5/2)\*x\*\*(3/2)) - 2\*b\*log(c)/(3\*d\*\*(5/2)\*x\*\*(3/2))

### 3.95 $\int (dx)^{5/2} (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=73

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} - \frac{8bn(dx)^{7/2} (a + b \log(cx^n))}{49d} + \frac{16b^2n^2(dx)^{7/2}}{343d}$$

[Out]  $16/343*b^2*n^2*(d*x)^{(7/2)}/d-8/49*b*n*(d*x)^{(7/2)*(a+b*\ln(c*x^n))/d+2/7*(d*x)^{(7/2)*(a+b*\ln(c*x^n))^2/d}$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2305, 2304}

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} - \frac{8bn(dx)^{7/2} (a + b \log(cx^n))}{49d} + \frac{16b^2n^2(dx)^{7/2}}{343d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(16*b^2*n^2*(d*x)^{(7/2)})/(343*d) - (8*b*n*(d*x)^{(7/2)*(a + b*Log[c*x^n])})/(49*d) + (2*(d*x)^{(7/2)*(a + b*Log[c*x^n])^2})/(7*d)$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int (dx)^{5/2} (a + b \log(cx^n))^2 dx &= \frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} - \frac{1}{7}(4bn) \int (dx)^{5/2} (a + b \log(cx^n)) dx \\ &= \frac{16b^2n^2(dx)^{7/2}}{343d} - \frac{8bn(dx)^{7/2} (a + b \log(cx^n))}{49d} + \frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 61, normalized size = 0.84

$$\frac{2}{343}x(dx)^{5/2}\left(49a^2 + 14b(7a - 2bn)\log(cx^n) - 28abn + 49b^2\log^2(cx^n) + 8b^2n^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out] (2\*x\*(d\*x)^(5/2)\*(49\*a^2 - 28\*a\*b\*n + 8\*b^2\*n^2 + 14\*b\*(7\*a - 2\*b\*n)\*Log[c\*x^n] + 49\*b^2\*Log[c\*x^n]^2))/343

**fricas [B]** time = 0.46, size = 141, normalized size = 1.93

$$\frac{2}{343}\left(49b^2d^2n^2x^3\log(x)^2 + 49b^2d^2x^3\log(c)^2 - 14\left(2b^2d^2n - 7abd^2\right)x^3\log(c) + \left(8b^2d^2n^2 - 28abd^2n + 49a^2d^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 2/343\*(49\*b^2\*d^2\*n^2\*x^3\*log(x)^2 + 49\*b^2\*d^2\*x^3\*log(c)^2 - 14\*(2\*b^2\*d^2\*n - 7\*a\*b\*d^2)\*x^3\*log(c) + (8\*b^2\*d^2\*n^2 - 28\*a\*b\*d^2\*n + 49\*a^2\*d^2)\*x^3 + 14\*(7\*b^2\*d^2\*n\*x^3\*log(c) - (2\*b^2\*d^2\*n^2 - 7\*a\*b\*d^2\*n)\*x^3)\*log(x))\*sqrt(d\*x)

**giac [C]** time = 1.31, size = 425, normalized size = 5.82

$$\left(\frac{1}{7}i + \frac{1}{7}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right)\log(x)^2 - \left(\frac{1}{7}i - \frac{1}{7}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\log(x)^2\sin\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right) - \left(\frac{4}{49}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] (1/7\*I + 1/7)\*sqrt(2)\*b^2\*d^2\*n^2\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x)^2 - (1/7\*I - 1/7)\*sqrt(2)\*b^2\*d^2\*n^2\*x^(7/2)\*sqrt(abs(d))\*log(x)^2\*sin(1/4\*pi\*sgn(d)) - (4/49\*I + 4/49)\*sqrt(2)\*b^2\*d^2\*n^2\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) + (2/7\*I + 2/7)\*sqrt(2)\*b^2\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(c)\*log(x) + (4/49\*I - 4/49)\*sqrt(2)\*b^2\*d^2\*n^2\*x^(7/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) - (2/7\*I - 2/7)\*sqrt(2)\*b^2\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*log(c)\*log(x)\*sin(1/4\*pi\*sgn(d)) + (8/343\*I + 8/343)\*sqrt(2)\*b^2\*d^2\*n^2\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d)) - (4/49\*I + 4/49)\*sqrt(2)\*b^2\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(c) + (2/7\*I + 2/7)\*sqrt(2)\*a\*b\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) - (8/343\*I - 8/343)\*sqrt(2)\*b^2\*d^2\*n^2\*x^(7/2)\*sqrt(abs(d))\*sin(1/4\*pi\*sgn(d)) + (4/49\*I - 4/49)\*sqrt(2)\*b^2\*d^2\*n\*x^(7/2)\*sqrt(abs(d))\*log(c)

) $\sin(1/4\pi\text{sgn}(d)) - (2/7I - 2/7)\sqrt{2}ab^2d^2n^2x^{7/2}\sqrt{\text{abs}(d)}$   
 $\log(x)\sin(1/4\pi\text{sgn}(d)) - (4/49I + 4/49)\sqrt{2}ab^2d^2n^2x^{7/2}\sqrt{\text{abs}(d)}$   
 $\cos(1/4\pi\text{sgn}(d)) + (4/49I - 4/49)\sqrt{2}ab^2d^2n^2x^{7/2}\sqrt{\text{abs}(d)}$   
 $\sin(1/4\pi\text{sgn}(d)) + 2/7b^2d^{5/2}x^{7/2}\log(c)^2 + 4/7ab^2d^{5/2}x^{7/2}\log(c)$   
 $+ 2/7a^2d^{5/2}x^{7/2}$

**maple [C]** time = 0.17, size = 716, normalized size = 9.81

$$\frac{2b^2d^3x^4\ln(x^n)^2}{7\sqrt{dx}} + \frac{2(-7i\pi b\text{csgn}(ic)\text{csgn}(ix^n)\text{csgn}(icx^n) + 7i\pi b\text{csgn}(ic)\text{csgn}(icx^n)^2 + 7i\pi b\text{csgn}(ix^n)\text{csgn}(icx^n))}{49\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)\*(b\*ln(c\*x^n)+a)^2,x)

[Out]  $2/7d^3x^4b^2/(d*x)^{1/2}\ln(x^n)^2 + 2/49d^3b^2x^4(-7I\pi b\text{csgn}(I*c)\text{csgn}(I*x^n)\text{csgn}(I*c*x^n) + 7I\pi b\text{csgn}(I*c)\text{csgn}(I*c*x^n)^2 + 7I\pi b\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 - 7I\pi b\text{csgn}(I*c*x^n)^3 - 4b^n + 14b\ln(c) + 14a)/(d*x)^{1/2}$   
 $+ \ln(x^n) + 1/686d^3(-49\pi^2b^2\text{csgn}(I*c)^2\text{csgn}(I*x^n)^2\text{csgn}(I*c*x^n)^2 - 196\pi^2b^2\text{csgn}(I*c)\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^4 + 98\pi^2b^2\text{csgn}(I*c)^2\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^3 + 98\pi^2b^2\text{csgn}(I*c)\text{csgn}(I*x^n)^2\text{csgn}(I*c*x^n)^3 + 196a^2 + 32b^2n^2 - 196I\pi a b\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) + 392ab\ln(c) - 112b^2n\ln(c) + 196b^2\ln(c)^2 - 49\pi^2b^2\text{csgn}(I*x^n)^2\text{csgn}(I*c*x^n)^4 + 98\pi^2b^2\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5 - 112abn - 49\pi^2b^2\text{csgn}(I*c*x^n)^6 + 98\pi^2b^2\text{csgn}(I*c)\text{csgn}(I*c*x^n)^5 - 49\pi^2b^2\text{csgn}(I*c)^2\text{csgn}(I*c*x^n)^4 + 56I\pi b^2n\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 196I\ln(c)\pi b^2\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) + 56I\pi b^2n\text{csgn}(I*c*x^n)^3 - 196I\ln(c)\pi b^2\text{csgn}(I*c*x^n)^3 - 196I\pi a b\text{csgn}(I*c*x^n)^3 + 196I\ln(c)\pi b^2\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 196I\pi a b\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) - 56I\pi b^2n\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 - 56I\pi b^2n\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 196I\ln(c)\pi b^2\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 196I\pi a b\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2)x^4/(d*x)^{1/2}$

**maxima [A]** time = 0.59, size = 102, normalized size = 1.40

$$\frac{2(dx)^{7/2}b^2\log(cx^n)^2}{7d} - \frac{8(dx)^{7/2}abn}{49d} + \frac{4(dx)^{7/2}ab\log(cx^n)}{7d} + \frac{2(dx)^{7/2}a^2}{7d} + \frac{8}{343}\left(\frac{2(dx)^{7/2}n^2}{d} - \frac{7(dx)^{7/2}n\log(cx^n)}{d}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out]  $2/7(d*x)^{7/2}b^2\log(c*x^n)^2/d - 8/49(d*x)^{7/2}a*b*n/d + 4/7(d*x)^{7/2}a*b*\log(c*x^n)/d + 2/7(d*x)^{7/2}a^2/d + 8/343(2*(d*x)^{7/2}*n^2/d - 7*(d*x)^{7/2}*n*\log(c*x^n)/d)*b^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^{5/2} (a + b \ln(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)\*(a + b\*log(c\*x^n))^2,x)

[Out] int((d\*x)^(5/2)\*(a + b\*log(c\*x^n))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

### 3.96 $\int (dx)^{3/2} (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=73

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} - \frac{8bn(dx)^{5/2} (a + b \log(cx^n))}{25d} + \frac{16b^2n^2(dx)^{5/2}}{125d}$$

[Out]  $16/125*b^2*n^2*(d*x)^{(5/2)}/d-8/25*b*n*(d*x)^{(5/2)*(a+b*\ln(c*x^n))/d+2/5*(d*x)^{(5/2)*(a+b*\ln(c*x^n))^2/d}$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2305, 2304}

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} - \frac{8bn(dx)^{5/2} (a + b \log(cx^n))}{25d} + \frac{16b^2n^2(dx)^{5/2}}{125d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(16*b^2*n^2*(d*x)^{(5/2)})/(125*d) - (8*b*n*(d*x)^{(5/2)*(a + b*Log[c*x^n])})/(25*d) + (2*(d*x)^{(5/2)*(a + b*Log[c*x^n])^2})/(5*d)$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int (dx)^{3/2} (a + b \log(cx^n))^2 dx &= \frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} - \frac{1}{5}(4bn) \int (dx)^{3/2} (a + b \log(cx^n)) dx \\ &= \frac{16b^2n^2(dx)^{5/2}}{125d} - \frac{8bn(dx)^{5/2} (a + b \log(cx^n))}{25d} + \frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} \end{aligned}$$



**Mathematica [A]** time = 0.02, size = 61, normalized size = 0.84

$$\frac{2}{125}x(dx)^{3/2} \left( 25a^2 + 10b(5a - 2bn) \log(cx^n) - 20abn + 25b^2 \log^2(cx^n) + 8b^2n^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out] (2\*x\*(d\*x)^(3/2)\*(25\*a^2 - 20\*a\*b\*n + 8\*b^2\*n^2 + 10\*b\*(5\*a - 2\*b\*n)\*Log[c\*x^n] + 25\*b^2\*Log[c\*x^n]^2))/125

**fricas [A]** time = 0.47, size = 121, normalized size = 1.66

$$\frac{2}{125} \left( 25b^2dn^2x^2 \log(x)^2 + 25b^2dx^2 \log(c)^2 - 10(2b^2dn - 5abd)x^2 \log(c) + (8b^2dn^2 - 20abdn + 25a^2d)x^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 2/125\*(25\*b^2\*d\*n^2\*x^2\*log(x)^2 + 25\*b^2\*d\*x^2\*log(c)^2 - 10\*(2\*b^2\*d\*n - 5\*a\*b\*d)\*x^2\*log(c) + (8\*b^2\*d\*n^2 - 20\*a\*b\*d\*n + 25\*a^2\*d)\*x^2 + 10\*(5\*b^2\*d\*n\*x^2\*log(c) - (2\*b^2\*d\*n^2 - 5\*a\*b\*d\*n)\*x^2)\*log(x))\*sqrt(d\*x)

**giac [C]** time = 1.42, size = 386, normalized size = 5.29

$$-\frac{1}{125} \left( -(25i + 25) \sqrt{2} b^2 n^2 x^{\frac{5}{2}} \sqrt{|d|} \cos\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) \log(x)^2 + (25i - 25) \sqrt{2} b^2 n^2 x^{\frac{5}{2}} \sqrt{|d|} \log(x)^2 \sin\left(\frac{1}{4} \pi \operatorname{sgn}(d)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] -1/125\*(-(25\*I + 25)\*sqrt(2)\*b^2\*n^2\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x)^2 + (25\*I - 25)\*sqrt(2)\*b^2\*n^2\*x^(5/2)\*sqrt(abs(d))\*log(x)^2\*sin(1/4\*pi\*sgn(d)) + (20\*I + 20)\*sqrt(2)\*b^2\*n^2\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) - (50\*I + 50)\*sqrt(2)\*b^2\*n\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(c)\*log(x) - (20\*I - 20)\*sqrt(2)\*b^2\*n^2\*x^(5/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) + (50\*I - 50)\*sqrt(2)\*b^2\*n\*x^(5/2)\*sqrt(abs(d))\*log(c)\*log(x)\*sin(1/4\*pi\*sgn(d)) - (8\*I + 8)\*sqrt(2)\*b^2\*n^2\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d)) + (20\*I + 20)\*sqrt(2)\*b^2\*n\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(c) - (50\*I + 50)\*sqrt(2)\*a\*b\*n\*x^(5/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) + (8\*I - 8)\*sqrt(2)\*b^2\*n^2\*x^(5/2)\*sqrt(abs(d))\*sin(1/4\*pi\*sgn(d)) - (20\*I - 20)\*sqrt(2)\*b^2\*n\*x^(5/2)\*sqrt(abs(d))\*log(c)\*sin(1/4\*pi\*sgn(d)) + (50\*I - 50)\*sqrt(2)\*a\*b\*n\*x^(5/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) + (20\*I + 20)\*sqrt(2)\*a\*b\*n\*x^(5/2)\*sqrt(abs(d))\*cos(1/4

$\pi \operatorname{sgn}(d) - (20I - 20) \sqrt{2} a b n x^{5/2} \sqrt{\operatorname{abs}(d)} \sin(1/4 \pi \operatorname{sgn}(d)) - 50 b^2 \sqrt{d} x^{5/2} \log(c)^2 - 100 a b \sqrt{d} x^{5/2} \log(c) - 50 a^2 \sqrt{d} x^{5/2} d$

**maple [C]** time = 0.17, size = 716, normalized size = 9.81

$$\frac{2b^2 d^2 x^3 \ln(x^n)^2}{5\sqrt{dx}} + \frac{2(-5i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 5i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 5i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2)}{25\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*(b*ln(c*x^n)+a)^2,x)`

[Out]  $\frac{2}{5} d^2 b^2 x^3 / (d x)^{1/2} \ln(x^n)^2 + \frac{2}{25} d^2 b^2 x^3 (5 I b \pi \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 - 5 I b \pi \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) - 5 I b \pi \operatorname{csgn}(I c x^n)^3 + 5 I b \pi \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) + 10 b \ln(c) - 4 b n + 10 a) / (d x)^{1/2} \ln(x^n) + \frac{1}{250} d^2 (-25 \pi^2 b^2 \operatorname{csgn}(I c)^2 \operatorname{csgn}(I x^n)^2 \operatorname{csgn}(I c x^n)^2 - 100 \pi^2 b^2 \operatorname{csgn}(I c) \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^4 + 50 \pi^2 b^2 \operatorname{csgn}(I c)^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^3 + 50 \pi^2 b^2 \operatorname{csgn}(I c) \operatorname{csgn}(I x^n)^2 \operatorname{csgn}(I c x^n)^3 + 100 a^2 + 32 b^2 n^2 - 100 I \ln(c) \pi b^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) + 200 a b \ln(c) - 80 b^2 n \ln(c) + 100 b^2 \ln(c)^2 - 25 \pi^2 b^2 \operatorname{csgn}(I x^n)^2 \operatorname{csgn}(I c x^n)^4 + 50 \pi^2 b^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^5 - 80 a b n - 25 \pi^2 b^2 \operatorname{csgn}(I c x^n)^6 + 50 \pi^2 b^2 \operatorname{csgn}(I c) \operatorname{csgn}(I c x^n)^5 - 25 \pi^2 b^2 \operatorname{csgn}(I c)^2 \operatorname{csgn}(I c x^n)^4 - 100 I \pi a b \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) + 40 I \pi b^2 n \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) - 100 I \ln(c) \pi b^2 \operatorname{csgn}(I c x^n)^3 - 100 I \pi a b \operatorname{csgn}(I c x^n)^3 + 40 I \pi b^2 n \operatorname{csgn}(I c x^n)^3 + 100 I \ln(c) \pi b^2 \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) + 100 I \pi a b \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 + 100 I \ln(c) \pi b^2 \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 + 100 I \pi a b \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) - 40 I \pi b^2 n \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 - 40 I \pi b^2 n \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c)) x^3 / (d x)^{1/2}$

**maxima [A]** time = 0.62, size = 102, normalized size = 1.40

$$\frac{2 (dx)^{5/2} b^2 \log(cx^n)^2}{5d} - \frac{8 (dx)^{5/2} abn}{25d} + \frac{4 (dx)^{5/2} ab \log(cx^n)}{5d} + \frac{2 (dx)^{5/2} a^2}{5d} + \frac{8}{125} \left( \frac{2 (dx)^{5/2} n^2}{d} - \frac{5 (dx)^{5/2} n \log(cx^n)}{d} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $\frac{2}{5} (d x)^{5/2} b^2 \log(c x^n)^2 / d - \frac{8}{25} (d x)^{5/2} a b n / d + \frac{4}{5} (d x)^{5/2} a^2 / d + \frac{8}{125} (2 (d x)^{5/2} n^2 / d - 5 (d x)^{5/2} n \log(c x^n) / d) b^2$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (d x)^{3/2} (a + b \ln(c x^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*(a + b*log(c*x^n))^2,x)`

[Out] `int((d*x)^(3/2)*(a + b*log(c*x^n))^2, x)`

**sympy** [B] time = 50.36, size = 216, normalized size = 2.96

$$\frac{2a^2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5} + \frac{4abd^{\frac{3}{2}}nx^{\frac{5}{2}}\log(x)}{5} - \frac{8abd^{\frac{3}{2}}nx^{\frac{5}{2}}}{25} + \frac{4abd^{\frac{3}{2}}x^{\frac{5}{2}}\log(c)}{5} + \frac{2b^2d^{\frac{3}{2}}n^2x^{\frac{5}{2}}\log(x)^2}{5} - \frac{8b^2d^{\frac{3}{2}}n^2x^{\frac{5}{2}}\log(x)}{25} + \frac{16b^2d^{\frac{3}{2}}n^2}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(3/2)*(a+b*ln(c*x**n))**2,x)`

[Out] `2*a**2*d**(3/2)*x**(5/2)/5 + 4*a*b*d**(3/2)*n*x**(5/2)*log(x)/5 - 8*a*b*d**(3/2)*n*x**(5/2)/25 + 4*a*b*d**(3/2)*x**(5/2)*log(c)/5 + 2*b**2*d**(3/2)*n**2*x**(5/2)*log(x)**2/5 - 8*b**2*d**(3/2)*n**2*x**(5/2)*log(x)/25 + 16*b**2*d**(3/2)*n**2*x**(5/2)/125 + 4*b**2*d**(3/2)*n*x**(5/2)*log(c)*log(x)/5 - 8*b**2*d**(3/2)*n*x**(5/2)*log(c)/25 + 2*b**2*d**(3/2)*x**(5/2)*log(c)**2/5`

### 3.97 $\int \sqrt{dx} (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=73

$$-\frac{8bn(dx)^{3/2}(a + b \log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a + b \log(cx^n))^2}{3d} + \frac{16b^2n^2(dx)^{3/2}}{27d}$$

[Out]  $16/27*b^2*n^2*(d*x)^{(3/2)}/d - 8/9*b*n*(d*x)^{(3/2)}*(a+b*\ln(c*x^n))/d + 2/3*(d*x)^{(3/2)}*(a+b*\ln(c*x^n))^2/d$

**Rubi [A]** time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2305, 2304}

$$-\frac{8bn(dx)^{3/2}(a + b \log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a + b \log(cx^n))^2}{3d} + \frac{16b^2n^2(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(16*b^2*n^2*(d*x)^{(3/2)})/(27*d) - (8*b*n*(d*x)^{(3/2)}*(a + b*Log[c*x^n]))/(9*d) + (2*(d*x)^{(3/2)}*(a + b*Log[c*x^n])^2)/(3*d)$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int \sqrt{dx} (a + b \log(cx^n))^2 dx &= \frac{2(dx)^{3/2}(a + b \log(cx^n))^2}{3d} - \frac{1}{3}(4bn) \int \sqrt{dx} (a + b \log(cx^n)) dx \\ &= \frac{16b^2n^2(dx)^{3/2}}{27d} - \frac{8bn(dx)^{3/2}(a + b \log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a + b \log(cx^n))^2}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 61, normalized size = 0.84

$$\frac{2}{27}x\sqrt{dx} \left(9a^2 + 6b(3a - 2bn) \log(cx^n) - 12abn + 9b^2 \log^2(cx^n) + 8b^2n^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2,x]

[Out] (2\*x\*Sqrt[d\*x]\*(9\*a^2 - 12\*a\*b\*n + 8\*b^2\*n^2 + 6\*b\*(3\*a - 2\*b\*n)\*Log[c\*x^n] + 9\*b^2\*Log[c\*x^n]^2))/27

**fricas [A]** time = 0.48, size = 99, normalized size = 1.36

$$\frac{2}{27} \left(9b^2n^2x \log(x)^2 + 9b^2x \log(c)^2 - 6(2b^2n - 3ab)x \log(c) + (8b^2n^2 - 12abn + 9a^2)x + 6(3b^2nx \log(c) - \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 2/27\*(9\*b^2\*n^2\*x\*log(x)^2 + 9\*b^2\*x\*log(c)^2 - 6\*(2\*b^2\*n - 3\*a\*b)\*x\*log(c) + (8\*b^2\*n^2 - 12\*a\*b\*n + 9\*a^2)\*x + 6\*(3\*b^2\*n\*x\*log(c) - (2\*b^2\*n^2 - 3\*a\*b\*n)\*x)\*log(x))\*sqrt(d\*x)

**giac [C]** time = 1.28, size = 383, normalized size = 5.25

$$\left(\frac{1}{3}i + \frac{1}{3}\right) \sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|} \cos\left(\frac{1}{4}\pi\text{sgn}(d)\right) \log(x)^2 - \left(\frac{1}{3}i - \frac{1}{3}\right) \sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|} \log(x)^2 \sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{4}{9}i + \frac{4}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] (1/3\*I + 1/3)\*sqrt(2)\*b^2\*n^2\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x)^2 - (1/3\*I - 1/3)\*sqrt(2)\*b^2\*n^2\*x^(3/2)\*sqrt(abs(d))\*log(x)^2\*sin(1/4\*pi\*sgn(d)) - (4/9\*I + 4/9)\*sqrt(2)\*b^2\*n^2\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) + (2/3\*I + 2/3)\*sqrt(2)\*b^2\*n\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(c)\*log(x) + (4/9\*I - 4/9)\*sqrt(2)\*b^2\*n^2\*x^(3/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) - (2/3\*I - 2/3)\*sqrt(2)\*b^2\*n\*x^(3/2)\*sqrt(abs(d))\*log(c)\*log(x)\*sin(1/4\*pi\*sgn(d)) + (8/27\*I + 8/27)\*sqrt(2)\*b^2\*n^2\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d)) - (4/9\*I + 4/9)\*sqrt(2)\*b^2\*n\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(c) + (2/3\*I + 2/3)\*sqrt(2)\*a\*b\*n\*x^(3/2)\*sqrt(abs(d))\*cos(1/4\*pi\*sgn(d))\*log(x) - (8/27\*I - 8/27)\*sqrt(2)\*b^2\*n^2\*x^(3/2)\*sqrt(abs(d))\*sin(1/4\*pi\*sgn(d)) + (4/9\*I - 4/9)\*sqrt(2)\*b^2\*n\*x^(3/2)\*sqrt(abs(d))\*log(c)\*sin(1/4\*pi\*sgn(d)) - (2/3\*I - 2/3)\*sqrt(2)\*a\*b\*n\*x^(3/2)\*sqrt(abs(d))\*log(x)\*sin(1/4\*pi\*sgn(d)) - (4/9\*I + 4/9)\*sqrt(2)\*a\*b\*n\*x^(

$3/2) * \sqrt{\text{abs}(d)} * \cos(1/4 * \pi * \text{sgn}(d)) + (4/9 * I - 4/9) * \sqrt{2} * a * b * n * x^{(3/2)} * \sqrt{\text{abs}(d)} * \sin(1/4 * \pi * \text{sgn}(d)) + 2/3 * b^2 * \sqrt{d} * x^{(3/2)} * \log(c)^2 + 4/3 * a * b * \sqrt{d} * x^{(3/2)} * \log(c) + 2/3 * a^2 * \sqrt{d} * x^{(3/2)}$

**maple [C]** time = 0.18, size = 710, normalized size = 9.73

$$\frac{2b^2 d x^2 \ln(x^n)^2}{3\sqrt{dx}} + \frac{2(-3i\pi b \text{csgn}(ic) \text{csgn}(ix^n) \text{csgn}(ic x^n) + 3i\pi b \text{csgn}(ic) \text{csgn}(ic x^n)^2 + 3i\pi b \text{csgn}(ix^n) \text{csgn}(ic x^n)^2)}{9\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(b\*ln(c\*x^n)+a)^2,x)

[Out]  $2/3 * d * b^2 * x^2 / (d * x)^{(1/2)} * \ln(x^n)^2 + 2/9 * d * b * x^2 * (3 * I * \pi * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 - 3 * I * \pi * b * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) - 3 * I * \pi * b * \text{csgn}(I * c * x^n)^3 + 3 * I * \pi * b * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^2 + 6 * b * \ln(c) - 4 * b * n + 6 * a) / (d * x)^{(1/2)} * \ln(x^n) + 1/54 * d * (-9 * \pi^2 * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^2 - 36 * \pi^2 * b^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^4 + 18 * \pi^2 * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^3 + 18 * \pi^2 * b^2 * \text{csgn}(I * c) * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^3 + 36 * a^2 + 32 * b^2 * n^2 + 24 * I * \pi * b^2 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) + 72 * a * b * \ln(c) - 48 * b^2 * n * \ln(c) + 36 * b^2 * \ln(c)^2 - 9 * \pi^2 * b^2 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 + 18 * \pi^2 * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^5 - 48 * a * b * n - 9 * \pi^2 * b^2 * \text{csgn}(I * c * x^n)^6 + 18 * \pi^2 * b^2 * \text{csgn}(I * c) * \text{csgn}(I * c * x^n)^5 - 9 * \pi^2 * b^2 * \text{csgn}(I * c)^2 * \text{csgn}(I * c * x^n)^4 - 36 * I * \pi * a * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) - 36 * I * \ln(c) * \pi * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) - 36 * I * \ln(c) * \pi * b^2 * \text{csgn}(I * c * x^n)^3 - 36 * I * \pi * a * b * \text{csgn}(I * c * x^n)^3 + 24 * I * \pi * b^2 * n * \text{csgn}(I * c * x^n)^3 - 24 * I * \pi * b^2 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 - 24 * I * \pi * b^2 * n * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) + 36 * I * \ln(c) * \pi * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 36 * I * \ln(c) * \pi * b^2 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) + 36 * I * \pi * a * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 36 * I * \pi * a * b * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c)) * x^2 / (d * x)^{(1/2)}$

**maxima [A]** time = 0.52, size = 102, normalized size = 1.40

$$\frac{2(dx)^{\frac{3}{2}} b^2 \log(cx^n)^2}{3d} - \frac{8(dx)^{\frac{3}{2}} abn}{9d} + \frac{4(dx)^{\frac{3}{2}} ab \log(cx^n)}{3d} + \frac{8}{27} \left( \frac{2(dx)^{\frac{3}{2}} n^2}{d} - \frac{3(dx)^{\frac{3}{2}} n \log(cx^n)}{d} \right) b^2 + \frac{2(dx)^{\frac{3}{2}} a^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out]  $2/3 * (d * x)^{(3/2)} * b^2 * \log(c * x^n)^2 / d - 8/9 * (d * x)^{(3/2)} * a * b * n / d + 4/3 * (d * x)^{(3/2)} * a * b * \log(c * x^n) / d + 8/27 * (2 * (d * x)^{(3/2)} * n^2 / d - 3 * (d * x)^{(3/2)} * n * \log(c * x^n) / d) * b^2 + 2/3 * (d * x)^{(3/2)} * a^2 / d$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{dx} (a + b \ln(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(1/2)*(a + b*log(c*x^n))^2,x)`

[Out] `int((d*x)^(1/2)*(a + b*log(c*x^n))^2, x)`

**sympy** [B] time = 2.46, size = 216, normalized size = 2.96

$$\frac{2a^2\sqrt{d}x^{\frac{3}{2}}}{3} + \frac{4ab\sqrt{d}nx^{\frac{3}{2}}\log(x)}{3} - \frac{8ab\sqrt{d}nx^{\frac{3}{2}}}{9} + \frac{4ab\sqrt{d}x^{\frac{3}{2}}\log(c)}{3} + \frac{2b^2\sqrt{d}n^2x^{\frac{3}{2}}\log(x)^2}{3} - \frac{8b^2\sqrt{d}n^2x^{\frac{3}{2}}\log(x)}{9} + \frac{16b^2\sqrt{d}n^2x^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(1/2)*(a+b*ln(c*x**n))**2,x)`

[Out] `2*a**2*sqrt(d)*x**(3/2)/3 + 4*a*b*sqrt(d)*n*x**(3/2)*log(x)/3 - 8*a*b*sqrt(d)*n*x**(3/2)/9 + 4*a*b*sqrt(d)*x**(3/2)*log(c)/3 + 2*b**2*sqrt(d)*n**2*x**(3/2)*log(x)**2/3 - 8*b**2*sqrt(d)*n**2*x**(3/2)*log(x)/9 + 16*b**2*sqrt(d)*n**2*x**(3/2)/27 + 4*b**2*sqrt(d)*n*x**(3/2)*log(c)*log(x)/3 - 8*b**2*sqrt(d)*n*x**(3/2)*log(c)/9 + 2*b**2*sqrt(d)*x**(3/2)*log(c)**2/3`

$$3.98 \quad \int \frac{(a+b \log(cx^n))^2}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=67

$$-\frac{8bn\sqrt{dx}(a+b \log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b \log(cx^n))^2}{d} + \frac{16b^2n^2\sqrt{dx}}{d}$$

[Out]  $16*b^2*n^2*(d*x)^{(1/2)}/d-8*b*n*(a+b*\ln(c*x^n))*(d*x)^{(1/2)}/d+2*(a+b*\ln(c*x^n))^2*(d*x)^{(1/2)}/d$

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2305, 2304}

$$-\frac{8bn\sqrt{dx}(a+b \log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b \log(cx^n))^2}{d} + \frac{16b^2n^2\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/Sqrt[d\*x], x]

[Out]  $(16*b^2*n^2*\text{Sqrt}[d*x])/d - (8*b*n*\text{Sqrt}[d*x]*(a + b*\text{Log}[c*x^n]))/d + (2*\text{Sqrt}[d*x]*(a + b*\text{Log}[c*x^n])^2)/d$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps



$$\int \frac{(a + b \log(cx^n))^2}{\sqrt{dx}} dx = \frac{2\sqrt{dx} (a + b \log(cx^n))^2}{d} - (4bn) \int \frac{a + b \log(cx^n)}{\sqrt{dx}} dx$$

$$= \frac{16b^2n^2\sqrt{dx}}{d} - \frac{8bn\sqrt{dx} (a + b \log(cx^n))}{d} + \frac{2\sqrt{dx} (a + b \log(cx^n))^2}{d}$$

**Mathematica [A]** time = 0.01, size = 54, normalized size = 0.81

$$\frac{2x(a^2 + 2b(a - 2bn)\log(cx^n) - 4abn + b^2\log^2(cx^n) + 8b^2n^2)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/Sqrt[d\*x], x]

[Out] (2\*x\*(a^2 - 4\*a\*b\*n + 8\*b^2\*n^2 + 2\*b\*(a - 2\*b\*n)\*Log[c\*x^n] + b^2\*Log[c\*x^n]^2))/Sqrt[d\*x]

**fricas [A]** time = 0.44, size = 87, normalized size = 1.30

$$\frac{2(b^2n^2\log(x)^2 + 8b^2n^2 + b^2\log(c)^2 - 4abn + a^2 - 2(2b^2n - ab)\log(c) - 2(2b^2n^2 - b^2n\log(c) - abn)\log(x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(1/2), x, algorithm="fricas")

[Out] 2\*(b^2\*n^2\*log(x)^2 + 8\*b^2\*n^2 + b^2\*log(c)^2 - 4\*a\*b\*n + a^2 - 2\*(2\*b^2\*n - a\*b)\*log(c) - 2\*(2\*b^2\*n^2 - b^2\*n\*log(c) - a\*b\*n)\*log(x))\*sqrt(d\*x)/d

**giac [A]** time = 0.43, size = 118, normalized size = 1.76

$$\frac{2((\sqrt{dx} \log(x)^2 - 4\sqrt{dx} \log(x) + 8\sqrt{dx})b^2n^2 + 2(\sqrt{dx} \log(x) - 2\sqrt{dx})b^2n \log(c) + \sqrt{dx} b^2 \log(c)^2 + 2(\sqrt{dx} \log(x) - 2\sqrt{dx})a*b*n + 2\sqrt{dx} a*b \log(c) + \sqrt{dx} a^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(1/2), x, algorithm="giac")

[Out] 2\*((sqrt(d\*x)\*log(x)^2 - 4\*sqrt(d\*x)\*log(x) + 8\*sqrt(d\*x))\*b^2\*n^2 + 2\*(sqrt(d\*x)\*log(x) - 2\*sqrt(d\*x))\*b^2\*n\*log(c) + sqrt(d\*x)\*b^2\*log(c)^2 + 2\*(sqrt(d\*x)\*log(x) - 2\*sqrt(d\*x))\*a\*b\*n + 2\*sqrt(d\*x)\*a\*b\*log(c) + sqrt(d\*x)\*a^2)/d

**maple [A]** time = 0.06, size = 107, normalized size = 1.60

$$\frac{16\sqrt{dx} b^2 n^2}{d} - \frac{8\sqrt{dx} b^2 n \ln(c e^{n \ln(x)})}{d} + \frac{2\sqrt{dx} b^2 \ln(c e^{n \ln(x)})^2}{d} - \frac{8\sqrt{dx} abn}{d} + \frac{4\sqrt{dx} ab \ln(c x^n)}{d} + \frac{2\sqrt{dx} a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^n)+a)^2/(d\*x)^(1/2), x)

[Out] 2/d\*b^2\*(d\*x)^(1/2)\*ln(c\*exp(n\*ln(x)))^2-8/d\*b^2\*n\*(d\*x)^(1/2)\*ln(c\*exp(n\*ln(x)))+16\*b^2\*n^2\*(d\*x)^(1/2)/d+4/d\*(d\*x)^(1/2)\*a\*b\*ln(c\*x^n)-8/d\*(d\*x)^(1/2)\*a\*b\*n+2/d\*(d\*x)^(1/2)\*a^2

**maxima [A]** time = 0.53, size = 102, normalized size = 1.52

$$\frac{2\sqrt{dx} b^2 \log(cx^n)^2}{d} + 8 \left( \frac{2\sqrt{dx} n^2}{d} - \frac{\sqrt{dx} n \log(cx^n)}{d} \right) b^2 - \frac{8\sqrt{dx} abn}{d} + \frac{4\sqrt{dx} ab \log(cx^n)}{d} + \frac{2\sqrt{dx} a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(1/2), x, algorithm="maxima")

[Out] 2\*sqrt(d\*x)\*b^2\*log(c\*x^n)^2/d + 8\*(2\*sqrt(d\*x)\*n^2/d - sqrt(d\*x)\*n\*log(c\*x^n)/d)\*b^2 - 8\*sqrt(d\*x)\*a\*b\*n/d + 4\*sqrt(d\*x)\*a\*b\*log(c\*x^n)/d + 2\*sqrt(d\*x)\*a^2/d

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(c x^n))^2}{\sqrt{d} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^2/(d\*x)^(1/2), x)

[Out] int((a + b\*log(c\*x^n))^2/(d\*x)^(1/2), x)

**sympy [B]** time = 1.46, size = 199, normalized size = 2.97

$$\frac{2a^2\sqrt{x}}{\sqrt{d}} + \frac{4abn\sqrt{x} \log(x)}{\sqrt{d}} - \frac{8abn\sqrt{x}}{\sqrt{d}} + \frac{4ab\sqrt{x} \log(c)}{\sqrt{d}} + \frac{2b^2n^2\sqrt{x} \log(x)^2}{\sqrt{d}} - \frac{8b^2n^2\sqrt{x} \log(x)}{\sqrt{d}} + \frac{16b^2n^2\sqrt{x}}{\sqrt{d}} + \frac{4b^2n\sqrt{x}}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/(d\*x)\*\*(1/2), x)

```
[Out] 2*a**2*sqrt(x)/sqrt(d) + 4*a*b*n*sqrt(x)*log(x)/sqrt(d) - 8*a*b*n*sqrt(x)/s
qrt(d) + 4*a*b*sqrt(x)*log(c)/sqrt(d) + 2*b**2*n**2*sqrt(x)*log(x)**2/sqrt(
d) - 8*b**2*n**2*sqrt(x)*log(x)/sqrt(d) + 16*b**2*n**2*sqrt(x)/sqrt(d) + 4*
b**2*n*sqrt(x)*log(c)*log(x)/sqrt(d) - 8*b**2*n*sqrt(x)*log(c)/sqrt(d) + 2*
b**2*sqrt(x)*log(c)**2/sqrt(d)
```

$$3.99 \quad \int \frac{(a+b \log(cx^n))^2}{(dx)^{3/2}} dx$$

Optimal. Leaf size=67

$$-\frac{8bn(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))^2}{d\sqrt{dx}} - \frac{16b^2n^2}{d\sqrt{dx}}$$

[Out]  $-16*b^2*n^2/d/(d*x)^{(1/2)}-8*b*n*(a+b*\ln(c*x^n))/d/(d*x)^{(1/2)}-2*(a+b*\ln(c*x^n))^2/d/(d*x)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2305, 2304}

$$-\frac{8bn(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))^2}{d\sqrt{dx}} - \frac{16b^2n^2}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/(d\*x)^(3/2), x]

[Out]  $(-16*b^2*n^2)/(d*\text{Sqrt}[d*x]) - (8*b*n*(a + b*\text{Log}[c*x^n]))/(d*\text{Sqrt}[d*x]) - (2*(a + b*\text{Log}[c*x^n])^2)/(d*\text{Sqrt}[d*x])$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps

$$\int \frac{(a + b \log(cx^n))^2}{(dx)^{3/2}} dx = -\frac{2(a + b \log(cx^n))^2}{d\sqrt{dx}} + (4bn) \int \frac{a + b \log(cx^n)}{(dx)^{3/2}} dx$$

$$= -\frac{16b^2n^2}{d\sqrt{dx}} - \frac{8bn(a + b \log(cx^n))}{d\sqrt{dx}} - \frac{2(a + b \log(cx^n))^2}{d\sqrt{dx}}$$

**Mathematica [A]** time = 0.01, size = 54, normalized size = 0.81

$$\frac{2x(a^2 + 2b(a + 2bn) \log(cx^n) + 4abn + b^2 \log^2(cx^n) + 8b^2n^2)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/(d\*x)^(3/2), x]

[Out] (-2\*x\*(a^2 + 4\*a\*b\*n + 8\*b^2\*n^2 + 2\*b\*(a + 2\*b\*n)\*Log[c\*x^n] + b^2\*Log[c\*x^n]^2))/(d\*x)^(3/2)

**fricas [A]** time = 0.44, size = 87, normalized size = 1.30

$$\frac{2(b^2n^2 \log(x)^2 + 8b^2n^2 + b^2 \log(c)^2 + 4abn + a^2 + 2(2b^2n + ab) \log(c) + 2(2b^2n^2 + b^2n \log(c) + abn) \log^2(x))}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(3/2), x, algorithm="fricas")

[Out] -2\*(b^2\*n^2\*log(x)^2 + 8\*b^2\*n^2 + b^2\*log(c)^2 + 4\*a\*b\*n + a^2 + 2\*(2\*b^2\*n + a\*b)\*log(c) + 2\*(2\*b^2\*n^2 + b^2\*n\*log(c) + a\*b\*n)\*log(x))\*sqrt(d\*x)/(d^2\*x)

**giac [B]** time = 0.41, size = 149, normalized size = 2.22

$$\frac{2\left(\frac{b^2n^2 \log(dx)^2}{\sqrt{dx}} - \frac{2(b^2n^2 \log(d) - 2b^2n^2 - b^2n \log(c) - abn) \log(dx)}{\sqrt{dx}} + \frac{b^2n^2 \log(d)^2 - 4b^2n^2 \log(d) - 2b^2n \log(c) \log(d) + 8b^2n^2 + 4b^2n \log(c) + b^2 \log(c)^2}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(3/2), x, algorithm="giac")

[Out] -2\*(b^2\*n^2\*log(dx)^2/sqrt(dx) - 2\*(b^2\*n^2\*log(d) - 2\*b^2\*n^2 - b^2\*n\*log(c) - a\*b\*n)\*log(dx)/sqrt(dx) + (b^2\*n^2\*log(d)^2 - 4\*b^2\*n^2\*log(d) - 2\*b^2\*n\*log(c)\*log(d) + 8\*b^2\*n^2 + 4\*b^2\*n\*log(c) + b^2\*log(c)^2)/sqrt(dx))/d

$*b^2*n*\log(c)*\log(d) + 8*b^2*n^2 + 4*b^2*n*\log(c) + b^2*\log(c)^2 - 2*a*b*n*\log(d) + 4*a*b*n + 2*a*b*\log(c) + a^2)/\sqrt{d*x})/d$

**maple** [C] time = 0.17, size = 707, normalized size = 10.55

$$\frac{2b^2 \ln(x^n)^2}{\sqrt{dx} d} \frac{2 \left( -i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n)^2 \right)}{\sqrt{dx} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^n)+a)^2/(d*x)^(3/2),x)`

[Out]  $-2/d*b^2/(d*x)^{(1/2)}*\ln(x^n)^2-2/d*b*(-I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)+I*\pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2+I*\pi*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-I*\pi*b*\operatorname{csgn}(I*c*x^n)^3+4*b*n+2*b*\ln(c)+2*a)/(d*x)^{(1/2)}*\ln(x^n)-1/2/d*(-\pi^2*b^2*\operatorname{csgn}(I*c)^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^2-4*\pi^2*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^4+2*\pi^2*b^2*\operatorname{csgn}(I*c)^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^3+2*\pi^2*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^3-4*I*\pi*a*b*\operatorname{csgn}(I*c*x^n)^3-4*I*\pi*b^2*\operatorname{csgn}(I*c*x^n)^3*\ln(c)+4*a^2+32*b^2*n^2+8*a*b*\ln(c)+16*b^2*n*\ln(c)+4*b^2*\ln(c)^2-\pi^2*b^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^4+2*\pi^2*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^5+16*a*b*n-\pi^2*b^2*\operatorname{csgn}(I*c*x^n)^6+2*\pi^2*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^5-\pi^2*b^2*\operatorname{csgn}(I*c)^2*\operatorname{csgn}(I*c*x^n)^4-8*I*\pi*b^2*n*\operatorname{csgn}(I*c*x^n)^3+4*I*\pi*a*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2+4*I*\pi*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2*\ln(c)+4*I*\pi*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2*\ln(c)+4*I*\pi*a*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-4*I*\pi*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\ln(c)+8*I*\pi*b^2*n*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)+8*I*\pi*b^2*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-8*I*\pi*b^2*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c)-4*I*\pi*a*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n))/(d*x)^{(1/2)}$

**maxima** [A] time = 0.63, size = 101, normalized size = 1.51

$$-8b^2 \left( \frac{2n^2}{\sqrt{dx}d} + \frac{n \log(cx^n)}{\sqrt{dx}d} \right) - \frac{2b^2 \log(cx^n)^2}{\sqrt{dx}d} - \frac{8abn}{\sqrt{dx}d} - \frac{4ab \log(cx^n)}{\sqrt{dx}d} - \frac{2a^2}{\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2/(d*x)^(3/2),x, algorithm="maxima")`

[Out]  $-8*b^2*(2*n^2/(\sqrt{d*x}*d) + n*\log(c*x^n)/(\sqrt{d*x}*d)) - 2*b^2*\log(c*x^n)^2/(\sqrt{d*x}*d) - 8*a*b*n/(\sqrt{d*x}*d) - 4*a*b*\log(c*x^n)/(\sqrt{d*x}*d) - 2*a^2/(\sqrt{d*x}*d)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(cx^n))^2}{(dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))^2/(d*x)^(3/2), x)`

[Out] `int((a + b*log(c*x^n))^2/(d*x)^(3/2), x)`

**sympy** [B] time = 2.90, size = 201, normalized size = 3.00

$$\frac{2a^2}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4abn \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{8abn}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4ab \log(c)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{2b^2n^2 \log(x)^2}{d^{\frac{3}{2}}\sqrt{x}} - \frac{8b^2n^2 \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{16b^2n^2}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4b^2n \log(c) \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{8b^2n}{d^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**2/(d*x)**(3/2), x)`

[Out] `-2*a**2/(d**(3/2)*sqrt(x)) - 4*a*b*n*log(x)/(d**(3/2)*sqrt(x)) - 8*a*b*n/(d**(3/2)*sqrt(x)) - 4*a*b*log(c)/(d**(3/2)*sqrt(x)) - 2*b**2*n**2*log(x)**2/(d**(3/2)*sqrt(x)) - 8*b**2*n**2*log(x)/(d**(3/2)*sqrt(x)) - 16*b**2*n**2/(d**(3/2)*sqrt(x)) - 4*b**2*n*log(c)*log(x)/(d**(3/2)*sqrt(x)) - 8*b**2*n*log(c)/(d**(3/2)*sqrt(x)) - 2*b**2*log(c)**2/(d**(3/2)*sqrt(x))`

$$3.100 \quad \int \frac{(a+b \log(cx^n))^2}{(dx)^{5/2}} dx$$

Optimal. Leaf size=73

$$-\frac{8bn(a+b \log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))^2}{3d(dx)^{3/2}} - \frac{16b^2n^2}{27d(dx)^{3/2}}$$

[Out]  $-16/27*b^2*n^2/d/(d*x)^{(3/2)} - 8/9*b*n*(a+b*\ln(c*x^n))/d/(d*x)^{(3/2)} - 2/3*(a+b*\ln(c*x^n))^2/d/(d*x)^{(3/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2305, 2304}

$$-\frac{8bn(a+b \log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))^2}{3d(dx)^{3/2}} - \frac{16b^2n^2}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/(d\*x)^(5/2), x]

[Out]  $(-16*b^2*n^2)/(27*d*(d*x)^{(3/2)}) - (8*b*n*(a + b*Log[c*x^n]))/(9*d*(d*x)^{(3/2)}) - (2*(a + b*Log[c*x^n])^2)/(3*d*(d*x)^{(3/2)})$

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rubi steps





$9*b^2*d^2*n^2*\log(d)^2 - 12*b^2*d^2*n^2*\log(d) - 18*b^2*d^2*n*\log(c)*\log(d)$   
 $+ 8*b^2*d^2*n^2 + 12*b^2*d^2*n*\log(c) + 9*b^2*d^2*\log(c)^2 - 18*a*b*d^2*n*$   
 $\log(d) + 12*a*b*d^2*n + 18*a*b*d^2*\log(c) + 9*a^2*d^2)/(\sqrt{d*x}*d*x))/d^3$

**maple** [C] time = 0.18, size = 716, normalized size = 9.81

$$\frac{2b^2 \ln(x^n)^2}{3\sqrt{dx} d^2x} - \frac{2(-3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n) + 3i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic x^n)^2 + 3i\pi b \operatorname{csgn}(ix^n) \operatorname{csgn}(ic x^n))}{9\sqrt{dx} d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^n)+a)^2/(d*x)^(5/2),x)`

[Out]  $-2/3/d^2*b^2/x/(d*x)^{(1/2)}*\ln(x^n)^2-2/9/d^2*b*(-3*I*Pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)+3*I*Pi*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2+3*I*Pi*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-3*I*Pi*b*\operatorname{csgn}(I*c*x^n)^3+4*b*n+6*b*\ln(c)+6*a)/x/(d*x)^{(1/2)}*\ln(x^n)-1/54/d^2*(-9*Pi^2*b^2*\operatorname{csgn}(I*c)^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^2-36*Pi^2*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^4+18*Pi^2*b^2*\operatorname{csgn}(I*c)^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^3+18*Pi^2*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^3+36*a^2+32*b^2*n^2+72*a*b*\ln(c)+48*b^2*n*\ln(c)+36*b^2*\ln(c)^2-9*Pi^2*b^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^4+18*Pi^2*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^5+48*a*b*n-9*Pi^2*b^2*\operatorname{csgn}(I*c*x^n)^6+18*Pi^2*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^5-9*Pi^2*b^2*\operatorname{csgn}(I*c)^2*\operatorname{csgn}(I*c*x^n)^4-36*I*Pi*a*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)-36*I*Pi*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\ln(c)-36*I*Pi*b^2*\operatorname{csgn}(I*c*x^n)^3*\ln(c)-36*I*Pi*a*b*\operatorname{csgn}(I*c*x^n)^3-24*I*Pi*b^2*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c)-24*I*Pi*b^2*n*\operatorname{csgn}(I*c*x^n)^3+24*I*Pi*b^2*n*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)+24*I*Pi*b^2*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2+36*I*Pi*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2*\ln(c)+36*I*Pi*b^2*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2*\ln(c)+36*I*Pi*a*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2+36*I*Pi*a*b*\operatorname{csgn}(I*c)*\operatorname{csgn}(I*c*x^n)^2)/x/(d*x)^{(1/2)}$

**maxima** [A] time = 0.66, size = 102, normalized size = 1.40

$$-\frac{8}{27}b^2\left(\frac{2n^2}{(dx)^{\frac{3}{2}}d} + \frac{3n\log(cx^n)}{(dx)^{\frac{3}{2}}d}\right) - \frac{2b^2\log(cx^n)^2}{3(dx)^{\frac{3}{2}}d} - \frac{8abn}{9(dx)^{\frac{3}{2}}d} - \frac{4ab\log(cx^n)}{3(dx)^{\frac{3}{2}}d} - \frac{2a^2}{3(dx)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2/(d*x)^(5/2),x, algorithm="maxima")`

[Out]  $-8/27*b^2*(2*n^2/((d*x)^{(3/2)}*d) + 3*n*\log(c*x^n)/((d*x)^{(3/2)}*d)) - 2/3*b^2*\log(c*x^n)^2/((d*x)^{(3/2)}*d) - 8/9*a*b*n/((d*x)^{(3/2)}*d) - 4/3*a*b*\log(c*x^n)/((d*x)^{(3/2)}*d) - 2/3*a^2/((d*x)^{(3/2)}*d)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(cx^n))^2}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^2/(d\*x)^(5/2), x)

[Out] int((a + b\*log(c\*x^n))^2/(d\*x)^(5/2), x)

**sympy [B]** time = 26.71, size = 218, normalized size = 2.99

$$\frac{2a^2}{3d^{5/2}x^{3/2}} - \frac{4abn \log(x)}{3d^{5/2}x^{3/2}} - \frac{8abn}{9d^{5/2}x^{3/2}} - \frac{4ab \log(c)}{3d^{5/2}x^{3/2}} - \frac{2b^2n^2 \log(x)^2}{3d^{5/2}x^{3/2}} - \frac{8b^2n^2 \log(x)}{9d^{5/2}x^{3/2}} - \frac{16b^2n^2}{27d^{5/2}x^{3/2}} - \frac{4b^2n \log(c) \log(x)}{3d^{5/2}x^{3/2}} - \frac{8b^2n}{9d^{5/2}x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/(d\*x)\*\*(5/2), x)

[Out]  $-2*a**2/(3*d**(5/2)*x**(3/2)) - 4*a*b*n*\log(x)/(3*d**(5/2)*x**(3/2)) - 8*a*b*n/(9*d**(5/2)*x**(3/2)) - 4*a*b*\log(c)/(3*d**(5/2)*x**(3/2)) - 2*b**2*n**2*\log(x)**2/(3*d**(5/2)*x**(3/2)) - 8*b**2*n**2*\log(x)/(9*d**(5/2)*x**(3/2)) - 16*b**2*n**2/(27*d**(5/2)*x**(3/2)) - 4*b**2*n*\log(c)*\log(x)/(3*d**(5/2)*x**(3/2)) - 8*b**2*n*\log(c)/(9*d**(5/2)*x**(3/2)) - 2*b**2*\log(c)**2/(3*d**(5/2)*x**(3/2))$

$$3.101 \quad \int \frac{(dx)^{5/2}}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=64

$$\frac{(dx)^{7/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

[Out]  $(d*x)^{(7/2)}*Ei(7/2*(a+b*\ln(c*x^n))/b/n)/b/d/\exp(7/2*a/b/n)/n/((c*x^n)^{(7/2/n)})$

**Rubi [A]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2178}

$$\frac{(dx)^{7/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^{(5/2)}/(a + b*\operatorname{Log}[c*x^n]), x]$

[Out]  $((d*x)^{(7/2)}*ExpIntegralEi[(7*(a + b*\operatorname{Log}[c*x^n]))/(2*b*n)]/(b*d*E^{((7*a)/(2*b*n))}*n*(c*x^n)^{(7/(2*n))}))$

Rule 2178

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d))})*ExpIntegralEi[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /; \operatorname{FreeQ}\{F, c, d, e, f, g, x\} \&\amp; \operatorname{!}\$UseGamma == True$

Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*b_.)^{(p_.)}*(d_.)*(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m + 1)}/(d*n*(c*x^n)^{(m + 1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m + 1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, x\}$

Rubi steps

$$\int \frac{(dx)^{5/2}}{a + b \log(cx^n)} dx = \frac{\left( (dx)^{7/2} (cx^n)^{-7/2/n} \right) \text{Subst} \left( \int \frac{e^{7x/2n}}{a+bx} dx, x, \log(cx^n) \right)}{dn}$$

$$= \frac{e^{-7a/2bn} (dx)^{7/2} (cx^n)^{-7/2/n} \text{Ei} \left( \frac{7(a+b \log(cx^n))}{2bn} \right)}{bdn}$$

**Mathematica** [A] time = 0.08, size = 62, normalized size = 0.97

$$\frac{x(dx)^{5/2} e^{-7a/2bn} (cx^n)^{-7/2/n} \text{Ei} \left( \frac{7(a+b \log(cx^n))}{2bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)/(a + b\*Log[c\*x^n]), x]

[Out] (x\*(d\*x)^(5/2)\*ExpIntegralEi[(7\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*E^((7\*a)/(2\*b\*n))\*n\*(c\*x^n)^(7/(2\*n)))

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx} d^2 x^2}{b \log(cx^n) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d^2\*x^2/(b\*log(c\*x^n) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{5/2}}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n)), x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)/(b\*log(c\*x^n) + a), x)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{b \ln(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)/(b\*ln(c\*x^n)+a), x)

[Out] int((d\*x)^(5/2)/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2bd^{\frac{5}{2}}n \int \frac{x^{\frac{5}{2}}}{7(b^2 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2 + 2(b^2 \log(c) + ab) \log(x^n))} dx + \frac{2d^{\frac{5}{2}}x^{\frac{7}{2}}}{7(b \log(c) + b \log(x^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*d^(5/2)\*n\*integrate(1/7\*x^(5/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n)), x) + 2/7\*d^(5/2)\*x^(7/2)/(b\*log(c) + b\*log(x^n) + a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx)^{5/2}}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)/(a + b\*log(c\*x^n)), x)

[Out] int((d\*x)^(5/2)/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral((d\*x)\*\*(5/2)/(a + b\*log(c\*x\*\*n)), x)

$$3.102 \quad \int \frac{(dx)^{3/2}}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=64

$$\frac{(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

[Out]  $(d*x)^{(5/2)*\operatorname{Ei}(5/2*(a+b*\ln(c*x^n))/b/n)/b/d/\exp(5/2*a/b/n)/n/((c*x^n)^{(5/2/n)})$

**Rubi [A]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2178}

$$\frac{(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^{(3/2)/(a + b*\operatorname{Log}[c*x^n])}, x]$

[Out]  $((d*x)^{(5/2)*\operatorname{ExpIntegralEi}[(5*(a + b*\operatorname{Log}[c*x^n]))/(2*b*n)]/(b*d*E^{((5*a)/(2*b*n))}*n*(c*x^n)^{(5/(2*n))})$

#### Rule 2178

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d))*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d]}/d, x] /; \operatorname{FreeQ}\{F, c, d, e, f, g, x\} \&\amp; \operatorname{!}\$UseGamma === \operatorname{True}$

#### Rule 2310

$\operatorname{Int}[((a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}])*(b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m + 1)/(d*n*(c*x^n)^{(m + 1)/n})}, \operatorname{Subst}[\operatorname{Int}[E^{((m + 1)*x/n)*(a + b*x)^p}, x], x, \operatorname{Log}[c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, x\}$

#### Rubi steps

$$\int \frac{(dx)^{3/2}}{a + b \log(cx^n)} dx = \frac{\left( (dx)^{5/2} (cx^n)^{-5/2/n} \right) \text{Subst} \left( \int \frac{e^{\frac{5x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{dn}$$

$$= \frac{e^{-\frac{5a}{2bn}} (dx)^{5/2} (cx^n)^{-5/2/n} \text{Ei} \left( \frac{5(a+b \log(cx^n))}{2bn} \right)}{bdn}$$

**Mathematica** [A] time = 0.07, size = 62, normalized size = 0.97

$$\frac{x(dx)^{3/2} e^{-\frac{5a}{2bn}} (cx^n)^{-5/2/n} \text{Ei} \left( \frac{5(a+b \log(cx^n))}{2bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)/(a + b\*Log[c\*x^n]),x]

[Out] (x\*(d\*x)^(3/2)\*ExpIntegralEi[(5\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*E^((5\*a)/(2\*b\*n))\*n\*(c\*x^n)^(5/(2\*n)))

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx} dx}{b \log(cx^n) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d\*x/(b\*log(c\*x^n) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)/(b\*log(c\*x^n) + a), x)



**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{b \ln(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(b\*ln(c\*x^n)+a), x)

[Out] int((d\*x)^(3/2)/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2bd^{\frac{3}{2}}n \int \frac{x^{\frac{3}{2}}}{5(b^2 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2 + 2(b^2 \log(c) + ab) \log(x^n))} dx + \frac{2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5(b \log(c) + b \log(x^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*d^(3/2)\*n\*integrate(1/5\*x^(3/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n)), x) + 2/5\*d^(3/2)\*x^(5/2)/(b\*log(c) + b\*log(x^n) + a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx)^{3/2}}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a + b\*log(c\*x^n)), x)

[Out] int((d\*x)^(3/2)/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral((d\*x)\*\*(3/2)/(a + b\*log(c\*x\*\*n)), x)

$$3.103 \quad \int \frac{\sqrt{dx}}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=64

$$\frac{(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2n}} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

[Out]  $(d*x)^{(3/2)}*Ei(3/2*(a+b*\ln(c*x^n))/b/n)/b/d/\exp(3/2*a/b/n)/n/((c*x^n)^{(3/2/n)})$

**Rubi [A]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2178}

$$\frac{(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2n}} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]/(a + b\*Log[c\*x^n]),x]

[Out]  $((d*x)^{(3/2)}*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(2*b*n)])/(b*d*E^{((3*a)/(2*b*n))}*n*(c*x^n)^{(3/(2*n))})$

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{\sqrt{dx}}{a + b \log(cx^n)} dx = \frac{\left( (dx)^{3/2} (cx^n)^{-\frac{3}{2}/n} \right) \text{Subst} \left( \int \frac{e^{\frac{3x}{2n}}}{a+bx} dx, x, \log(cx^n) \right)}{dn}$$

$$= \frac{e^{-\frac{3a}{2bn}} (dx)^{3/2} (cx^n)^{-\frac{3}{2}/n} \text{Ei} \left( \frac{3(a+b \log(cx^n))}{2bn} \right)}{bdn}$$

**Mathematica** [A] time = 0.08, size = 62, normalized size = 0.97

$$\frac{x \sqrt{dx} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \text{Ei} \left( \frac{3(a+b \log(cx^n))}{2bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]/(a + b\*Log[c\*x^n]), x]

[Out] (x\*Sqrt[d\*x]\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*E^((3\*a)/(2\*b\*n))\*n\*(c\*x^n)^(3/(2\*n)))

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx}}{b \log(cx^n) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*log(c\*x^n) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n)), x, algorithm="giac")

[Out] integrate(sqrt(d\*x)/(b\*log(c\*x^n) + a), x)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{b \ln(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(b\*ln(c\*x^n)+a), x)

[Out] int((d\*x)^(1/2)/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2b\sqrt{d}n \int \frac{\sqrt{x}}{3(b^2 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2 + 2(b^2 \log(c) + ab) \log(x^n))} dx + \frac{2\sqrt{d}x^{\frac{3}{2}}}{3(b \log(c) + b \log(x^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*sqrt(d)\*n\*integrate(1/3\*sqrt(x)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n)), x) + 2/3\*sqrt(d)\*x^(3/2)/(b\*log(c) + b\*log(x^n) + a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{dx}}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a + b\*log(c\*x^n)), x)

[Out] int((d\*x)^(1/2)/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral(sqrt(d\*x)/(a + b\*log(c\*x\*\*n)), x)

$$3.104 \quad \int \frac{1}{\sqrt{dx} (a+b \log(cx^n))} dx$$

Optimal. Leaf size=64

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2}/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bdn}$$

[Out] Ei(1/2\*(a+b\*ln(c\*x^n))/b/n)\*(d\*x)^(1/2)/b/d/exp(1/2\*a/b/n)/n/((c\*x^n)^(1/2/n))

**Rubi [A]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2178}

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2}/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])),x]

[Out] (Sqrt[d\*x]\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*d\*E^(a/(2\*b\*n))\*n\*(c\*x^n)^(1/(2\*n)))

Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{1}{\sqrt{dx} (a + b \log(cx^n))} dx = \frac{\left(\sqrt{dx} (cx^n)^{-\frac{1}{2n}}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{dn}$$

$$= \frac{e^{-\frac{a}{2bn}} \sqrt{dx} (cx^n)^{-\frac{1}{2n}} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bdn}$$

**Mathematica** [A] time = 0.07, size = 62, normalized size = 0.97

$$\frac{xe^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2n}} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bn\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])),x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*E^(a/(2\*b\*n))\*n\*Sqrt[d\*x]\*(c\*x^n)^(1/(2\*n)))

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{bdx \log(cx^n) + adx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d\*x\*log(c\*x^n) + a\*d\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(d\*x)\*(b\*log(c\*x^n) + a)),x, algorithm="giac")

[Out] integrate(1/(sqrt(d\*x)\*(b\*log(c\*x^n) + a)), x)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (b \ln(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(1/2)/(b\*ln(c\*x^n)+a), x)

[Out] int(1/(d\*x)^(1/2)/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2^{bn} \int \frac{1}{(b^2 \sqrt{d} \log(c)^2 + b^2 \sqrt{d} \log(x^n)^2 + 2ab\sqrt{d} \log(c) + a^2 \sqrt{d} + 2(b^2 \sqrt{d} \log(c) + ab\sqrt{d}) \log(x^n)) \sqrt{x}} dx + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*n\*integrate(1/((b^2\*sqrt(d)\*log(c)^2 + b^2\*sqrt(d)\*log(x^n)^2 + 2\*a\*b\*sqrt(d)\*log(c) + a^2\*sqrt(d) + 2\*(b^2\*sqrt(d)\*log(c) + a\*b\*sqrt(d))\*log(x^n))\*sqrt(x)), x) + 2\*sqrt(x)/(b\*sqrt(d)\*log(c) + b\*sqrt(d)\*log(x^n) + a\*sqrt(d))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{dx} (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d\*x)^(1/2)\*(a + b\*log(c\*x^n))), x)

[Out] int(1/((d\*x)^(1/2)\*(a + b\*log(c\*x^n))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(1/2)/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral(1/(sqrt(d\*x)\*(a + b\*log(c\*x\*\*n))), x)

$$3.105 \quad \int \frac{1}{(dx)^{3/2}(a+b \log(cx^n))} dx$$

Optimal. Leaf size=67

$$\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(\frac{-a-b \log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

[Out]  $\exp(1/2*a/b/n)*(c*x^n)^{(1/2/n)}*\operatorname{Ei}(1/2*(-a-b*\ln(c*x^n))/b/n)/b/d/n/(d*x)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((d*x)^{(3/2)}*(a + b*\operatorname{Log}[c*x^n])), x]$

[Out]  $(E^{(a/(2*b*n))}*(c*x^n)^{(1/(2*n))}*\operatorname{ExpIntegralEi}[-(a + b*\operatorname{Log}[c*x^n])/(2*b*n)])/(b*d*n*\operatorname{Sqrt}[d*x])$

Rule 2178

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d))}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ \! \$UseGamma == True$

Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m + 1)}/(d*n*(c*x^n)^{(m + 1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m + 1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

Rubi steps



$$\int \frac{1}{(dx)^{3/2} (a + b \log(cx^n))} dx = \frac{(cx^n)^{1/2n} \text{Subst} \left( \int \frac{e^{-\frac{x}{2n}}}{a+bx} dx, x, \log(cx^n) \right)}{dn\sqrt{dx}}$$

$$= \frac{e^{\frac{a}{2bn}} (cx^n)^{1/2n} \text{Ei} \left( -\frac{a+b \log(cx^n)}{2bn} \right)}{bdn\sqrt{dx}}$$

**Mathematica** [A] time = 0.08, size = 62, normalized size = 0.93

$$\frac{xe^{\frac{a}{2bn}} (cx^n)^{1/2n} \text{Ei} \left( -\frac{a+b \log(cx^n)}{2bn} \right)}{bn(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(3/2)\*(a + b\*Log[c\*x^n])),x]

[Out] (E^(a/(2\*b\*n))\*x\*(c\*x^n)^(1/(2\*n))\*ExpIntegralEi[-1/2\*(a + b\*Log[c\*x^n])/(b\*n)])/(b\*n\*(d\*x)^(3/2))

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx}}{bd^2x^2 \log(cx^n) + ad^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d^2\*x^2\*log(c\*x^n) + a\*d^2\*x^2), x)

**giac** [A] time = 0.39, size = 49, normalized size = 0.73

$$\frac{c^{\frac{1}{2n}} \text{Ei} \left( -\frac{\log(c)}{2n} - \frac{a}{2bn} - \frac{1}{2} \log(x) \right) e^{\left( \frac{a}{2bn} \right)}}{bd^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] c^(1/2/n)\*Ei(-1/2\*log(c)/n - 1/2\*a/(b\*n) - 1/2\*log(x))\*e^(1/2\*a/(b\*n))/(b\*d^(3/2)\*n)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (b \ln(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(3/2)/(b\*ln(c\*x^n)+a),x)

[Out] int(1/(d\*x)^(3/2)/(b\*ln(c\*x^n)+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2bn \int \frac{1}{\left(b^2 d^{\frac{3}{2}} \log(c)^2 + b^2 d^{\frac{3}{2}} \log(x^n)^2 + 2abd^{\frac{3}{2}} \log(c) + a^2 d^{\frac{3}{2}} + 2\left(b^2 d^{\frac{3}{2}} \log(c) + abd^{\frac{3}{2}}\right) \log(x^n)\right) x^{\frac{3}{2}}} dx - \frac{1}{\left(bd^{\frac{3}{2}} \log(c) + bd^{\frac{3}{2}} \log(x^n) + a d^{\frac{3}{2}}\right) \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -2\*b\*n\*integrate(1/((b^2\*d^(3/2)\*log(c)^2 + b^2\*d^(3/2)\*log(x^n)^2 + 2\*a\*b\*d^(3/2)\*log(c) + a^2\*d^(3/2) + 2\*(b^2\*d^(3/2)\*log(c) + a\*b\*d^(3/2))\*log(x^n))\*x^(3/2)), x) - 2/((b\*d^(3/2)\*log(c) + b\*d^(3/2)\*log(x^n) + a\*d^(3/2))\*sqrt(x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d\*x)^(3/2)\*(a + b\*log(c\*x^n))),x)

[Out] int(1/((d\*x)^(3/2)\*(a + b\*log(c\*x^n))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/((d\*x)\*\*(3/2)\*(a + b\*log(c\*x\*\*n))), x)

$$3.106 \quad \int \frac{1}{(dx)^{5/2}(a+b \log(cx^n))} dx$$

Optimal. Leaf size=64

$$\frac{e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

[Out]  $\exp(3/2*a/b/n)*(c*x^n)^{(3/2/n)}*\operatorname{Ei}(-3/2*(a+b*\ln(c*x^n))/b/n)/b/d/n/(d*x)^{(3/2)}$

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((d*x)^{(5/2)}*(a + b*\operatorname{Log}[c*x^n])), x]$

[Out]  $(E^{((3*a)/(2*b*n))}*(c*x^n)^{(3/(2*n))}*\operatorname{ExpIntegralEi}[(-3*(a + b*\operatorname{Log}[c*x^n]))/(2*b*n)])/(b*d*n*(d*x)^{(3/2)})$

Rule 2178

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d))}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}*(b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m + 1)}/(d*n*(c*x^n)^{((m + 1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m + 1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{1}{(dx)^{5/2} (a + b \log(cx^n))} dx = \frac{(cx^n)^{\frac{3}{2}/n} \text{Subst} \left( \int \frac{e^{-\frac{3x}{2n}}}{a+bx} dx, x, \log(cx^n) \right)}{dn(dx)^{3/2}}$$

$$= \frac{e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2}/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{2bn} \right)}{bdn(dx)^{3/2}}$$

**Mathematica** [A] time = 0.08, size = 62, normalized size = 0.97

$$\frac{x e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2}/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{2bn} \right)}{bn(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(5/2)\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((3\*a)/(2\*b\*n))\*x\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n])/(2\*b\*n))]/(b\*n\*(d\*x)^(5/2))

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{dx}}{bd^3x^3 \log(cx^n) + ad^3x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d^3\*x^3\*log(c\*x^n) + a\*d^3\*x^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} (b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((d\*x)^(5/2)\*(b\*log(c\*x^n) + a)), x)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} (b \ln(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(5/2)/(b\*ln(c\*x^n)+a), x)

[Out] int(1/(d\*x)^(5/2)/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2bn \int \frac{1}{3 \left( b^2 d^{\frac{5}{2}} \log(c)^2 + b^2 d^{\frac{5}{2}} \log(x^n)^2 + 2abd^{\frac{5}{2}} \log(c) + a^2 d^{\frac{5}{2}} + 2 \left( b^2 d^{\frac{5}{2}} \log(c) + abd^{\frac{5}{2}} \right) \log(x^n) \right) x^{\frac{5}{2}}} dx - \frac{1}{3 \left( b^2 d^{\frac{5}{2}} \log(c)^2 + b^2 d^{\frac{5}{2}} \log(x^n)^2 + 2abd^{\frac{5}{2}} \log(c) + a^2 d^{\frac{5}{2}} + 2 \left( b^2 d^{\frac{5}{2}} \log(c) + abd^{\frac{5}{2}} \right) \log(x^n) \right) x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] -2\*b\*n\*integrate(1/3/((b^2\*d^(5/2)\*log(c)^2 + b^2\*d^(5/2)\*log(x^n)^2 + 2\*a\*b\*d^(5/2)\*log(c) + a^2\*d^(5/2) + 2\*(b^2\*d^(5/2)\*log(c) + a\*b\*d^(5/2))\*log(x^n))\*x^(5/2)), x) - 2/3/((b\*d^(5/2)\*log(c) + b\*d^(5/2)\*log(x^n) + a\*d^(5/2))\*x^(3/2))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(dx)^{\frac{5}{2}} (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d\*x)^(5/2)\*(a + b\*log(c\*x^n))), x)

[Out] int(1/((d\*x)^(5/2)\*(a + b\*log(c\*x^n))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(5/2)/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral(1/((d\*x)\*\*(5/2)\*(a + b\*log(c\*x\*\*n))), x)

$$3.107 \quad \int \frac{(dx)^{5/2}}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{7(dx)^{7/2}e^{-\frac{7a}{2bn}}(cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{7/2}}{bdn(a+b \log(cx^n))}$$

[Out]  $7/2*(d*x)^{(7/2)}*Ei(7/2*(a+b*\ln(c*x^n))/b/n)/b^2/d/\exp(7/2*a/b/n)/n^2/((c*x^n)^{(7/2/n)})-(d*x)^{(7/2)}/b/d/n/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2306, 2310, 2178}

$$\frac{7(dx)^{7/2}e^{-\frac{7a}{2bn}}(cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{7/2}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] `Int[(d*x)^(5/2)/(a + b*Log[c*x^n])^2,x]`

[Out]  $(7*(d*x)^{(7/2)}*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(2*b^2*d*E^((7*a)/(2*b*n))*n^2*(c*x^n)^{(7/(2*n))}) - (d*x)^{(7/2)}/(b*d*n*(a + b*Log[c*x^n]))$

#### Rule 2178

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True`

#### Rule 2306

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[(((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

#### Rule 2310

`Int[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)`

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(dx)^{5/2}}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{7/2}}{bdn(a + b \log(cx^n))} + \frac{7 \int \frac{(dx)^{5/2}}{a+b \log(cx^n)} dx}{2bn} \\ &= -\frac{(dx)^{7/2}}{bdn(a + b \log(cx^n))} + \frac{\left(7(dx)^{7/2} (cx^n)^{-7/2/n}\right) \text{Subst}\left(\int \frac{e^{7x/2n}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{7e^{-\frac{7a}{2bn}} (dx)^{7/2} (cx^n)^{-7/2/n} \text{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{7/2}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 84, normalized size = 0.86

$$\frac{x(dx)^{5/2} \left(7e^{-\frac{7a}{2bn}} (cx^n)^{-7/2/n} \text{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)}\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)/(a + b\*Log[c\*x^n])^2,x]

[Out] (x\*(d\*x)^(5/2)\*((7\*ExpIntegralEi[(7\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(E^((7\*a)/(2\*b\*n))\*(c\*x^n)^(7/(2\*n))) - (2\*b\*n)/(a + b\*Log[c\*x^n])))/(2\*b^2\*n^2)

**fricas [F]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx} d^2x^2}{b^2 \log(cx^n)^2 + 2ab \log(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d^2\*x^2/(b^2\*log(c\*x^n)^2 + 2\*a\*b\*log(c\*x^n) + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{5/2}}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)/(b\*log(c\*x^n) + a)^2, x)

**maple** [F] time = 5.10, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{(b \ln(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)/(b\*ln(c\*x^n)+a)^2,x)

[Out] int((d\*x)^(5/2)/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4bd^{\frac{5}{2}}n \int \frac{x^{\frac{5}{2}}}{7(b^3 \log(c)^3 + b^3 \log(x^n)^3 + 3ab^2 \log(c)^2 + 3a^2b \log(c) + a^3 + 3(b^3 \log(c) + ab^2) \log(x^n)^2 + 3(b^3 \log(c)^2 + 2ab^2 \log(c) + a^2b) \log(x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 4\*b\*d^(5/2)\*n\*integrate(1/7\*x^(5/2)/(b^3\*log(c)^3 + b^3\*log(x^n)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3 + 3\*(b^3\*log(c) + a\*b^2)\*log(x^n)^2 + 3\*(b^3\*log(c)^2 + 2\*a\*b^2\*log(c) + a^2\*b)\*log(x^n)), x) + 2/7\*d^(5/2)\*x^(7/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx)^{5/2}}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)/(a + b\*log(c\*x^n))^2,x)

[Out] int((d\*x)^(5/2)/(a + b\*log(c\*x^n))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{(a + b \log(cx^n))^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*log(c*x**n))**2, x)
```

$$3.108 \quad \int \frac{(dx)^{3/2}}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{5(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{5/2}}{bdn(a+b \log(cx^n))}$$

[Out]  $5/2*(d*x)^{(5/2)*\operatorname{Ei}(5/2*(a+b*\ln(c*x^n))/b/n)/b^2/d/\exp(5/2*a/b/n)/n^2/((c*x^n)^{(5/2/n))}-(d*x)^{(5/2)/b/d/n/(a+b*\ln(c*x^n))}$

**Rubi [A]** time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2306, 2310, 2178}

$$\frac{5(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{5/2}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] `Int[(d*x)^(3/2)/(a + b*Log[c*x^n])^2,x]`

[Out]  $(5*(d*x)^{(5/2)*\operatorname{ExpIntegralEi}[(5*(a + b*\operatorname{Log}[c*x^n]))/(2*b*n)])/(2*b^2*d*\operatorname{E}((5*a)/(2*b*n))*n^2*(c*x^n)^{(5/(2*n))}) - (d*x)^{(5/2)/(b*d*n*(a + b*\operatorname{Log}[c*x^n])})$

**Rule 2178**

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True`

**Rule 2306**

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

**Rule 2310**

`Int[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x)`

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(dx)^{3/2}}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{5/2}}{bdn(a + b \log(cx^n))} + \frac{5 \int \frac{(dx)^{3/2}}{a+b \log(cx^n)} dx}{2bn} \\ &= -\frac{(dx)^{5/2}}{bdn(a + b \log(cx^n))} + \frac{\left(5(dx)^{5/2} (cx^n)^{-5/2/n}\right) \text{Subst}\left(\int \frac{e^{5x/2n}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{5e^{-\frac{5a}{2bn}} (dx)^{5/2} (cx^n)^{-5/2/n} \text{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{5/2}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 84, normalized size = 0.86

$$\frac{x(dx)^{3/2} \left(5e^{-\frac{5a}{2bn}} (cx^n)^{-5/2/n} \text{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)}\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)/(a + b\*Log[c\*x^n])^2,x]

[Out] (x\*(d\*x)^(3/2)\*((5\*ExpIntegralEi[(5\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(E^((5\*a)/(2\*b\*n))\*(c\*x^n)^(5/(2\*n)))) - (2\*b\*n)/(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2)

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx} dx}{b^2 \log(cx^n)^2 + 2ab \log(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d\*x/(b^2\*log(c\*x^n)^2 + 2\*a\*b\*log(c\*x^n) + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)/(b\*log(c\*x^n) + a)^2, x)

**maple** [F] time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(b \ln(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(b\*ln(c\*x^n)+a)^2,x)

[Out] int((d\*x)^(3/2)/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4bd^{\frac{3}{2}}n \int \frac{x^{\frac{3}{2}}}{5(b^3 \log(c)^3 + b^3 \log(x^n)^3 + 3ab^2 \log(c)^2 + 3a^2b \log(c) + a^3 + 3(b^3 \log(c) + ab^2) \log(x^n)^2 + 3(b^3 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2) \log(x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 4\*b\*d^(3/2)\*n\*integrate(1/5\*x^(3/2)/(b^3\*log(c)^3 + b^3\*log(x^n)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3 + 3\*(b^3\*log(c) + a\*b^2)\*log(x^n)^2 + 3\*(b^3\*log(c)^2 + 2\*a\*b^2\*log(c) + a^2\*b)\*log(x^n)), x) + 2/5\*d^(3/2)\*x^(5/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx)^{3/2}}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a + b\*log(c\*x^n))^2,x)

[Out] int((d\*x)^(3/2)/(a + b\*log(c\*x^n))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*log(c*x**n))**2, x)
```

$$3.109 \quad \int \frac{\sqrt{dx}}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{3(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2n}} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{3/2}}{bdn(a+b \log(cx^n))}$$

[Out]  $3/2*(d*x)^{(3/2)}*Ei(3/2*(a+b*\ln(c*x^n))/b/n)/b^2/d/\exp(3/2*a/b/n)/n^2/((c*x^n)^{(3/2/n)})-(d*x)^{(3/2)}/b/d/n/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2306, 2310, 2178}

$$\frac{3(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2n}} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{3/2}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[d*x]/(a + b*Log[c*x^n])^2, x]`

[Out]  $(3*(d*x)^{(3/2)}*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(2*b*n)])/(2*b^2*d*E^((3*a)/(2*b*n))*n^2*(c*x^n)^{(3/(2*n))}) - (d*x)^{(3/2)}/(b*d*n*(a + b*Log[c*x^n]))$

**Rule 2178**

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True`

**Rule 2306**

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[(((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]`

**Rule 2310**

`Int[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)`

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{dx}}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{3/2}}{bdn(a + b \log(cx^n))} + \frac{3 \int \frac{\sqrt{dx}}{a + b \log(cx^n)} dx}{2bn} \\ &= -\frac{(dx)^{3/2}}{bdn(a + b \log(cx^n))} + \frac{\left(3(dx)^{3/2} (cx^n)^{-\frac{3}{2}/n}\right) \text{Subst}\left(\int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{3e^{-\frac{3a}{2bn}} (dx)^{3/2} (cx^n)^{-\frac{3}{2}/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{3/2}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica** [A] time = 0.17, size = 84, normalized size = 0.86

$$\frac{x\sqrt{dx} \left(3e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)}\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]/(a + b\*Log[c\*x^n])^2, x]

[Out] (x\*Sqrt[d\*x]\*((3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))]/(2\*b\*n)))/(E^((3\*a)/(2\*b\*n))\*(c\*x^n)^(3/(2\*n))) - (2\*b\*n)/(a + b\*Log[c\*x^n])))/(2\*b^2\*n^2)

**fricas** [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2 \log(cx^n)^2 + 2ab \log(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n))^2, x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*log(c\*x^n)^2 + 2\*a\*b\*log(c\*x^n) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(sqrt(d\*x)/(b\*log(c\*x^n) + a)^2, x)

maple [F] time = 5.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(b \ln(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(b\*ln(c\*x^n)+a)^2,x)

[Out] int((d\*x)^(1/2)/(b\*ln(c\*x^n)+a)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4b\sqrt{d}n \int \frac{\sqrt{x}}{3(b^3 \log(c)^3 + b^3 \log(x^n)^3 + 3ab^2 \log(c)^2 + 3a^2b \log(c) + a^3 + 3(b^3 \log(c) + ab^2) \log(x^n)^2 + 3(b^3 \log(c)^2 + 2ab^2 \log(c) + a^2b) \log(x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 4\*b\*sqrt(d)\*n\*integrate(1/3\*sqrt(x)/(b^3\*log(c)^3 + b^3\*log(x^n)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3 + 3\*(b^3\*log(c) + a\*b^2)\*log(x^n)^2 + 3\*(b^3\*log(c)^2 + 2\*a\*b^2\*log(c) + a^2\*b)\*log(x^n)), x) + 2/3\*sqrt(d)\*x^(3/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{dx}}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a + b\*log(c\*x^n))^2,x)

[Out] int((d\*x)^(1/2)/(a + b\*log(c\*x^n))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(a + b \log(cx^n))^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*log(c*x**n))**2, x)
```

$$3.110 \quad \int \frac{1}{\sqrt{dx} (a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2n}} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2dn^2} - \frac{\sqrt{dx}}{bdn (a+b \log(cx^n))}$$

[Out]  $1/2*\operatorname{Ei}(1/2*(a+b*\ln(c*x^n))/b/n)*(d*x)^{(1/2)}/b^2/d/\exp(1/2*a/b/n)/n^2/((c*x^n)^{(1/2/n)})-(d*x)^{(1/2)}/b/d/n/(a+b*\ln(c*x^n))$

**Rubi [A]** time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2306, 2310, 2178}

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2n}} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2dn^2} - \frac{\sqrt{dx}}{bdn (a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2), x]

[Out] (Sqrt[d\*x]\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(2\*b^2\*d\*E^(a/(2\*b\*n))\*n^2\*(c\*x^n)^(1/(2\*n))) - Sqrt[d\*x]/(b\*d\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2178

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)

$/n)*(a + b*x)^p, x], x, \text{Log}[c*x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{dx} (a + b \log(cx^n))^2} dx &= -\frac{\sqrt{dx}}{bdn (a + b \log(cx^n))} + \frac{\int \frac{1}{\sqrt{dx} (a + b \log(cx^n))} dx}{2bn} \\ &= -\frac{\sqrt{dx}}{bdn (a + b \log(cx^n))} + \frac{\left(\sqrt{dx} (cx^n)^{-\frac{1}{2/n}}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{e^{-\frac{a}{2bn}} \sqrt{dx} (cx^n)^{-\frac{1}{2/n}} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2dn^2} - \frac{\sqrt{dx}}{bdn (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 83, normalized size = 0.85

$$\frac{x \left( e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2/n}} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)} \right)}{2b^2n^2 \sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2), x]

[Out] (x\*(ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)]/(E^(a/(2\*b\*n))\*(c\*x^n)^(1/(2\*n)))) - (2\*b\*n)/(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2\*Sqrt[d\*x])

**fricas [F]** time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2 dx \log(cx^n)^2 + 2 ab dx \log(cx^n) + a^2 dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n))^2, x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d\*x\*log(c\*x^n)^2 + 2\*a\*b\*d\*x\*log(c\*x^n) + a^2\*d\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(1/(sqrt(d\*x)\*(b\*log(c\*x^n) + a)^2), x)

**maple** [C] time = 2.00, size = 427, normalized size = 4.36

$$\frac{2x}{\sqrt{dx} \left( -i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ie^{n \ln(x)}) \operatorname{csgn}(ic e^{n \ln(x)}) + i\pi b \operatorname{csgn}(ic) \operatorname{csgn}(ic e^{n \ln(x)})^2 + i\pi b \operatorname{csgn}(ie^{n \ln(x)}) \operatorname{csgn}(ic) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(1/2)/(b\*ln(c\*x^n)+a)^2,x)

[Out]  $-2/b/n*x/(d*x)^{(1/2)}/(2*a+2*b*\ln(c)+2*b*\ln(\exp(n*\ln(x))))+I*b*Pi*\operatorname{csgn}(I*\exp(n*\ln(x)))*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^2-I*b*Pi*\operatorname{csgn}(I*\exp(n*\ln(x)))*\operatorname{csgn}(I*c*\exp(n*\ln(x)))*\operatorname{csgn}(I*c)-I*b*Pi*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^3+I*b*Pi*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^2*\operatorname{csgn}(I*c)-1/2/d/b^2/n^2*\exp(1/4*I*(b*Pi*\operatorname{csgn}(I*\exp(n*\ln(x)))*\operatorname{csgn}(I*c*\exp(n*\ln(x)))*\operatorname{csgn}(I*c)-b*Pi*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^2*\operatorname{csgn}(I*c)-b*Pi*\operatorname{csgn}(I*\exp(n*\ln(x)))*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^2+b*Pi*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^3+2*I*b*n*(\ln(x)-\ln(d*x))+2*I*b*\ln(c)+2*I*b*(\ln(\exp(n*\ln(x)))-n*\ln(x))+2*I*a)/b/n)*Ei(1,-1/2*\ln(d*x)+1/4*I*(b*Pi*\operatorname{csgn}(I*\exp(n*\ln(x)))*\operatorname{csgn}(I*c*\exp(n*\ln(x)))*\operatorname{csgn}(I*c)-b*Pi*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^2*\operatorname{csgn}(I*c)-b*Pi*\operatorname{csgn}(I*\exp(n*\ln(x)))*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^2+b*Pi*\operatorname{csgn}(I*c*\exp(n*\ln(x)))^3+2*I*b*n*(\ln(x)-\ln(d*x))+2*I*b*\ln(c)+2*I*b*(\ln(\exp(n*\ln(x)))-n*\ln(x))+2*I*a)/b/n)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4bn \int \frac{1}{(b^3\sqrt{d} \log(c)^3 + b^3\sqrt{d} \log(x^n)^3 + 3ab^2\sqrt{d} \log(c)^2 + 3a^2b\sqrt{d} \log(c) + a^3\sqrt{d} + 3(b^3\sqrt{d} \log(c) + ab^2\sqrt{d})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out]  $4*b*n*\operatorname{integrate}(1/((b^3*\sqrt{d})*\log(c)^3 + b^3*\sqrt{d})*\log(x^n)^3 + 3*a*b^2*\sqrt{d}*\log(c)^2 + 3*a^2*b*\sqrt{d}*\log(c) + a^3*\sqrt{d} + 3*(b^3*\sqrt{d})*1$

```
og(c) + a*b^2*sqrt(d))*log(x^n)^2 + 3*(b^3*sqrt(d)*log(c)^2 + 2*a*b^2*sqrt(
d)*log(c) + a^2*b*sqrt(d))*log(x^n))*sqrt(x)), x) + 2*sqrt(x)/(b^2*sqrt(d)*
log(c)^2 + b^2*sqrt(d)*log(x^n)^2 + 2*a*b*sqrt(d)*log(c) + a^2*sqrt(d) + 2*
(b^2*sqrt(d)*log(c) + a*b*sqrt(d))*log(x^n))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{dx} (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a + b*log(c*x^n))^2), x)
```

```
[Out] int(1/((d*x)^(1/2)*(a + b*log(c*x^n))^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx} (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(a+b*ln(c*x**n))**2, x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*log(c*x**n))**2), x)
```

$$3.111 \quad \int \frac{1}{(dx)^{3/2} (a+b \log(cx^n))^2} dx$$

Optimal. Leaf size=101

$$\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(\frac{-a-b \log(cx^n)}{2bn}\right)}{2b^2 dn^2 \sqrt{dx}} - \frac{1}{bdn \sqrt{dx} (a+b \log(cx^n))}$$

[Out]  $-1/2*\exp(1/2*a/b/n)*(c*x^n)^{(1/2/n)}*Ei(1/2*(-a-b*\ln(c*x^n))/b/n)/b^2/d/n^2/(d*x)^{(1/2)}-1/b/d/n/(a+b*\ln(c*x^n))/(d*x)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2306, 2310, 2178}

$$\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(\frac{-a+b \log(cx^n)}{2bn}\right)}{2b^2 dn^2 \sqrt{dx}} - \frac{1}{bdn \sqrt{dx} (a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/((d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2),x]

[Out]  $-(E^{a/(2*b*n)}*(c*x^n)^{(1/(2*n))}*ExpIntegralEi[-(a + b*Log[c*x^n])/(2*b*n)])/((2*b^2*d*n^2*sqrt[d*x]) - 1/(b*d*n*sqrt[d*x]*(a + b*Log[c*x^n])))$

### Rule 2178

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)

$/n)*(a + b*x)^p, x], x, \text{Log}[c*x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{(dx)^{3/2} (a + b \log(cx^n))^2} dx &= -\frac{1}{bdn\sqrt{dx} (a + b \log(cx^n))} - \frac{\int \frac{1}{(dx)^{3/2} (a+b \log(cx^n))} dx}{2bn} \\ &= -\frac{1}{bdn\sqrt{dx} (a + b \log(cx^n))} - \frac{(cx^n)^{\frac{1}{2}/n} \text{Subst}\left(\int \frac{e^{-x}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2\sqrt{dx}} \\ &= -\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2}/n} \text{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2dn^2\sqrt{dx}} - \frac{1}{bdn\sqrt{dx} (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 93, normalized size = 0.92

$$\frac{x \left( e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2}/n} (a + b \log(cx^n)) \text{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right) + 2bn \right)}{2b^2n^2(dx)^{3/2} (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2),x]

[Out] -1/2\*(x\*(2\*b\*n + E^(a/(2\*b\*n))\*(c\*x^n)^(1/(2\*n))\*ExpIntegralEi[-1/2\*(a + b\*Log[c\*x^n])/(b\*n)]\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]))

**fricas [F]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2d^2x^2 \log(cx^n)^2 + 2abd^2x^2 \log(cx^n) + a^2d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d^2\*x^2\*log(c\*x^n)^2 + 2\*a\*b\*d^2\*x^2\*log(c\*x^n) + a^2\*d^2\*x^2), x)





mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(dx)^{3/2} (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d\*x)^(3/2)\*(a + b\*log(c\*x^n))^2), x)

[Out] int(1/((d\*x)^(3/2)\*(a + b\*log(c\*x^n))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n))\*\*2, x)

[Out] Integral(1/((d\*x)\*\*(3/2)\*(a + b\*log(c\*x\*\*n))\*\*2), x)

$$3.112 \quad \int \frac{1}{(dx)^{5/2} (a + b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{3e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2 (dx)^{3/2}} - \frac{1}{bdn (dx)^{3/2} (a + b \log(cx^n))}$$

[Out]  $-3/2 * \exp(3/2 * a/b/n) * (c * x^n)^{(3/2)/n} * \operatorname{Ei}(-3/2 * (a + b * \ln(c * x^n))/b/n) / b^2/d/n^2 / (d * x)^{(3/2) - 1/b/d/n} / (d * x)^{(3/2)} / (a + b * \ln(c * x^n))$

**Rubi [A]** time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {2306, 2310, 2178}

$$\frac{3e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2 (dx)^{3/2}} - \frac{1}{bdn (dx)^{3/2} (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((d*x)^{(5/2)}*(a + b*\operatorname{Log}[c*x^n])^2), x]$

[Out]  $(-3 * E^{((3*a)/(2*b*n))} * (c*x^n)^{(3/(2*n))} * \operatorname{ExpIntegralEi}[(-3*(a + b*\operatorname{Log}[c*x^n]))/(2*b*n)]) / (2*b^2*d*n^2*(d*x)^{(3/2)}) - 1/(b*d*n*(d*x)^{(3/2)}*(a + b*\operatorname{Log}[c*x^n]))$

Rule 2178

$\operatorname{Int}[(F_)^((g_.) * ((e_.) + (f_.) * (x_)))) / ((c_.) + (d_.) * (x_)), x\_Symbol] := \operatorname{Simp}[(F^{(g*(e - (c*f)/d)}) * \operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$   
 $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \ \&\& \ \! \$UseGamma == True$

Rule 2306

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.) * (x_)^{(n_.)}] * (b_.)^{(p_.)} * ((d_.) * (x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)} * (a + b*\operatorname{Log}[c*x^n])^{(p+1)} / (b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m * (a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \operatorname{NeQ}[m, -1] \ \&\& \ \operatorname{LtQ}[p, -1]$

Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.) * (x_)^{(n_.)}] * (b_.)^{(p_.)} * ((d_.) * (x_))^{(m_.)}, x\_Symbol] := \operatorname{Dist}[(d*x)^{(m+1)} / (d*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x}$

/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(dx)^{5/2} (a + b \log(cx^n))^2} dx &= -\frac{1}{bdn(dx)^{3/2} (a + b \log(cx^n))} - \frac{3 \int \frac{1}{(dx)^{5/2} (a + b \log(cx^n))} dx}{2bn} \\ &= -\frac{1}{bdn(dx)^{3/2} (a + b \log(cx^n))} - \frac{\left(3 (cx^n)^{\frac{3}{2n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{3x}{2n}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2(dx)^{3/2}} \\ &= -\frac{3e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \text{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}} - \frac{1}{bdn(dx)^{3/2} (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 94, normalized size = 0.96

$$-\frac{x \left( 3e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} (a + b \log(cx^n)) \text{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right) + 2bn \right)}{2b^2n^2(dx)^{5/2} (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2),x]

[Out] -1/2\*(x\*(2\*b\*n + 3\*E^((3\*a)/(2\*b\*n))\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)]\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*(d\*x)^(5/2)\*(a + b\*Log[c\*x^n]))

**fricas [F]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2d^3x^3 \log(cx^n)^2 + 2abd^3x^3 \log(cx^n) + a^2d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d^3\*x^3\*log(c\*x^n)^2 + 2\*a\*b\*d^3\*x^3\*log(c\*x^n) + a^2\*d^3\*x^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} (b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x)^(5/2)\*(b\*log(c\*x^n) + a)^2), x)

**maple** [F] time = 5.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} (b \ln(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(5/2)/(b\*ln(c\*x^n)+a)^2,x)

[Out] int(1/(d\*x)^(5/2)/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-4bn \int \frac{1}{3 \left( b^3 d^{\frac{5}{2}} \log(c)^3 + b^3 d^{\frac{5}{2}} \log(x^n)^3 + 3ab^2 d^{\frac{5}{2}} \log(c)^2 + 3a^2 b d^{\frac{5}{2}} \log(c) + a^3 d^{\frac{5}{2}} + 3 \left( b^3 d^{\frac{5}{2}} \log(c) + ab^2 d^{\frac{5}{2}} \right) \log(x^n) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -4\*b\*n\*integrate(1/3/((b^3\*d^(5/2)\*log(c)^3 + b^3\*d^(5/2)\*log(x^n)^3 + 3\*a\*b^2\*d^(5/2)\*log(c)^2 + 3\*a^2\*b\*d^(5/2)\*log(c) + a^3\*d^(5/2) + 3\*(b^3\*d^(5/2)\*log(c) + a\*b^2\*d^(5/2))\*log(x^n))^2), x) - 2/3/((b^2\*d^(5/2)\*log(c)^2 + b^2\*d^(5/2)\*log(x^n)^2 + 2\*a\*b\*d^(5/2)\*log(c) + a^2\*d^(5/2) + 2\*(b^2\*d^(5/2)\*log(c) + a\*b\*d^(5/2))\*log(x^n))\*x^(3/2))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(dx)^{5/2} (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d*x)^(5/2)*(a + b*log(c*x^n))^2), x)`

[Out] `int(1/((d*x)^(5/2)*(a + b*log(c*x^n))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}} (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x)**(5/2)/(a+b*ln(c*x**n))**2, x)`

[Out] `Integral(1/((d*x)**(5/2)*(a + b*log(c*x**n))**2), x)`

### 3.113 $\int \sqrt{a + b \log(cx^n)} dx$

Optimal. Leaf size=85

$$x\sqrt{a + b \log(cx^n)} - \frac{1}{2}\sqrt{\pi} \sqrt{b} \sqrt{n} x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b} \sqrt{n}}\right)$$

[Out]  $-1/2*x*\operatorname{erfi}((a+b*\ln(c*x^n))^{(1/2)}/b^{(1/2)}/n^{(1/2)})*b^{(1/2)}*n^{(1/2)}*\pi^{(1/2)}/\exp(a/b/n)/((c*x^n)^{(1/n)})+x*(a+b*\ln(c*x^n))^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2296, 2300, 2180, 2204}

$$x\sqrt{a + b \log(cx^n)} - \frac{1}{2}\sqrt{\pi} \sqrt{b} \sqrt{n} x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b} \sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*Log[c*x^n]], x]`

[Out]  $-(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi]*x*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]]]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[n]))/(2*E^{(a/(b*n))}*(c*x^n)^{n(-1)} + x*\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]])$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`  
`> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /;`  
`FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=`  
`Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /;`  
`FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2296

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :=`  
`Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;`  
`FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]`

#### Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] := Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + b \log(cx^n)} dx &= x\sqrt{a + b \log(cx^n)} - \frac{1}{2}(bn) \int \frac{1}{\sqrt{a + b \log(cx^n)}} dx \\ &= x\sqrt{a + b \log(cx^n)} - \frac{1}{2}(bx(cx^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{a + bx}} dx, x, \log(cx^n)\right) \\ &= x\sqrt{a + b \log(cx^n)} - (x(cx^n)^{-1/n}) \text{Subst}\left(\int e^{-\frac{a}{bn} + \frac{x^2}{bn}} dx, x, \sqrt{a + b \log(cx^n)}\right) \\ &= -\frac{1}{2}\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n} \text{erfi}\left(\frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + x\sqrt{a + b \log(cx^n)} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 85, normalized size = 1.00

$$x\sqrt{a + b \log(cx^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n} \text{erfi}\left(\frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Log[c*x^n]], x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[a + b*Log[c*x^n]]]/(Sqrt[b]*Sqrt[
n]))/(E^(a/(b*n))*(c*x^n)^n^(-1)) + x*Sqrt[a + b*Log[c*x^n]]
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^(1/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*log(c\*x^n) + a), x)

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \sqrt{b \ln(c x^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^n)+a)^(1/2),x)

[Out] int((b\*ln(c\*x^n)+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*log(c\*x^n) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \ln(c x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^(1/2),x)

[Out] int((a + b\*log(c\*x^n))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*log(c\*x\*\*n)), x)



### 3.114 $\int x^3 \sqrt{\log(ax^n)} dx$

Optimal. Leaf size=64

$$\frac{1}{4}x^4\sqrt{\log(ax^n)} - \frac{1}{16}\sqrt{\pi}\sqrt{n}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-1/16*x^4*\operatorname{erfi}(2*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*n^{(1/2)}*\pi^{(1/2)}/((a*x^n)^{(4/n)})+1/4*x^4*\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{4}x^4\sqrt{\log(ax^n)} - \frac{1}{16}\sqrt{\pi}\sqrt{n}x^4(ax^n)^{-4/n}\operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In] `Int[x^3*Sqrt[Log[a*x^n]],x]`

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(16*(a*x^n)^{(4/n)})+(x^4*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/4$

#### Rule 2180

`Int[(F_)^((g_.)*(e_.)+(f_.)*(x_))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e-(c*f)/d)+(f*g*x^2)/d), x], x, Sqrt[c+d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True`

#### Rule 2204

`Int[(F_)^((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c+d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2305

`Int[((a_.)+Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[((d*x)^(m+1)*(a+b*Log[c*x^n])^p)/(d*(m+1)), x] - Dist[(b*n*p)/(m+1), Int[(d*x)^m*(a+b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]`

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int x^3 \sqrt{\log(ax^n)} dx &= \frac{1}{4} x^4 \sqrt{\log(ax^n)} - \frac{1}{8} n \int \frac{x^3}{\sqrt{\log(ax^n)}} dx \\ &= \frac{1}{4} x^4 \sqrt{\log(ax^n)} - \frac{1}{8} \left( x^4 (ax^n)^{-4/n} \right) \text{Subst} \left( \int \frac{e^{\frac{4x}{n}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\ &= \frac{1}{4} x^4 \sqrt{\log(ax^n)} - \frac{1}{4} \left( x^4 (ax^n)^{-4/n} \right) \text{Subst} \left( \int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\ &= -\frac{1}{16} \sqrt{n} \sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + \frac{1}{4} x^4 \sqrt{\log(ax^n)} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 61, normalized size = 0.95

$$\frac{1}{16} x^4 \left( 4\sqrt{\log(ax^n)} - \sqrt{\pi} \sqrt{n} (ax^n)^{-4/n} \operatorname{erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[Log[a*x^n]],x]
```

```
[Out] (x^4*(-((Sqrt[n]*Sqrt[Pi]*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(4/n)
)+ 4*Sqrt[Log[a*x^n]]))/16
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x^3\*sqrt(log(a\*x^n)), x)

**maple** [F] time = 0.65, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(a\*x^n)^(1/2),x)

[Out] int(x^3\*ln(a\*x^n)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3\*sqrt(log(a\*x^n)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*log(a\*x^n)^(1/2),x)

[Out] int(x^3\*log(a\*x^n)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(x\*\*3\*sqrt(log(a\*x\*\*n)), x)

### 3.115 $\int x^2 \sqrt{\log(ax^n)} dx$

Optimal. Leaf size=72

$$\frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{6}\sqrt{\frac{\pi}{3}}\sqrt{n}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-1/18*x^3*\operatorname{erfi}(3^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*n^{(1/2)}*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/((a*x^n)^{(3/n))+1/3*x^3*\ln(a*x^n)^{(1/2)})$

**Rubi [A]** time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{6}\sqrt{\frac{\pi}{3}}\sqrt{n}x^3(ax^n)^{-3/n}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]], x]$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}/3]*x^3*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(6*(a*x^n)^{(3/n)}) + (x^3*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/3$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :$   
 $> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!}\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

#### Rule 2305

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]]*(b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(m+1), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{GtQ}[p, 0]$

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol
] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)*x)
/n]*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int x^2 \sqrt{\log(ax^n)} dx &= \frac{1}{3} x^3 \sqrt{\log(ax^n)} - \frac{1}{6} n \int \frac{x^2}{\sqrt{\log(ax^n)}} dx \\ &= \frac{1}{3} x^3 \sqrt{\log(ax^n)} - \frac{1}{6} (x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int \frac{e^{\frac{3x}{n}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\ &= \frac{1}{3} x^3 \sqrt{\log(ax^n)} - \frac{1}{3} (x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\ &= -\frac{1}{6} \sqrt{n} \sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) + \frac{1}{3} x^3 \sqrt{\log(ax^n)} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 67, normalized size = 0.93

$$\frac{1}{18} x^3 \left( 6 \sqrt{\log(ax^n)} - \sqrt{3\pi} \sqrt{n} (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[Log[a*x^n]],x]
```

```
[Out] (x^3*(-((Sqrt[n]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^
n)^(3/n)) + 6*Sqrt[Log[a*x^n]]))/18
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(log(a*x^n)), x)
```

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a*x^n)^(1/2),x)
```

```
[Out] int(x^2*ln(a*x^n)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(log(a*x^n)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(a*x^n)^(1/2),x)
```

```
[Out] int(x^2*log(a*x^n)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(log(a*x**n)), x)
```

### 3.116 $\int x \sqrt{\log(ax^n)} dx$

Optimal. Leaf size=72

$$\frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{n}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-1/8*x^2*\operatorname{erfi}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*n^{(1/2)}*2^{(1/2)}*\pi^{(1/2)}/((a*x^n)^{(2/n))+1/2*x^2*\ln(a*x^n)^{(1/2)})$

**Rubi [A]** time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{n}x^2(ax^n)^{-2/n}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[Log[a*x^n]],x]`

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi/2]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(4*(a*x^n)^{(2/n)} + (x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]))/2$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e-(c*f)/d)+(f*g*x^2)/d), x], x, Sqrt[c+d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c+d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2305

`Int[((a_.)+Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[((d*x)^(m+1)*(a+b*Log[c*x^n])^p)/(d*(m+1)), x] - Dist[(b*n*p)/(m+1), Int[(d*x)^m*(a+b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]`

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int x\sqrt{\log(ax^n)} dx &= \frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}n \int \frac{x}{\sqrt{\log(ax^n)}} dx \\ &= \frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}(x^2(ax^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right) \\ &= \frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{2}(x^2(ax^n)^{-2/n}) \operatorname{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right) \\ &= -\frac{1}{4}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2}x^2\sqrt{\log(ax^n)} \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 67, normalized size = 0.93

$$\frac{1}{8}x^2\left(4\sqrt{\log(ax^n)} - \sqrt{2\pi}\sqrt{n}(ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Log[a*x^n]], x]
```

```
[Out] (x^2*((Sqrt[n]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(2/n)) + 4*Sqrt[Log[a*x^n]])/8
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x^n)^(1/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\log(ax^n)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(a*x^n)^(1/2),x, algorithm="giac")`

[Out] `integrate(x*sqrt(log(a*x^n)), x)`

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x\sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*ln(a*x^n)^(1/2),x)`

[Out] `int(x*ln(a*x^n)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(a*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x*sqrt(log(a*x^n)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*log(a*x^n)^(1/2),x)`

[Out] `int(x*log(a*x^n)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(a*x**n)**(1/2),x)`

[Out] `Integral(x*sqrt(log(a*x**n)), x)`

### 3.117 $\int \sqrt{\log(ax^n)} dx$

Optimal. Leaf size=56

$$x\sqrt{\log(ax^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{n}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-1/2*x*\operatorname{erfi}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*n^{(1/2)}*\pi^{(1/2)}/((a*x^n)^{(1/n)})+x*\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2296, 2300, 2180, 2204}

$$x\sqrt{\log(ax^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{n}x(ax^n)^{-1/n}\operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Log[a*x^n]], x]`

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi]*x*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(2*(a*x^n)^n)+x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`  
`> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2296

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p, x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]`

#### Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] := Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \sqrt{\log(ax^n)} dx &= x\sqrt{\log(ax^n)} - \frac{1}{2}n \int \frac{1}{\sqrt{\log(ax^n)}} dx \\
 &= x\sqrt{\log(ax^n)} - \frac{1}{2} \left( x(ax^n)^{-1/n} \right) \text{Subst} \left( \int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
 &= x\sqrt{\log(ax^n)} - \left( x(ax^n)^{-1/n} \right) \text{Subst} \left( \int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
 &= -\frac{1}{2}\sqrt{n} \sqrt{\pi} x(ax^n)^{-1/n} \operatorname{erfi} \left( \frac{\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + x\sqrt{\log(ax^n)}
 \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 56, normalized size = 1.00

$$x\sqrt{\log(ax^n)} - \frac{1}{2}\sqrt{\pi} \sqrt{n} x(ax^n)^{-1/n} \operatorname{erfi} \left( \frac{\sqrt{\log(ax^n)}}{\sqrt{n}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[a*x^n]], x]
```

```
[Out] -1/2*(Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(a*x^n)^n^(-1) + x
*Sqrt[Log[a*x^n]]
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(log(a*x^n)), x)
```

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*x^n)^(1/2),x)
```

```
[Out] int(ln(a*x^n)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(log(a*x^n)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*x^n)^(1/2),x)
```

```
[Out] int(log(a*x^n)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(log(a*x**n)), x)
```

$$3.118 \quad \int \frac{\sqrt{\log(ax^n)}}{x} dx$$

Optimal. Leaf size=17

$$\frac{2 \log^{\frac{3}{2}}(ax^n)}{3n}$$

[Out]  $2/3 * \ln(a * x^n)^{(3/2)} / n$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{2 \log^{\frac{3}{2}}(ax^n)}{3n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a\*x^n]]/x,x]

[Out] (2\*Log[a\*x^n]^(3/2))/(3\*n)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\log(ax^n)}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{x} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{2 \log^{\frac{3}{2}}(ax^n)}{3n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{2 \log^{\frac{3}{2}}(ax^n)}{3n}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Log[a\*x^n]]/x,x]

[Out] (2\*Log[a\*x^n]^(3/2))/(3\*n)

**fricas** [A] time = 0.45, size = 14, normalized size = 0.82

$$\frac{2 \left( n \log(x) + \log(a) \right)^{\frac{3}{2}}}{3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x,x, algorithm="fricas")

[Out] 2/3\*(n\*log(x) + log(a))^(3/2)/n

**giac** [A] time = 0.36, size = 14, normalized size = 0.82

$$\frac{2 \left( n \log(x) + \log(a) \right)^{\frac{3}{2}}}{3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x,x, algorithm="giac")

[Out] 2/3\*(n\*log(x) + log(a))^(3/2)/n

**maple** [A] time = 0.03, size = 14, normalized size = 0.82

$$\frac{2 \ln(a x^n)^{\frac{3}{2}}}{3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(1/2)/x,x)

[Out] 2/3\*ln(a\*x^n)^(3/2)/n

**maxima** [A] time = 0.64, size = 13, normalized size = 0.76

$$\frac{2 \log(ax^n)^{\frac{3}{2}}}{3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x,x, algorithm="maxima")

[Out] 2/3\*log(a\*x^n)^(3/2)/n

**mupad [B]** time = 3.54, size = 13, normalized size = 0.76

$$\frac{2 \ln(ax^n)^{3/2}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(1/2)/x,x)

[Out] (2\*log(a\*x^n)^(3/2))/(3\*n)

**sympy [A]** time = 1.17, size = 29, normalized size = 1.71

$$-\begin{cases} -\sqrt{\log(a)} \log(x) & \text{for } n = 0 \\ -\frac{2 \log(ax^n)^{3/2}}{3n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(1/2)/x,x)

[Out] -Piecewise((-sqrt(log(a))\*log(x), Eq(n, 0)), (-2\*log(a\*x\*\*n)\*\*(3/2)/(3\*n), True))

$$3.119 \quad \int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Optimal. Leaf size=59

$$\frac{\sqrt{\pi} \sqrt{n} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x} - \frac{\sqrt{\log(ax^n)}}{x}$$

[Out]  $1/2*(a*x^n)^{(1/n)*\operatorname{erf}(\ln(a*x^n)^{(1/2)/n^{(1/2)}})*n^{(1/2)*\pi^{(1/2)}/x-\ln(a*x^n)^{(1/2)}/x}$

**Rubi [A]** time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{\sqrt{\pi} \sqrt{n} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x} - \frac{\sqrt{\log(ax^n)}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a\*x^n]]/x^2,x]

[Out]  $(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi]*(a*x^n)^{n^{-1}}*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(2*x) - \operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/x$

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]



Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol]
  := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x)
/n]*(a + b*x)^p, x], x, Log[c*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\log(ax^n)}}{x^2} dx &= -\frac{\sqrt{\log(ax^n)}}{x} + \frac{1}{2^n} \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx \\ &= -\frac{\sqrt{\log(ax^n)}}{x} + \frac{(ax^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{2x} \\ &= -\frac{\sqrt{\log(ax^n)}}{x} + \frac{(ax^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{x} \\ &= \frac{\sqrt{n} \sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x} - \frac{\sqrt{\log(ax^n)}}{x} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 65, normalized size = 1.10

$$-\frac{2 \log(ax^n) + n (ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right)}{2x \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[a*x^n]]/x^2,x]
```

```
[Out] -1/2*(2*Log[a*x^n] + n*(a*x^n)^n^(-1)*Gamma[1/2, Log[a*x^n]/n]*Sqrt[Log[a*x^n]/n])/(x*Sqrt[Log[a*x^n]])
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(log(a\*x^n))/x^2, x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\ln(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(1/2)/x^2,x)

[Out] int(ln(a\*x^n)^(1/2)/x^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(log(a\*x^n))/x^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\ln(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(1/2)/x^2,x)

[Out] int(log(a\*x^n)^(1/2)/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*x**n)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(log(a*x**n))/x**2, x)
```

$$3.120 \quad \int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Optimal. Leaf size=72

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{n} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}$$

[Out] 1/8\*(a\*x^n)^(2/n)\*erf(2^(1/2)\*ln(a\*x^n)^(1/2)/n^(1/2))\*n^(1/2)\*2^(1/2)\*Pi^(1/2)/x^2-1/2\*ln(a\*x^n)^(1/2)/x^2

**Rubi [A]** time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{n} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a\*x^n]]/x^3,x]

[Out] (Sqrt[n]\*Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(4\*x^2) - Sqrt[Log[a\*x^n]]/(2\*x^2)

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol]
  :-> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\log(ax^n)}}{x^3} dx &= -\frac{\sqrt{\log(ax^n)}}{2x^2} + \frac{1}{4}n \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx \\
&= -\frac{\sqrt{\log(ax^n)}}{2x^2} + \frac{(ax^n)^{2/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{4x^2} \\
&= -\frac{\sqrt{\log(ax^n)}}{2x^2} + \frac{(ax^n)^{2/n} \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{2x^2} \\
&= \frac{\sqrt{n} \sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 73, normalized size = 1.01

$$-\frac{4 \log(ax^n) + \sqrt{2} n (ax^n)^{2/n} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{2 \log(ax^n)}{n}\right)}{8x^2 \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[a*x^n]]/x^3, x]
```

```
[Out] -1/8*(4*Log[a*x^n] + Sqrt[2]*n*(a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])/n]*Sqrt[Log[a*x^n]/n])/(x^2*Sqrt[Log[a*x^n]])
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x^3, x, algorithm="fricas")
```

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(log(a\*x^n))/x^3, x)

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\ln(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(1/2)/x^3,x)

[Out] int(ln(a\*x^n)^(1/2)/x^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(log(a\*x^n))/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\ln(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(1/2)/x^3,x)

[Out] int(log(a\*x^n)^(1/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(1/2)/x\*\*3, x)

[Out] Integral(sqrt(log(a\*x\*\*n))/x\*\*3, x)

### 3.121 $\int x^3 \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=82

$$\frac{3}{128} \sqrt{\pi} n^{3/2} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{4} x^4 \log^{\frac{3}{2}}(ax^n) - \frac{3}{32} n x^4 \sqrt{\log(ax^n)}$$

[Out]  $\frac{1}{4} x^4 \ln(a x^n)^{(3/2)} + \frac{3}{128} n^{(3/2)} x^4 \operatorname{erfi}\left(\frac{2 \ln(a x^n)^{(1/2)}}{n^{(1/2)}}\right) \sqrt{\pi} - \frac{3}{32} n x^4 \ln(a x^n)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{3}{128} \sqrt{\pi} n^{3/2} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{4} x^4 \log^{\frac{3}{2}}(ax^n) - \frac{3}{32} n x^4 \sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3 \operatorname{Log}[a x^n]^{(3/2)}, x]$

[Out]  $(3 n^{(3/2)} \sqrt{\pi} x^4 \operatorname{Erfi}[(2 \sqrt{\operatorname{Log}[a x^n]}) / \sqrt{n}]) / (128 (a x^n)^{(4/n)}) - (3 n x^4 \sqrt{\operatorname{Log}[a x^n]}) / 32 + (x^4 \operatorname{Log}[a x^n]^{(3/2)}) / 4$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))} / \sqrt{(c_.) + (d_.)*(x_)}], x\_Symbol] :$   
 $> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \sqrt{c + d*x}], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ \! \$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a \sqrt{\pi} \operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]) / (2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{PosQ}[b]$

#### Rule 2305

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}] * (b_.)]^{(p_.)} * ((d_.)*(x_))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)} * (a + b*\operatorname{Log}[c*x^n])^p / (d*(m+1)), x] - \operatorname{Dist}[(b^n * p) / (m+1), \operatorname{Int}[(d*x)^m * (a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\amp; \ \operatorname{NeQ}[m, -1] \ \&\amp; \ \operatorname{GtQ}[p, 0]$

#### Rule 2310



```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol]
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \log^{\frac{3}{2}}(ax^n) dx &= \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) - \frac{1}{8}(3n) \int x^3 \sqrt{\log(ax^n)} dx \\
&= -\frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) + \frac{1}{64}(3n^2) \int \frac{x^3}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) + \frac{1}{64}(3nx^4 (ax^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right) \\
&= -\frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) + \frac{1}{32}(3nx^4 (ax^n)^{-4/n}) \text{Subst}\left(\int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right) \\
&= \frac{3}{128}n^{3/2}\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 73, normalized size = 0.89

$$\frac{1}{128}x^4 \left( 3\sqrt{\pi} n^{3/2} (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + 4\sqrt{\log(ax^n)} (8 \log(ax^n) - 3n) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[a*x^n]^(3/2),x]
```

```
[Out] (x^4*((3*n^(3/2)*Sqrt[Pi]*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(4/n)
+ 4*Sqrt[Log[a*x^n]]*(-3*n + 8*Log[a*x^n]))) / 128
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^3\*log(a\*x^n)^(3/2), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^3 \ln(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(a\*x^n)^(3/2),x)

[Out] int(x^3\*ln(a\*x^n)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3\*log(a\*x^n)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \ln(ax^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*log(a\*x^n)^(3/2),x)

[Out] int(x^3\*log(a\*x^n)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x\*\*3\*log(a\*x\*\*n)\*\*(3/2), x)

### 3.122 $\int x^2 \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=90

$$\frac{1}{12} \sqrt{\frac{\pi}{3}} n^{3/2} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)}$$

[Out]  $\frac{1}{3} x^3 \ln(a x^n)^{(3/2)} + \frac{1}{36} n^{(3/2)} x^3 \operatorname{erfi}\left(\frac{3^{(1/2)} \ln(a x^n)^{(1/2)}}{n^{(1/2)}}\right) * 3^{(1/2)} * \pi^{(1/2)} / ((a x^n)^{(3/n)}) - \frac{1}{6} n x^3 \ln(a x^n)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{12} \sqrt{\frac{\pi}{3}} n^{3/2} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Log[a\*x^n]^(3/2),x]

[Out]  $(n^{(3/2)} * \operatorname{Sqrt}[\pi/3] * x^3 * \operatorname{Erfi}[(\operatorname{Sqrt}[3] * \operatorname{Sqrt}[\log[a*x^n]]) / \operatorname{Sqrt}[n]]) / (12 * (a*x^n)^{(3/n)}) - (n*x^3 * \operatorname{Sqrt}[\log[a*x^n]]) / 6 + (x^3 * \log[a*x^n]^{(3/2)}) / 3$

#### Rule 2180

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \log^{\frac{3}{2}}(ax^n) dx &= \frac{1}{3}x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{2}n \int x^2 \sqrt{\log(ax^n)} dx \\
&= -\frac{1}{6}nx^3 \sqrt{\log(ax^n)} + \frac{1}{3}x^3 \log^{\frac{3}{2}}(ax^n) + \frac{1}{12}n^2 \int \frac{x^2}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{1}{6}nx^3 \sqrt{\log(ax^n)} + \frac{1}{3}x^3 \log^{\frac{3}{2}}(ax^n) + \frac{1}{12} \left( nx^3 (ax^n)^{-3/n} \right) \text{Subst} \left( \int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= -\frac{1}{6}nx^3 \sqrt{\log(ax^n)} + \frac{1}{3}x^3 \log^{\frac{3}{2}}(ax^n) + \frac{1}{6} \left( nx^3 (ax^n)^{-3/n} \right) \text{Subst} \left( \int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= \frac{1}{12}n^{3/2} \sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) - \frac{1}{6}nx^3 \sqrt{\log(ax^n)} + \frac{1}{3}x^3 \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 76, normalized size = 0.84

$$\frac{1}{36}x^3 \left( \sqrt{3\pi} n^{3/2} (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) - 6(n - 2 \log(ax^n)) \sqrt{\log(ax^n)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a*x^n]^(3/2),x]
```

```
[Out] (x^3*((n^(3/2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(3/n) - 6*(n - 2*Log[a*x^n])*Sqrt[Log[a*x^n]]))/36
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^2\*log(a\*x^n)^(3/2), x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int x^2 \ln(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(a\*x^n)^(3/2),x)

[Out] int(x^2\*ln(a\*x^n)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^2\*log(a\*x^n)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \ln(ax^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*log(a\*x^n)^(3/2),x)

[Out] int(x^2\*log(a\*x^n)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x\*\*2\*log(a\*x\*\*n)\*\*(3/2), x)

### 3.123 $\int x \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=90

$$\frac{3}{16} \sqrt{\frac{\pi}{2}} n^{3/2} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2} x^2 \log^{\frac{3}{2}}(ax^n) - \frac{3}{8} n x^2 \sqrt{\log(ax^n)}$$

[Out]  $\frac{1}{2} x^2 \ln(a x^n)^{(3/2)} + \frac{3}{32} n^{(3/2)} x^2 \operatorname{erfi}\left(\frac{2^{(1/2)} \ln(a x^n)^{(1/2)}}{n^{(1/2)}}\right) \sqrt{2} \sqrt{\pi} / ((a x^n)^{(2/n)}) - \frac{3}{8} n x^2 \ln(a x^n)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{3}{16} \sqrt{\frac{\pi}{2}} n^{3/2} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2} x^2 \log^{\frac{3}{2}}(ax^n) - \frac{3}{8} n x^2 \sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In] `Int[x*Log[a*x^n]^(3/2), x]`

[Out]  $(3 n^{(3/2)} \sqrt{\pi/2} x^2 \operatorname{Erfi}[\sqrt{2} \sqrt{\log[a x^n]}] / \sqrt{n}) / (16 (a x^n)^{(2/n)}) - (3 n x^2 \sqrt{\log[a x^n]}) / 8 + (x^2 \log[a x^n]^{(3/2)}) / 2$

#### Rule 2180

`Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)) / Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]]) / (2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2305

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] :> Simp[((d*x)^(m+1)*(a + b*Log[c*x^n])^p) / (d*(m+1)), x] - Dist[(b*n*p) / (m+1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]`

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol]
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned}
 \int x \log^{\frac{3}{2}}(ax^n) dx &= \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) - \frac{1}{4}(3n) \int x \sqrt{\log(ax^n)} dx \\
 &= -\frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) + \frac{1}{16}(3n^2) \int \frac{x}{\sqrt{\log(ax^n)}} dx \\
 &= -\frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) + \frac{1}{16}(3nx^2 (ax^n)^{-2/n}) \text{Subst}\left(\int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right) \\
 &= -\frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) + \frac{1}{8}(3nx^2 (ax^n)^{-2/n}) \text{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right) \\
 &= \frac{3}{16}n^{3/2} \sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n)
 \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 79, normalized size = 0.88

$$\frac{1}{32}x^2 \left( 3\sqrt{2\pi} n^{3/2} (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) + 4\sqrt{\log(ax^n)} (4 \log(ax^n) - 3n) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a*x^n]^(3/2), x]
```

```
[Out] (x^2*((3*n^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]]))/(a*x^n)^(2/n) + 4*Sqrt[Log[a*x^n]]*(-3*n + 4*Log[a*x^n]))/32
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x^n)^(3/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x\*log(a\*x^n)^(3/2), x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int x \ln(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(a\*x^n)^(3/2),x)

[Out] int(x\*ln(a\*x^n)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x\*log(a\*x^n)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \ln(ax^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*log(a\*x^n)^(3/2),x)

[Out] int(x\*log(a\*x^n)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x\*log(a\*x\*\*n)\*\*(3/2), x)



### 3.124 $\int \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=72

$$\frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+x\log^{\frac{3}{2}}(ax^n)-\frac{3}{2}nx\sqrt{\log(ax^n)}$$

[Out]  $x*\ln(a*x^n)^{(3/2)}+3/4*n^{(3/2)}*x*erfi(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*Pi^{(1/2)}/((a*x^n)^{(1/n)})-3/2*n*x*\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2296, 2300, 2180, 2204}

$$\frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n}\operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+x\log^{\frac{3}{2}}(ax^n)-\frac{3}{2}nx\sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(3/2), x]

[Out]  $(3*n^{(3/2)}*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(4*(a*x^n)^n(-1)) - (3*n*x*Sqrt[Log[a*x^n]])/2 + x*Log[a*x^n]^(3/2)$

#### Rule 2180

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] := Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

### Rubi steps

$$\begin{aligned}
\int \log^{\frac{3}{2}}(ax^n) dx &= x \log^{\frac{3}{2}}(ax^n) - \frac{1}{2}(3n) \int \sqrt{\log(ax^n)} dx \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n) + \frac{1}{4}(3n^2) \int \frac{1}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n) + \frac{1}{4}(3nx(ax^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right) \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n) + \frac{1}{2}(3nx(ax^n)^{-1/n}) \text{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right) \\
&= \frac{3}{4}n^{3/2}\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 72, normalized size = 1.00

$$\frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + x \log^{\frac{3}{2}}(ax^n) - \frac{3}{2}nx\sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*x^n]^(3/2), x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(4*(a*x^n)^n^(-1)) -
(3*n*x*Sqrt[Log[a*x^n]])/2 + x*Log[a*x^n]^(3/2)
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(3/2), x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(3/2), x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \ln(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2),x)

[Out] int(ln(a\*x^n)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln(ax^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(3/2),x)

[Out] int(log(a\*x^n)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(log(a\*x\*\*n)\*\*(3/2), x)

$$3.125 \quad \int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx$$

Optimal. Leaf size=17

$$\frac{2 \log^{\frac{5}{2}}(ax^n)}{5n}$$

[Out]  $2/5 * \ln(a * x^n)^{(5/2)} / n$

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{2 \log^{\frac{5}{2}}(ax^n)}{5n}$$

Antiderivative was successfully verified.

[In] `Int[Log[a*x^n]^(3/2)/x,x]`

[Out] `(2*Log[a*x^n]^(5/2))/(5*n)`

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2302

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]`

Rubi steps

$$\begin{aligned} \int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx &= \frac{\text{Subst}\left(\int x^{3/2} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{2 \log^{\frac{5}{2}}(ax^n)}{5n} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{2 \log^{\frac{5}{2}}(ax^n)}{5n}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2)/x,x]

[Out] (2\*Log[a\*x^n]^(5/2))/(5\*n)

**fricas [B]** time = 0.46, size = 34, normalized size = 2.00

$$\frac{2 \left( n^2 \log(x)^2 + 2 n \log(a) \log(x) + \log(a)^2 \right) \sqrt{n \log(x) + \log(a)}}{5 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x,x, algorithm="fricas")

[Out] 2/5\*(n^2\*log(x)^2 + 2\*n\*log(a)\*log(x) + log(a)^2)\*sqrt(n\*log(x) + log(a))/n

**giac [B]** time = 0.40, size = 72, normalized size = 4.24

$$\frac{2 \left( 3 \left( n \log(x) + \log(a) \right)^{\frac{5}{2}} - 10 \left( n \log(x) + \log(a) \right)^{\frac{3}{2}} \log(a) + 30 \sqrt{n \log(x) + \log(a)} \log(a)^2 + 10 \left( n \log(x) + \log(a) \right) \right)}{15 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x,x, algorithm="giac")

[Out] 2/15\*(3\*(n\*log(x) + log(a))^(5/2) - 10\*(n\*log(x) + log(a))^(3/2)\*log(a) + 30\*sqrt(n\*log(x) + log(a))\*log(a)^2 + 10\*((n\*log(x) + log(a))^(3/2) - 3\*sqrt(n\*log(x) + log(a))\*log(a))\*log(a))/n

**maple [A]** time = 0.03, size = 14, normalized size = 0.82

$$\frac{2 \ln(ax^n)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2)/x,x)

[Out] 2/5\*ln(a\*x^n)^(5/2)/n

**maxima [A]** time = 0.61, size = 13, normalized size = 0.76

$$\frac{2 \log(ax^n)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x,x, algorithm="maxima")

[Out] 2/5\*log(a\*x^n)^(5/2)/n

**mupad [B]** time = 3.53, size = 13, normalized size = 0.76

$$\frac{2 \ln(ax^n)^{5/2}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(3/2)/x,x)

[Out] (2\*log(a\*x^n)^(5/2))/(5\*n)

**sympy [A]** time = 21.92, size = 75, normalized size = 4.41

$$\begin{cases} \frac{2n\sqrt{n\log(x)+\log(a)}\log(x)^2}{5} + \frac{4\sqrt{n\log(x)+\log(a)}\log(a)\log(x)}{5} + \frac{2\sqrt{n\log(x)+\log(a)}\log(a)^2}{5n} & \text{for } n \neq 0 \\ \log(a)^{\frac{3}{2}}\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2)/x,x)

[Out] Piecewise((2\*n\*sqrt(n\*log(x) + log(a))\*log(x)\*\*2/5 + 4\*sqrt(n\*log(x) + log(a))\*log(a)\*log(x)/5 + 2\*sqrt(n\*log(x) + log(a))\*log(a)\*\*2/(5\*n), Ne(n, 0)), (log(a)\*\*(3/2)\*log(x), True))

$$3.126 \quad \int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2} dx$$

**Optimal.** Leaf size=77

$$\frac{3\sqrt{\pi} n^{3/2} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} - \frac{3n\sqrt{\log(ax^n)}}{2x}$$

[Out]  $-\ln(a*x^n)^{(3/2)}/x+3/4*n^{(3/2)}*(a*x^n)^{(1/n)}*\operatorname{erf}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*P$   
 $i^{(1/2)}/x-3/2*n*\ln(a*x^n)^{(1/2)}/x$

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00,  
 number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} =$   
 0.286, Rules used = {2305, 2310, 2180, 2205}

$$\frac{3\sqrt{\pi} n^{3/2} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} - \frac{3n\sqrt{\log(ax^n)}}{2x}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(3/2)/x^2,x]

[Out]  $(3*n^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(4*x) - ($   
 $3*n*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/(2*x) - \operatorname{Log}[a*x^n]^{(3/2)}/x$

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :  
 > Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqr  
 t[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; Fr  
 eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbo  
 l] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n  
 \*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b,  
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol
] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2} dx &= -\frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{1}{2}(3n) \int \frac{\sqrt{\log(ax^n)}}{x^2} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{1}{4}(3n^2) \int \frac{1}{x^2\sqrt{\log(ax^n)}} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{(3n(ax^n)^{\frac{1}{n}}) \text{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{4x} \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{(3n(ax^n)^{\frac{1}{n}}) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{2x} \\
&= \frac{3n^{3/2}\sqrt{\pi}(ax^n)^{\frac{1}{n}} \text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x}
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 79, normalized size = 1.03

$$\frac{3n^2(ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) + 4\log^2(ax^n) + 6n\log(ax^n)}{4x\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2)/x^2, x]

[Out] -1/4\*(6\*n\*Log[a\*x^n] + 4\*Log[a\*x^n]^2 + 3\*n^2\*(a\*x^n)^n^(-1)\*Gamma[1/2, Log[a\*x^n]/n]\*Sqrt[Log[a\*x^n]/n])/(x\*Sqrt[Log[a\*x^n]])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(log(a\*x^n)^(3/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^2,x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(3/2)/x^2, x)

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\ln(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2)/x^2,x)

[Out] int(ln(a\*x^n)^(3/2)/x^2, x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(3/2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(ax^n)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(3/2)/x^2,x)

[Out] int(log(a\*x^n)^(3/2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2)/x\*\*2,x)

[Out] Integral(log(a\*x\*\*n)\*\*(3/2)/x\*\*2, x)

$$3.127 \quad \int \frac{\log^2(ax^n)}{x^3} dx$$

**Optimal.** Leaf size=90

$$\frac{3\sqrt{\frac{\pi}{2}} n^{3/2} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{\log^2(ax^n)}{2x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2}$$

[Out]  $-1/2*\ln(a*x^n)^{(3/2)}/x^2+3/32*n^{(3/2)}*(a*x^n)^{(2/n)}*\operatorname{erf}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/x^2-3/8*n*\ln(a*x^n)^{(1/2)}/x^2$

**Rubi [A]** time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}} n^{3/2} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{\log^2(ax^n)}{2x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(3/2)/x^3, x]

[Out]  $(3*n^{(3/2)}*\operatorname{Sqrt}[\pi/2]*(a*x^n)^{(2/n)}*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\log[a*x^n]])/\operatorname{Sqrt}[n]])/(16*x^2) - (3*n*\operatorname{Sqrt}[\log[a*x^n]])/(8*x^2) - \log[a*x^n]^{(3/2)}/(2*x^2)$

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol
] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} dx &= -\frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{1}{4}(3n) \int \frac{\sqrt{\log(ax^n)}}{x^3} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{1}{16}(3n^2) \int \frac{1}{x^3\sqrt{\log(ax^n)}} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{(3n(ax^n)^{2/n}) \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{16x^2} \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{(3n(ax^n)^{2/n}) \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{8x^2} \\
&= \frac{3n^{3/2}\sqrt{\frac{\pi}{2}}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 88, normalized size = 0.98

$$-\frac{3\sqrt{2}n^2(ax^n)^{2/n}\sqrt{\frac{\log(ax^n)}{n}}\Gamma\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right) + 4\log(ax^n)(4\log(ax^n) + 3n)}{32x^2\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2)/x^3, x]

[Out] -1/32\*(3\*Sqrt[2]\*n^2\*(a\*x^n)^(2/n)\*Gamma[1/2, (2\*Log[a\*x^n])/n]\*Sqrt[Log[a\*x^n]/n] + 4\*Log[a\*x^n]\*(3\*n + 4\*Log[a\*x^n]))/(x^2\*Sqrt[Log[a\*x^n]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^3,x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(3/2)/x^3, x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\ln(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2)/x^3,x)

[Out] int(ln(a\*x^n)^(3/2)/x^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(3/2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(ax^n)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(a\*x^n)^(3/2)/x^3,x)

[Out] int(log(a\*x^n)^(3/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2)/x\*\*3,x)

[Out] Integral(log(a\*x\*\*n)\*\*(3/2)/x\*\*3, x)

$$3.128 \quad \int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=46

$$\frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

[Out]  $1/2*x^4*\operatorname{erfi}(2*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/((a*x^n)^{(4/n)}/n^{(1/2)})$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

Antiderivative was successfully verified.

[In] `Int[x^3/Sqrt[Log[a*x^n]], x]`

[Out] `(Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(2*Sqrt[n]*(a*x^n)^(4/n))`

#### Rule 2180

`Int[(F_)^((g_)*(e_) + (f_)*(x_))/Sqrt[(c_) + (d_)*(x_)], x_Symbol] :`  
`> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]]`, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

`Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2310

`Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)*((d_)*(x_))^(m_), x_Symbol] :`  
`> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)/n)*(a + b*x)^p, x], x, Log[c*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{\log(ax^n)}} dx &= \frac{(x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\
&= \frac{(2x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\
&= \frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi]\*x^4\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(2\*Sqrt[n]\*(a\*x^n)^(4/n))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(1/2),x, algorithm="giac")



[Out] integrate(x^3/sqrt(log(a\*x^n)), x)

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(a\*x^n)^(1/2), x)

[Out] int(x^3/ln(a\*x^n)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(x^3/sqrt(log(a\*x^n)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/log(a\*x^n)^(1/2), x)

[Out] int(x^3/log(a\*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/ln(a\*x\*\*n)\*\*(1/2), x)

[Out] Integral(x\*\*3/sqrt(log(a\*x\*\*n)), x)

$$3.129 \quad \int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=51

$$\frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

[Out]  $1/3*x^3*\operatorname{erfi}(3^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*3^{(1/2)}*Pi^{(1/2)}/((a*x^n)^{(3/n)}/n^{(1/2)})$

**Rubi [A]** time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] `Int[x^2/Sqrt[Log[a*x^n]],x]`

[Out] `(Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[n]*(a*x^n)^(3/n))`

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`  
`> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /;` `FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /;` `FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2310

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /;` `FreeQ[{a, b, c, d, m, n, p}, x]`

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{\log(ax^n)}} dx &= \frac{(x^3 (ax^n)^{-3/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\
&= \frac{(2x^3 (ax^n)^{-3/n}) \operatorname{Subst}\left(\int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\
&= \frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 51, normalized size = 1.00

$$\frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi/3]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(3/n))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/sqrt(log(a\*x^n)), x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(a\*x^n)^(1/2),x)

[Out] int(x^2/ln(a\*x^n)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(log(a\*x^n)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/log(a\*x^n)^(1/2),x)

[Out] int(x^2/log(a\*x^n)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(x\*\*2/sqrt(log(a\*x\*\*n)), x)

$$3.130 \quad \int \frac{x}{\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=51

$$\frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

[Out]  $1/2*x^2*\operatorname{erfi}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*Pi^{(1/2)}/((a*x^n)^{(2/n)}/n^{(1/2)})$

**Rubi [A]** time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi/2]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(2/n))

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{\log(ax^n)}} dx &= \frac{(x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{(2x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 51, normalized size = 1.00

$$\frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi/2]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(2/n))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(1/2), x, algorithm="giac")

[Out] integrate(x/sqrt(log(a\*x^n)), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(a\*x^n)^(1/2), x)

[Out] int(x/ln(a\*x^n)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(x/sqrt(log(a\*x^n)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/log(a\*x^n)^(1/2), x)

[Out] int(x/log(a\*x^n)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(a\*x\*\*n)\*\*(1/2), x)

[Out] Integral(x/sqrt(log(a\*x\*\*n)), x)

$$3.131 \quad \int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Optimal. Leaf size=40

$$\frac{\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

[Out]  $x \operatorname{erfi}(\ln(a*x^n)^{(1/2)}/n^{(1/2)}) * \pi^{(1/2)} / ((a*x^n)^{(1/n)}) / n^{(1/2)}$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {2300, 2180, 2204}

$$\frac{\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[Log[a*x^n]], x]`

[Out] `(Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1))`

Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`  
`> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /;`  
`FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True`

Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :=`  
`Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /;`  
`FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2300

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p, x_Symbol] :=`  
`Dist[x/(n*(c*x^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /;`  
`FreeQ[{a, b, c, n, p}, x]`

Rubi steps



$$\begin{aligned} \int \frac{1}{\sqrt{\log(ax^n)}} dx &= \frac{(x(ax^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{(2x(ax^n)^{-1/n}) \operatorname{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 40, normalized size = 1.00

$$\frac{\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^n^(-1))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sqrt(log(a\*x^n))^(1/2), x, algorithm="giac")

[Out] integrate(1/sqrt(log(a\*x^n)), x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(a\*x^n)^(1/2), x)

[Out] int(1/ln(a\*x^n)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(log(a\*x^n)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/log(a\*x^n)^(1/2), x)

[Out] int(1/log(a\*x^n)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(a\*x\*\*n)\*\*(1/2), x)

[Out] Integral(1/sqrt(log(a\*x\*\*n)), x)

$$3.132 \quad \int \frac{1}{x\sqrt{\log(ax^n)}} dx$$

Optimal. Leaf size=15

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

[Out]  $2*\ln(a*x^n)^{(1/2)}/n$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Log[a\*x^n]]), x]

[Out] (2\*Sqrt[Log[a\*x^n]])/n

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{\log(ax^n)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{2\sqrt{\log(ax^n)}}{n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Log[a\*x^n]]),x]

[Out] (2\*Sqrt[Log[a\*x^n]])/n

**fricas** [A] time = 0.41, size = 14, normalized size = 0.93

$$\frac{2\sqrt{n\log(x) + \log(a)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] 2\*sqrt(n\*log(x) + log(a))/n

**giac** [A] time = 0.21, size = 14, normalized size = 0.93

$$\frac{2\sqrt{n\log(x) + \log(a)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] 2\*sqrt(n\*log(x) + log(a))/n

**maple** [A] time = 0.03, size = 14, normalized size = 0.93

$$\frac{2\sqrt{\ln(ax^n)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(a\*x^n)^(1/2),x)

[Out] 2\*ln(a\*x^n)^(1/2)/n

**maxima** [A] time = 0.55, size = 13, normalized size = 0.87

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(log(a\*x^n))/n

mupad [B] time = 3.58, size = 13, normalized size = 0.87

$$\frac{2\sqrt{\ln(ax^n)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*log(a*x^n)^(1/2)),x)`

[Out] `(2*log(a*x^n)^(1/2))/n`

sympy [A] time = 2.08, size = 24, normalized size = 1.60

$$\begin{cases} \frac{2\sqrt{n\log(x)+\log(a)}}{n} & \text{for } n \neq 0 \\ \frac{\log(x)}{\sqrt{\log(a)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(a*x**n)**(1/2),x)`

[Out] `Piecewise((2*sqrt(n*log(x) + log(a))/n, Ne(n, 0)), (log(x)/sqrt(log(a)), True))`

$$3.133 \quad \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Optimal. Leaf size=40

$$\frac{\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} x}$$

[Out]  $(a*x^n)^{(1/n)}*\operatorname{erf}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/x/n^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2205}

$$\frac{\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} x}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*Sqrt[Log[a*x^n]]),x]`

[Out] `(Sqrt[Pi]*(a*x^n)^n^(-1)*Erf[Sqrt[Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*x)`

Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :`  
`> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*`  
`x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True`

Rule 2205

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr`  
`t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]]/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr`  
`eeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 2310

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)])*(b_.)^((p_.)*((d_.)*(x_)^(m_.)), x_Symbol`  
`] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x`  
`/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx &= \frac{(ax^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{nx} \\
&= \frac{\left(2(ax^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{nx} \\
&= \frac{\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} x}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 52, normalized size = 1.30

$$\frac{(ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right)}{x \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[Log[a\*x^n]]),x]

[Out] -(((a\*x^n)^n^(-1)\*Gamma[1/2, Log[a\*x^n]/n]\*Sqrt[Log[a\*x^n]/n])/(x\*Sqrt[Log[a\*x^n]]))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(x^2\*sqrt(log(a\*x^n))), x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(a\*x^n)^(1/2), x)

[Out] int(1/x^2/ln(a\*x^n)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(x^2\*sqrt(log(a\*x^n))), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*log(a\*x^n)^(1/2)), x)

[Out] int(1/(x^2\*log(a\*x^n)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(a\*x\*\*n)\*\*(1/2), x)

[Out] Integral(1/(x\*\*2\*sqrt(log(a\*x\*\*n))), x)



$$3.134 \quad \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} x^2}$$

[Out]  $1/2*(a*x^n)^{(2/n)}*\operatorname{erf}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/x^2/n^{(1/2)}$

**Rubi [A]** time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Sqrt[Log[a\*x^n]]),x]

[Out] (Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*x^2)

#### Rule 2180

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx &= \frac{(ax^n)^{2/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{nx^2} \\ &= \frac{(2(ax^n)^{2/n}) \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{nx^2} \\ &= \frac{\sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n} x^2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 1.18

$$-\frac{(ax^n)^{2/n} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{2 \log(ax^n)}{n}\right)}{\sqrt{2} x^2 \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[Log[a\*x^n]]),x]

[Out] -(((a\*x^n)^(2/n)\*Gamma[1/2, (2\*Log[a\*x^n])/n]\*Sqrt[Log[a\*x^n]/n])/(Sqrt[2]\*x^2\*Sqrt[Log[a\*x^n]]))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(x^3\*sqrt(log(a\*x^n))), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(a\*x^n)^(1/2), x)

[Out] int(1/x^3/ln(a\*x^n)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(x^3\*sqrt(log(a\*x^n))), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*log(a\*x^n)^(1/2)), x)

[Out] int(1/(x^3\*log(a\*x^n)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(a\*x\*\*n)\*\*(1/2), x)

[Out] Integral(1/(x\*\*3\*sqrt(log(a\*x\*\*n))), x)

$$3.135 \quad \int \frac{x^3}{\log^2(ax^n)} dx$$

Optimal. Leaf size=63

$$\frac{4\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^4}{n\sqrt{\log(ax^n)}}$$

[Out]  $4*x^4*\operatorname{erfi}(2*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/n^{(3/2)}/((a*x^n)^{(4/n)})-2*x^4/n/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{4\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^4}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[a\*x^n]^(3/2), x]

[Out]  $(4*\operatorname{Sqrt}[\operatorname{Pi}]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(3/2)}*(a*x^n)^{(4/n)}) - (2*x^4)/(n*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

Rule 2180

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[(((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x]

;/ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^4}{n\sqrt{\log(ax^n)}} + \frac{8 \int \frac{x^3}{\sqrt{\log(ax^n)}} dx}{n} \\ &= -\frac{2x^4}{n\sqrt{\log(ax^n)}} + \frac{(8x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\ &= -\frac{2x^4}{n\sqrt{\log(ax^n)}} + \frac{(16x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\ &= \frac{4\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^4}{n\sqrt{\log(ax^n)}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 73, normalized size = 1.16

$$\frac{2x^4 (ax^n)^{-4/n} \left( (ax^n)^{4/n} - 2\sqrt{-\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, -\frac{4\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[a\*x^n]^(3/2), x]

[Out] (-2\*x^4\*((a\*x^n)^(4/n) - 2\*Gamma[1/2, (-4\*Log[a\*x^n])/n]\*Sqrt[-(Log[a\*x^n]/n)]))/(n\*(a\*x^n)^(4/n)\*Sqrt[Log[a\*x^n]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/log(a*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/log(a*x^n)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^3/log(a*x^n)^(3/2), x)`

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/ln(a*x^n)^(3/2),x)`

[Out] `int(x^3/ln(a*x^n)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/log(a*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^3/log(a*x^n)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/log(a*x^n)^(3/2),x)`

[Out] `int(x^3/log(a*x^n)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/ln(a*x**n)**(3/2),x)`

[Out] `Integral(x**3/log(a*x**n)**(3/2), x)`

$$3.136 \quad \int \frac{x^2}{\log^2(ax^n)} dx$$

Optimal. Leaf size=69

$$\frac{2\sqrt{3\pi} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}$$

[Out]  $2*x^3*\operatorname{erfi}(3^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*3^{(1/2)}*Pi^{(1/2)}/n^{(3/2)}/((a*x^n)^{(3/n)})-2*x^3/n/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{2\sqrt{3\pi} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[a\*x^n]^(3/2), x]

[Out]  $(2*\operatorname{Sqrt}[3*Pi]*x^3*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(3/2)}*(a*x^n)^{(3/n)}) - (2*x^3)/(n*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x]



;/ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{6 \int \frac{x^2}{\sqrt{\log(ax^n)}} dx}{n} \\ &= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{(6x^3 (ax^n)^{-3/n}) \text{Subst}\left(\int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\ &= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{(12x^3 (ax^n)^{-3/n}) \text{Subst}\left(\int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\ &= \frac{2\sqrt{3\pi} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 1.13

$$\frac{2x^3 (ax^n)^{-3/n} \left( (ax^n)^{3/n} - \sqrt{3} \sqrt{-\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, -\frac{3\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[a\*x^n]^(3/2), x]

[Out] (-2\*x^3\*((a\*x^n)^(3/n) - Sqrt[3]\*Gamma[1/2, (-3\*Log[a\*x^n])/n]\*Sqrt[-(Log[a\*x^n]/n)])/(n\*(a\*x^n)^(3/n)\*Sqrt[Log[a\*x^n]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(a*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(a*x^n)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^2/log(a*x^n)^(3/2), x)`

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/ln(a*x^n)^(3/2),x)`

[Out] `int(x^2/ln(a*x^n)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(a*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^2/log(a*x^n)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/log(a*x^n)^(3/2),x)
```

```
[Out] int(x^2/log(a*x^n)^(3/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(a*x**n)**(3/2),x)
```

```
[Out] Integral(x**2/log(a*x**n)**(3/2), x)
```

$$3.137 \quad \int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx$$

Optimal. Leaf size=69

$$\frac{2\sqrt{2\pi} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}$$

[Out]  $2*x^2*\operatorname{erfi}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*Pi^{(1/2)}/n^{(3/2)}/((a*x^n)^{(2/n)})-2*x^2/n/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{2\sqrt{2\pi} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x/Log[a\*x^n]^(3/2), x]

[Out]  $(2*\operatorname{Sqrt}[2*Pi]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(3/2)}*(a*x^n)^{(2/n)}) - (2*x^2)/(n*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x]

;/ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{4 \int \frac{x}{\sqrt{\log(ax^n)}} dx}{n} \\
 &= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{(4x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\
 &= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{(8x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
 &= \frac{2\sqrt{2\pi} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 1.13

$$\frac{2x^2 (ax^n)^{-2/n} \left( (ax^n)^{2/n} - \sqrt{2} \sqrt{-\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, -\frac{2\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[a\*x^n]^(3/2), x]

[Out] (-2\*x^2\*((a\*x^n)^(2/n) - Sqrt[2]\*Gamma[1/2, (-2\*Log[a\*x^n])/n]\*Sqrt[-(Log[a\*x^n]/n)]))/(n\*(a\*x^n)^(2/n)\*Sqrt[Log[a\*x^n]])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/log(a*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/log(a*x^n)^(3/2),x, algorithm="giac")`

[Out] `integrate(x/log(a*x^n)^(3/2), x)`

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x}{\ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/ln(a*x^n)^(3/2),x)`

[Out] `int(x/ln(a*x^n)^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/log(a*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x/log(a*x^n)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/log(a*x^n)^(3/2),x)`

[Out] `int(x/log(a*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/ln(a*x**n)**(3/2), x)`

[Out] `Integral(x/log(a*x**n)**(3/2), x)`

$$3.138 \quad \int \frac{1}{\log^2(ax^n)} dx$$

**Optimal.** Leaf size=58

$$\frac{2\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}$$

[Out]  $2*x*\operatorname{erfi}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/n^{(3/2)}/((a*x^n)^{(1/n)})-2*x/n/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2297, 2300, 2180, 2204}

$$\frac{2\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Log}[a*x^n]^{(-3/2)}, x]$

[Out]  $(2*\operatorname{Sqrt}[\operatorname{Pi}]*x*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(n^{(3/2)}*(a*x^n)^n^{(-1)}) - (2*x)/(n*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :$   
 $> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!}\$UseGamma == True$

Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

Rule 2297

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}, x\_Symbol] :> \operatorname{Simp}[(x*(a + b*\operatorname{Log}[c*x^n])^{(p + 1)})/(b*n*(p + 1)), x] - \operatorname{Dist}[1/(b*n*(p + 1)), \operatorname{Int}[(a + b*\operatorname{Log}[c*x^n])^{(p + 1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, n\}, x\} \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{IntegerQ}[2*p]$



Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x}{n\sqrt{\log(ax^n)}} + \frac{2 \int \frac{1}{\sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2x}{n\sqrt{\log(ax^n)}} + \frac{(2x(ax^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\
&= -\frac{2x}{n\sqrt{\log(ax^n)}} + \frac{(4x(ax^n)^{-1/n}) \operatorname{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
&= \frac{2\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 69, normalized size = 1.19

$$\frac{2x(ax^n)^{-1/n} \left( (ax^n)^{\frac{1}{n}} - \sqrt{-\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, -\frac{\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*x^n]^(-3/2), x]
```

```
[Out] (-2*x*((a*x^n)^n^(-1) - Gamma[1/2, -(Log[a*x^n]/n)]*Sqrt[-(Log[a*x^n]/n)]))
/(n*(a*x^n)^n^(-1)*Sqrt[Log[a*x^n]])
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(a*x^n)^(3/2), x, algorithm="fricas")
```

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(-3/2), x)

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(a\*x^n)^(3/2),x)

[Out] int(1/ln(a\*x^n)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/log(a\*x^n)^(3/2),x)

[Out] int(1/log(a\*x^n)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(a*x**n)**(3/2), x)
```

```
[Out] Integral(log(a*x**n)**(-3/2), x)
```

$$3.139 \quad \int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx$$

Optimal. Leaf size=15

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

[Out]  $-2/n/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Log}[a*x^n]^{(3/2)}), x]$

[Out]  $-2/(n*\text{Sqrt}[\text{Log}[a*x^n]])$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] :> \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 2302

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))^{(p_.)}/(x_), x\_Symbol] :> \text{Dist}[1/(b*n), \text{Subst}[\text{Int}[x^p, x], x, a + b*\text{Log}[c*x^n]], x] /; \text{FreeQ}\{a, b, c, n, p\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, \log(ax^n)\right)}{n} \\ &= -\frac{2}{n\sqrt{\log(ax^n)}} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[a\*x^n]^(3/2)),x]

[Out] -2/(n\*Sqrt[Log[a\*x^n]])

**fricas** [A] time = 0.46, size = 24, normalized size = 1.60

$$\frac{2\sqrt{n\log(x) + \log(a)}}{n^2\log(x) + n\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] -2\*sqrt(n\*log(x) + log(a))/(n^2\*log(x) + n\*log(a))

**giac** [A] time = 0.27, size = 14, normalized size = 0.93

$$-\frac{2}{\sqrt{n\log(x) + \log(a)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] -2/(sqrt(n\*log(x) + log(a))\*n)

**maple** [A] time = 0.03, size = 14, normalized size = 0.93

$$-\frac{2}{n\sqrt{\ln(ax^n)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(a\*x^n)^(3/2),x)

[Out] -2/n/ln(a\*x^n)^(1/2)

**maxima** [A] time = 0.70, size = 13, normalized size = 0.87

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] -2/(n\*sqrt(log(a\*x^n)))

mupad [B] time = 3.45, size = 13, normalized size = 0.87

$$-\frac{2}{n\sqrt{\ln(ax^n)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*log(a\*x^n)^(3/2)),x)

[Out] -2/(n\*log(a\*x^n)^(1/2))

sympy [A] time = 93.98, size = 41, normalized size = 2.73

$$\begin{cases} \infty \log(x) & \text{for } a = 1 \wedge n = 0 \\ \frac{\log(x)^3}{\log(a)^2} & \text{for } n = 0 \\ \infty \log(x) & \text{for } a = e^{-n \log(x)} \\ -\frac{2}{n\sqrt{n \log(x) + \log(a)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Piecewise((zoo\*log(x), Eq(a, 1) & Eq(n, 0)), (log(x)/log(a)\*\*(3/2), Eq(n, 0)), (zoo\*log(x), Eq(a, exp(-n\*log(x)))), (-2/(n\*sqrt(n\*log(x) + log(a))), True))

$$3.140 \quad \int \frac{1}{x^2 \log^2(ax^n)} dx$$

Optimal. Leaf size=60

$$\frac{2\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}$$

[Out]  $-2*(a*x^n)^{(1/n)}*\operatorname{erf}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/n^{(3/2)}/x-2/n/x/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$\frac{2\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^2*\operatorname{Log}[a*x^n]^{(3/2)}), x]$

[Out]  $(-2*\operatorname{Sqrt}[\operatorname{Pi}]*(a*x^n)^{n^{-1}}*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(n^{(3/2)*x}) - 2/(n*x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma === \operatorname{True}$

Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^{a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2])], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{NegQ}[b]$

Rule 2306

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}*(b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}], x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{LtQ}[p, -1]$

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol
] := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{2 \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx}{n} \\ &= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{\left(2(ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2 x} \\ &= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{\left(4(ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2 x} \\ &= -\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}} \text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2} x} - \frac{2}{nx\sqrt{\log(ax^n)}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.97

$$\frac{2 \left( (ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) - 1 \right)}{nx\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Log[a*x^n]^(3/2)),x]
```

```
[Out] (2*(-1 + (a*x^n)^n^(-1)*Gamma[1/2, Log[a*x^n]/n]*Sqrt[Log[a*x^n]/n]))/(n*x*
Sqrt[Log[a*x^n]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(a*x^n)^(3/2),x, algorithm="fricas")
```



[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/(x^2\*log(a\*x^n)^(3/2)), x)

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(a\*x^n)^(3/2),x)

[Out] int(1/x^2/ln(a\*x^n)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x^2\*log(a\*x^n)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*log(a\*x^n)^(3/2)),x)

[Out] int(1/(x^2\*log(a\*x^n)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(a\*x\*\*n)\*\*(3/2), x)

[Out] Integral(1/(x\*\*2\*log(a\*x\*\*n)\*\*(3/2)), x)

$$3.141 \quad \int \frac{1}{x^3 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=69

$$-\frac{2\sqrt{2\pi} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2} x^2} - \frac{2}{nx^2 \sqrt{\log(ax^n)}}$$

[Out]  $-2*(a*x^n)^{(2/n)}*\operatorname{erf}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/n^{(3/2)}/x^2-2/n/x^2/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$-\frac{2\sqrt{2\pi} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2} x^2} - \frac{2}{nx^2 \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^3*\operatorname{Log}[a*x^n]^{(3/2)}), x]$

[Out]  $(-2*\operatorname{Sqrt}[2*\operatorname{Pi}]* (a*x^n)^{(2/n)}*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(3/2)}*x^2) - 2/(n*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& !\$UseGamma == True$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2))}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{NegQ}[b]$

#### Rule 2306

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)^{(p_.)*((d_.)*(x_)^{(m_.)})}, x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x]$

```
;/ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^ (p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2}{nx^2 \sqrt{\log(ax^n)}} - \frac{4 \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx}{n} \\ &= -\frac{2}{nx^2 \sqrt{\log(ax^n)}} - \frac{(4(ax^n)^{2/n}) \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2 x^2} \\ &= -\frac{2}{nx^2 \sqrt{\log(ax^n)}} - \frac{(8(ax^n)^{2/n}) \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2 x^2} \\ &= -\frac{2\sqrt{2\pi} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2} x^2} - \frac{2}{nx^2 \sqrt{\log(ax^n)}} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 66, normalized size = 0.96

$$\frac{2 \left( \sqrt{2} (ax^n)^{2/n} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{2 \log(ax^n)}{n}\right) - 1 \right)}{nx^2 \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Log[a*x^n]^(3/2)), x]
```

```
[Out] (2*(-1 + Sqrt[2]*(a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])/n]*Sqrt[Log[a*x^n]
/n]))/(n*x^2*Sqrt[Log[a*x^n]])
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(a*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(a*x^n)^(3/2),x, algorithm="giac")`

[Out] `integrate(1/(x^3*log(a*x^n)^(3/2)), x)`

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/ln(a*x^n)^(3/2),x)`

[Out] `int(1/x^3/ln(a*x^n)^(3/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(a*x^n)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/(x^3*log(a*x^n)^(3/2)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*log(a*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^3*log(a*x^n)^(3/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(a*x**n)**(3/2),x)
```

```
[Out] Integral(1/(x**3*log(a*x**n)**(3/2)), x)
```

$$3.142 \quad \int \frac{x^3}{\log^2(ax^n)} dx$$

Optimal. Leaf size=87

$$\frac{32\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{16x^4}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3*x^4/n/\ln(a*x^n)^{(3/2)}+32/3*x^4*\operatorname{erfi}(2*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\Pi^{(1/2)}/n^{(5/2)}/((a*x^n)^{(4/n)}-16/3*x^4/n^2/\ln(a*x^n)^{(1/2)})$

**Rubi [A]** time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{32\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{16x^4}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/\operatorname{Log}[a*x^n]^{(5/2)}, x]$

[Out]  $(32*\operatorname{Sqrt}[\Pi]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(3*n^{(5/2)}*(a*x^n)^{(4/n)}) - (2*x^4)/(3*n*\operatorname{Log}[a*x^n]^{(3/2)}) - (16*x^4)/(3*n^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\& \ !\$UseGamma == True$

Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2))}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\Pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{PosQ}[b]$

Rule 2306

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)^{(p_.)*((d_.)*(x_)^{(m_.)})}, x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x]$

/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_) \* ((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{8 \int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
 &= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}} + \frac{64 \int \frac{x^3}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
 &= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}} + \frac{(64x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3} \\
 &= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}} + \frac{(128x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
 &= \frac{32\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 87, normalized size = 1.00

$$\frac{2x^4 (ax^n)^{-4/n} \left( (ax^n)^{4/n} (8 \log(ax^n) + n) + 16n \left( -\frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{4 \log(ax^n)}{n}\right) \right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[a\*x^n]^(5/2), x]

[Out] (-2\*x^4\*(16\*n\*Gamma[1/2, (-4\*Log[a\*x^n])/n])\*(-(Log[a\*x^n]/n))^(3/2) + (a\*x^n)^(4/n)\*(n + 8\*Log[a\*x^n]))/(3\*n^2\*(a\*x^n)^(4/n)\*Log[a\*x^n]^(3/2))



**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(x^3/log(a\*x^n)^(5/2), x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(a\*x^n)^(5/2),x)

[Out] int(x^3/ln(a\*x^n)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(x^3/log(a\*x^n)^(5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\ln(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/log(a*x^n)^(5/2),x)`

[Out] `int(x^3/log(a*x^n)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/ln(a*x**n)**(5/2),x)`

[Out] `Integral(x**3/log(a*x**n)**(5/2), x)`

$$3.143 \quad \int \frac{x^2}{\log^2(ax^n)} dx$$

Optimal. Leaf size=89

$$\frac{4\sqrt{3\pi} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} - \frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3*x^3/n/\ln(a*x^n)^{(3/2)}+4*x^3*\operatorname{erfi}(3^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*3^{(1/2)}*\pi^{(1/2)}/n^{(5/2)}/((a*x^n)^{(3/n)})-4*x^3/n^2/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{4\sqrt{3\pi} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} - \frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[a\*x^n]^(5/2), x]

[Out]  $(4*\operatorname{Sqrt}[3*\pi]*x^3*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(5/2)}*(a*x^n)^{(3/n)}) - (2*x^3)/(3*n*\operatorname{Log}[a*x^n]^{(3/2)}) - (4*x^3)/(n^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x]

/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_) \* ((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{2 \int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)} dx}{n} \\
 &= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}} + \frac{12 \int \frac{x^2}{\sqrt{\log(ax^n)}} dx}{n^2} \\
 &= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}} + \frac{(12x^3 (ax^n)^{-3/n}) \text{Subst}\left(\int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^3} \\
 &= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}} + \frac{(24x^3 (ax^n)^{-3/n}) \text{Subst}\left(\int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^3} \\
 &= \frac{4\sqrt{3\pi} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 92, normalized size = 1.03

$$\frac{2x^3 (ax^n)^{-3/n} \left( (ax^n)^{3/n} (6 \log(ax^n) + n) + 6\sqrt{3} n \left( -\frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{3 \log(ax^n)}{n}\right) \right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[a\*x^n]^(5/2), x]

[Out]  $(-2*x^3*(6*\text{Sqrt}[3]*n*\text{Gamma}[1/2, (-3*\text{Log}[a*x^n])/n])*(-\text{Log}[a*x^n]/n))^{(3/2)} + (a*x^n)^{(3/n)}*(n + 6*\text{Log}[a*x^n]))/(3*n^2*(a*x^n)^{(3/n)}*\text{Log}[a*x^n]^{(3/2)})$

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(a*x^n)^(5/2),x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(a*x^n)^(5/2),x, algorithm="giac")`

[Out] `integrate(x^2/log(a*x^n)^(5/2), x)`

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/ln(a*x^n)^(5/2),x)`

[Out] `int(x^2/ln(a*x^n)^(5/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/log(a*x^n)^(5/2),x, algorithm="maxima")`

[Out] `integrate(x^2/log(a*x^n)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\ln(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/log(a\*x^n)^(5/2), x)

[Out] int(x^2/log(a\*x^n)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\log(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(a\*x\*\*n)\*\*(5/2), x)

[Out] Integral(x\*\*2/log(a\*x\*\*n)\*\*(5/2), x)

$$3.144 \quad \int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx$$

Optimal. Leaf size=93

$$\frac{8\sqrt{2\pi} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} - \frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3*x^2/n/\ln(a*x^n)^{(3/2)}+8/3*x^2*\operatorname{erfi}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/n^{(5/2)}/((a*x^n)^{(2/n)})-8/3*x^2/n^2/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{8\sqrt{2\pi} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} - \frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x/Log[a\*x^n]^(5/2), x]

[Out]  $(8*\operatorname{Sqrt}[2*\pi]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(3*n^{(5/2)}*(a*x^n)^{(2/n)}) - (2*x^2)/(3*n*\operatorname{Log}[a*x^n]^{(3/2)}) - (8*x^2)/(3*n^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2180

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] -

Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^(m\*(a + b\*Log[c\*x^n]))^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{4 \int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
 &= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} + \frac{16 \int \frac{x}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
 &= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} + \frac{(16x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3} \\
 &= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} + \frac{(32x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
 &= \frac{8\sqrt{2\pi} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 92, normalized size = 0.99

$$\frac{2x^2 (ax^n)^{-2/n} \left( (ax^n)^{2/n} (4 \log(ax^n) + n) + 4\sqrt{2} n \left( -\frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{2 \log(ax^n)}{n}\right) \right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[a\*x^n]^(5/2), x]



```
[Out] (-2*x^2*(4*Sqrt[2]*n*Gamma[1/2, (-2*Log[a*x^n])/n]*(-(Log[a*x^n])/n))^(3/2)
+ (a*x^n)^(2/n)*(n + 4*Log[a*x^n])))/(3*n^2*(a*x^n)^(2/n)*Log[a*x^n]^(3/2))
```

```
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(a*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(a*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/log(a*x^n)^(5/2), x)
```

```
maple [F] time = 0.29, size = 0, normalized size = 0.00
```

$$\int \frac{x}{\ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/ln(a*x^n)^(5/2),x)
```

```
[Out] int(x/ln(a*x^n)^(5/2),x)
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(a*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/log(a*x^n)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\ln(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/log(a\*x^n)^(5/2), x)

[Out] int(x/log(a\*x^n)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\log(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(a\*x\*\*n)\*\*(5/2), x)

[Out] Integral(x/log(a\*x\*\*n)\*\*(5/2), x)

$$3.145 \quad \int \frac{1}{\frac{5}{\log^2(ax^n)}} dx$$

**Optimal.** Leaf size=80

$$\frac{4\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} - \frac{2x}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3*x/n/\ln(a*x^n)^{(3/2)}+4/3*x*\operatorname{erfi}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\Pi^{(1/2)}/n^{(5/2)}/((a*x^n)^{(1/n)})-4/3*x/n^2/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {2297, 2300, 2180, 2204}

$$\frac{4\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} - \frac{2x}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(-5/2), x]

[Out]  $(4*\operatorname{Sqrt}[\Pi]*x*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(3*n^{(5/2)}*(a*x^n)^{-1}) - (2*x)/(3*n*\operatorname{Log}[a*x^n]^{(3/2)}) - (4*x)/(3*n^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte

gerQ[2\*p]

Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] :> Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{2 \int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
 &= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} + \frac{4 \int \frac{1}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
 &= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} + \frac{(4x(ax^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3} \\
 &= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} + \frac{(8x(ax^n)^{-1/n}) \text{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
 &= \frac{4\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica** [A] time = 0.06, size = 83, normalized size = 1.04

$$\frac{2x(ax^n)^{-1/n} \left( (ax^n)^{\frac{1}{n}} (2 \log(ax^n) + n) + 2n \left( -\frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{\log(ax^n)}{n}\right) \right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(-5/2), x]

[Out] (-2\*x\*(2\*n\*Gamma[1/2, -(Log[a\*x^n]/n)]\*(-(Log[a\*x^n]/n))^(3/2) + (a\*x^n)^n^(-1)\*(n + 2\*Log[a\*x^n]))/(3\*n^2\*(a\*x^n)^n^(-1)\*Log[a\*x^n]^(3/2))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(-5/2), x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(a\*x^n)^(5/2),x)

[Out] int(1/ln(a\*x^n)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(-5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\ln(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/log(a*x^n)^(5/2), x)`

[Out] `int(1/log(a*x^n)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/ln(a*x**n)**(5/2), x)`

[Out] `Integral(log(a*x**n)**(-5/2), x)`

$$3.146 \quad \int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx$$

Optimal. Leaf size=17

$$-\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3/n/\ln(a*x^n)^{(3/2)}$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$-\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Log}[a*x^n]^{(5/2)}), x]$

[Out]  $-2/(3*n*\text{Log}[a*x^n]^{(3/2)})$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 2302

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}/(x_), x\_Symbol] \rightarrow \text{Dist}[1/(b*n), \text{Subst}[\text{Int}[x^p, x], x, a + b*\text{Log}[c*x^n]], x] /; \text{FreeQ}[\{a, b, c, n, p\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}} dx, x, \log(ax^n)\right)}{n} \\ &= -\frac{2}{3n \log^{\frac{3}{2}}(ax^n)} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$-\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[a\*x^n]^(5/2)),x]

[Out] -2/(3\*n\*Log[a\*x^n]^(3/2))

**fricas [B]** time = 0.45, size = 37, normalized size = 2.18

$$\frac{2\sqrt{n\log(x) + \log(a)}}{3(n^3\log(x)^2 + 2n^2\log(a)\log(x) + n\log(a)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] -2/3\*sqrt(n\*log(x) + log(a))/(n^3\*log(x)^2 + 2\*n^2\*log(a)\*log(x) + n\*log(a)^2)

**giac [A]** time = 0.25, size = 14, normalized size = 0.82

$$-\frac{2}{3(n\log(x) + \log(a))^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] -2/3/((n\*log(x) + log(a))^(3/2)\*n)

**maple [A]** time = 0.03, size = 14, normalized size = 0.82

$$-\frac{2}{3n \ln(ax^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(a\*x^n)^(5/2),x)

[Out] -2/3/n/ln(a\*x^n)^(3/2)



**maxima [A]** time = 0.81, size = 13, normalized size = 0.76

$$-\frac{2}{3n \log(ax^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] -2/3/(n\*log(a\*x^n)^(3/2))

**mupad [B]** time = 3.43, size = 13, normalized size = 0.76

$$-\frac{2}{3n \ln(ax^n)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*log(a\*x^n)^(5/2)),x)

[Out] -2/(3\*n\*log(a\*x^n)^(3/2))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Timed out

$$3.147 \quad \int \frac{1}{x^2 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=84

$$\frac{4\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3/n/x/\ln(a*x^n)^{(3/2)}+4/3*(a*x^n)^{(1/n)}*\operatorname{erf}(\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/n^{(5/2)}/x+4/3/n^2/x/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$\frac{4\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Log[a\*x^n]^(5/2)),x]

[Out]  $(4*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(3*n^{(5/2)*x}) - 2/(3*n*x*\operatorname{Log}[a*x^n]^{(3/2)}) + 4/(3*n^2*x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x]

;/ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} - \frac{2 \int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
 &= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}} + \frac{4 \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx}{3n^2} \\
 &= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}} + \frac{\left(4 (ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3 x} \\
 &= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}} + \frac{\left(8 (ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3 x} \\
 &= \frac{4\sqrt{\pi} (ax^n)^{\frac{1}{n}} \text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2} x} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 70, normalized size = 0.83

$$\frac{2 \left( -2 \log(ax^n) + 2n (ax^n)^{\frac{1}{n}} \left( \frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) + n \right)}{3n^2 x \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[a\*x^n]^(5/2)), x]

[Out] (-2\*(n - 2\*Log[a\*x^n] + 2\*n\*(a\*x^n)^(1/n)\*Gamma[1/2, Log[a\*x^n]/n]\*(Log[a\*x^n]/n)^(3/2))/(3\*n^2\*x\*Log[a\*x^n]^(3/2))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(1/(x^2\*log(a\*x^n)^(5/2)), x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(a\*x^n)^(5/2),x)

[Out] int(1/x^2/ln(a\*x^n)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x^2\*log(a\*x^n)^(5/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*log(a*x^n)^(5/2)),x)`

[Out] `int(1/(x^2*log(a*x^n)^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/ln(a*x**n)**(5/2),x)`

[Out] `Integral(1/(x**2*log(a*x**n)**(5/2)), x)`

$$3.148 \quad \int \frac{1}{x^3 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=93

$$\frac{8\sqrt{2\pi} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3/n/x^2/\ln(a*x^n)^{(3/2)}+8/3*(a*x^n)^{(2/n)}*\operatorname{erf}(2^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/n^{(5/2)}/x^2+8/3/n^2/x^2/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$\frac{8\sqrt{2\pi} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*Log[a*x^n]^(5/2)),x]`

[Out]  $(8*\operatorname{Sqrt}[2*\pi]*(a*x^n)^{(2/n)}*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(3*n^{(5/2)}*x^2) - 2/(3*n*x^2*\operatorname{Log}[a*x^n]^{(3/2)}) + 8/(3*n^2*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

**Rule 2180**

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

**Rule 2205**

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

**Rule 2306**

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] - Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]`

/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} - \frac{4 \int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
 &= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}} + \frac{16 \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx}{3n^2} \\
 &= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}} + \frac{(16(ax^n)^{2/n}) \text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3 x^2} \\
 &= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}} + \frac{(32(ax^n)^{2/n}) \text{Subst}\left(\int e^{-\frac{2x}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3 x^2} \\
 &= \frac{8\sqrt{2\pi} (ax^n)^{2/n} \text{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2} x^2} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 78, normalized size = 0.84

$$\frac{2 \left( -4 \log(ax^n) + 4\sqrt{2} n (ax^n)^{2/n} \left( \frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, \frac{2 \log(ax^n)}{n}\right) + n \right)}{3n^2 x^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[a\*x^n]^(5/2)), x]

[Out] (-2\*(n - 4\*Log[a\*x^n] + 4\*Sqrt[2]\*n\*(a\*x^n)^(2/n)\*Gamma[1/2, (2\*Log[a\*x^n])/n]\*(Log[a\*x^n]/n)^(3/2)))/(3\*n^2\*x^2\*Log[a\*x^n]^(3/2))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(1/(x^3\*log(a\*x^n)^(5/2)), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(a\*x^n)^(5/2),x)

[Out] int(1/x^3/ln(a\*x^n)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x^3\*log(a\*x^n)^(5/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \ln(ax^n)^{\frac{5}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*log(a*x^n)^(5/2)),x)`

[Out] `int(1/(x^3*log(a*x^n)^(5/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/ln(a*x**n)**(5/2),x)`

[Out] `Integral(1/(x**3*log(a*x**n)**(5/2)), x)`

$$3.149 \quad \int (dx)^m \left( a + \frac{a(1+m) \log(cx^n)}{n} \right) dx$$

Optimal. Leaf size=21

$$\frac{a(dx)^{m+1} \log(cx^n)}{dn}$$

[Out] a\*(d\*x)^(1+m)\*ln(c\*x^n)/d/n

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {2303}

$$\frac{a(dx)^{m+1} \log(cx^n)}{dn}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + (a\*(1 + m)\*Log[c\*x^n])/n), x]

[Out] (a\*(d\*x)^(1 + m)\*Log[c\*x^n])/(d\*n)

Rule 2303

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :>  
Simp[(b\*(d\*x)^(m + 1)\*Log[c\*x^n])/(d\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && EqQ[a\*(m + 1) - b\*n, 0]

Rubi steps

$$\int (dx)^m \left( a + \frac{a(1+m) \log(cx^n)}{n} \right) dx = \frac{a(dx)^{1+m} \log(cx^n)}{dn}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$\frac{ax(dx)^m \log(cx^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + (a\*(1 + m)\*Log[c\*x^n])/n), x]

[Out] (a\*x\*(d\*x)^m\*Log[c\*x^n])/n

**fricas [A]** time = 0.47, size = 26, normalized size = 1.24

$$\frac{(anx \log(x) + ax \log(c))e^{(m \log(d) + m \log(x))}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+a\*(1+m)\*log(c\*x^n)/n),x, algorithm="fricas")

[Out] (a\*n\*x\*log(x) + a\*x\*log(c))\*e^(m\*log(d) + m\*log(x))/n

**giac [B]** time = 0.46, size = 214, normalized size = 10.19

$$\frac{ad^{\frac{1}{d}} mxx^m |d|^{2m} \log(c)}{(d^2m + d^2)n} + \frac{ad^{\frac{1}{d}} xx^m |d|^{2m}}{d^2m + d^2} + \frac{ad^{\frac{1}{d}} xx^m |d|^{2m} \log(c)}{(d^2m + d^2)n} + \frac{ad^m m^2 xx^m \log(x)}{m^2 + 2m + 1} + \frac{2ad^m mxx^m \log(x)}{m^2 + 2m + 1} - \frac{a}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+a\*(1+m)\*log(c\*x^n)/n),x, algorithm="giac")

[Out] a\*d^2\*(1/d)^m\*m\*x\*x^m\*abs(d)^(2\*m)\*log(c)/((d^2\*m + d^2)\*n) + a\*d^2\*(1/d)^m\*x\*x^m\*abs(d)^(2\*m)/(d^2\*m + d^2) + a\*d^2\*(1/d)^m\*x\*x^m\*abs(d)^(2\*m)\*log(c)/((d^2\*m + d^2)\*n) + a\*d^m\*m^2\*x\*x^m\*log(x)/(m^2 + 2\*m + 1) + 2\*a\*d^m\*m\*x\*x^m\*log(x)/(m^2 + 2\*m + 1) - a\*d^m\*m\*x\*x^m/(m^2 + 2\*m + 1) + a\*d^m\*x\*x^m\*log(x)/(m^2 + 2\*m + 1) - a\*d^m\*x\*x^m/(m^2 + 2\*m + 1)

**maple [C]** time = 0.17, size = 260, normalized size = 12.38

$$ax e^{\frac{(-i\pi \operatorname{csgn}(id)\operatorname{csgn}(ix)\operatorname{csgn}(idx)+i\pi \operatorname{csgn}(id)\operatorname{csgn}(idx)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(idx)^2-i\pi \operatorname{csgn}(idx)^3+2\ln(d)+2\ln(x))m}{2}} \ln(x^n) + \frac{(-i\pi \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ix^n))m}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+a\*(m+1)\*ln(c\*x^n)/n),x)

[Out] a/n\*x\*exp(1/2\*m\*(-I\*Pi\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)+I\*Pi\*csgn(I\*d)\*csgn(I\*d\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*d\*x)^2-I\*Pi\*csgn(I\*d\*x)^3+2\*ln(d)+2\*ln(x)))\*ln(x^n)+1/2\*a\*(I\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*Pi\*csgn(I\*c\*x^n)^3+I\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*ln(c))\*x/n\*exp(1/2\*m\*(-I\*Pi\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)+I\*Pi\*csgn(I\*d)\*csgn(I\*d\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*d\*x)^2-I\*Pi\*csgn(I\*d\*x)^3+2\*ln(d)+2\*ln(x)))

**maxima [B]** time = 0.61, size = 102, normalized size = 4.86

$$-\frac{ad^m mxx^m}{(m+1)^2} - \frac{ad^m xx^m}{(m+1)^2} + \frac{(dx)^{m+1} am \log(cx^n)}{d(m+1)n} + \frac{(dx)^{m+1} a}{d(m+1)} + \frac{(dx)^{m+1} a \log(cx^n)}{d(m+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+a\*(1+m)\*log(c\*x^n)/n),x, algorithm="maxima")

[Out] -a\*d^m\*m\*x\*x^m/(m + 1)^2 - a\*d^m\*x\*x^m/(m + 1)^2 + (d\*x)^(m + 1)\*a\*m\*log(c\*x^n)/(d\*(m + 1)\*n) + (d\*x)^(m + 1)\*a/(d\*(m + 1)) + (d\*x)^(m + 1)\*a\*log(c\*x^n)/(d\*(m + 1)\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.05

$$\int (dx)^m \left( a + \frac{a \ln(c x^n) (m + 1)}{n} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + (a\*log(c\*x^n)\*(m + 1))/n),x)

[Out] int((d\*x)^m\*(a + (a\*log(c\*x^n)\*(m + 1))/n), x)

**sympy** [A] time = 0.98, size = 27, normalized size = 1.29

$$ad^m x x^m \log(x) + \frac{ad^m x x^m \log(c)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+a\*(1+m)\*ln(c\*x\*\*n)/n),x)

[Out] a\*d\*\*m\*x\*x\*\*m\*log(x) + a\*d\*\*m\*x\*x\*\*m\*log(c)/n

### 3.150 $\int (dx)^m \left( a + b \log(cx^n) \right)^3 dx$

**Optimal.** Leaf size=116

$$\frac{6b^2n^2(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)^3} + \frac{(dx)^{m+1} (a + b \log(cx^n))^3}{d(m+1)} - \frac{3bn(dx)^{m+1} (a + b \log(cx^n))^2}{d(m+1)^2} - \frac{6b^3n^3(dx)^{m+1}}{d(m+1)^4}$$

[Out]  $-6*b^3*n^3*(d*x)^{(1+m)}/d/(1+m)^4+6*b^2*n^2*(d*x)^{(1+m)}*(a+b*\ln(c*x^n))/d/(1+m)^3-3*b*n*(d*x)^{(1+m)}*(a+b*\ln(c*x^n))^2/d/(1+m)^2+(d*x)^{(1+m)}*(a+b*\ln(c*x^n))^3/d/(1+m)$

**Rubi [A]** time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2305, 2304}

$$\frac{6b^2n^2(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)^3} + \frac{(dx)^{m+1} (a + b \log(cx^n))^3}{d(m+1)} - \frac{3bn(dx)^{m+1} (a + b \log(cx^n))^2}{d(m+1)^2} - \frac{6b^3n^3(dx)^{m+1}}{d(m+1)^4}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^3,x]

[Out]  $(-6*b^3*n^3*(d*x)^{(1+m)})/(d*(1+m)^4) + (6*b^2*n^2*(d*x)^{(1+m)}*(a + b*\text{Log}[c*x^n]))/(d*(1+m)^3) - (3*b*n*(d*x)^{(1+m)}*(a + b*\text{Log}[c*x^n])^2)/(d*(1+m)^2) + ((d*x)^{(1+m)}*(a + b*\text{Log}[c*x^n])^3)/(d*(1+m))$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :=  
Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :=  
Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps



$$^2 + 3*a^2*b*m + a^2*b)*n)*x)*\log(x))*e^{(m*\log(d) + m*\log(x))}/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)$$

**giac** [B] time = 0.85, size = 1133, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out]  $b^3*d^m*m^3*n^3*x*x^m*\log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*b^3*d^m*m^2*n^3*x*x^m*\log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) - 3*b^3*d^m*m^2*n^3*x*x^m*\log(x)^2/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*b^3*d^m*m^2*n^2*x*x^m*\log(c)*\log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + 3*b^3*d^m*m*n^3*x*x^m*\log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*a*b^2*d^m*m^2*n^2*x*x^m*\log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 6*b^3*d^m*m*n^3*x*x^m*\log(x)^2/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 6*b^3*d^m*m*n^2*x*x^m*\log(c)*\log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + b^3*d^m*n^3*x*x^m*\log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 6*b^3*d^m*m*n^3*x*x^m*\log(x)/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) - 6*b^3*d^m*m*n^2*x*x^m*\log(c)*\log(x)/(m^3 + 3*m^2 + 3*m + 1) + 3*b^3*d^m*m*n*x*x^m*\log(c)^2*\log(x)/(m^2 + 2*m + 1) + 6*a*b^2*d^m*m*n^2*x*x^m*\log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 3*b^3*d^m*n^3*x*x^m*\log(x)^2/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*b^3*d^m*n^2*x*x^m*\log(c)*\log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 6*a*b^2*d^m*m*n^2*x*x^m*\log(x)/(m^3 + 3*m^2 + 3*m + 1) + 6*b^3*d^m*m*n^3*x*x^m*\log(x)/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 6*a*b^2*d^m*m*n*x*x^m*\log(c)*\log(x)/(m^2 + 2*m + 1) - 6*b^3*d^m*n^2*x*x^m*\log(c)*\log(x)/(m^3 + 3*m^2 + 3*m + 1) + 3*b^3*d^m*n*x*x^m*\log(c)^2*\log(x)/(m^2 + 2*m + 1) + 3*a*b^2*d^m*m*n^2*x*x^m*\log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 6*b^3*d^m*n^2*x*x^m*\log(c)/(m^3 + 3*m^2 + 3*m + 1) - 3*b^3*d^m*n*x*x^m*\log(c)^2/(m^2 + 2*m + 1) + 3*a^2*b*d^m*m*n*x*x^m*\log(x)/(m^2 + 2*m + 1) - 6*a*b^2*d^m*n^2*x*x^m*\log(x)/(m^3 + 3*m^2 + 3*m + 1) + 6*a*b^2*d^m*n*x*x^m*\log(c)*\log(x)/(m^2 + 2*m + 1) + 6*a*b^2*d^m*m*n^2*x*x^m/(m^3 + 3*m^2 + 3*m + 1) - 6*a*b^2*d^m*n*x*x^m*\log(c)/(m^2 + 2*m + 1) + (d*x)^m*b^3*x*\log(c)^3/(m + 1) + 3*a^2*b*d^m*n*x*x^m*\log(x)/(m^2 + 2*m + 1) - 3*a^2*b*d^m*n*x*x^m/(m^2 + 2*m + 1) + 3*(d*x)^m*a*b^2*x*\log(c)^2/(m + 1) + 3*(d*x)^m*a^2*b*x*\log(c)/(m + 1) + (d*x)^m*a^3*x/(m + 1)$

**maple** [C] time = 0.78, size = 9684, normalized size = 83.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(b\*ln(c\*x^n)+a)^3,x)

[Out] result too large to display





$$\begin{aligned}
& c)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*a^{**2}*b*d^{**m}*n*x*x^{**m}*log(x)/(m^{**4} \\
& + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 3*a^{**2}*b*d^{**m}*n*x*x^{**m}/(m^{**4} + 4*m^{**3} + 6*m \\
& **2 + 4*m + 1) + 3*a^{**2}*b*d^{**m}*x*x^{**m}*log(c)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m \\
& + 1) + 3*a*b^{**2}*d^{**m}*m^{**3}*n*x*x^{**m}*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4 \\
& *m + 1) + 6*a*b^{**2}*d^{**m}*m^{**3}*n*x*x^{**m}*log(c)*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} \\
& + 4*m + 1) + 3*a*b^{**2}*d^{**m}*m^{**3}*x*x^{**m}*log(c)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + \\
& 4*m + 1) + 9*a*b^{**2}*d^{**m}*m^{**2}*n*x*x^{**m}*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{** \\
& 2 + 4*m + 1) - 6*a*b^{**2}*d^{**m}*m^{**2}*n*x*x^{**m}*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{** \\
& 2 + 4*m + 1) + 18*a*b^{**2}*d^{**m}*m^{**2}*n*x*x^{**m}*log(c)*log(x)/(m^{**4} + 4*m^{**3} + \\
& 6*m^{**2} + 4*m + 1) - 6*a*b^{**2}*d^{**m}*m^{**2}*n*x*x^{**m}*log(c)/(m^{**4} + 4*m^{**3} + 6*m \\
& **2 + 4*m + 1) + 9*a*b^{**2}*d^{**m}*m^{**2}*x*x^{**m}*log(c)**2/(m^{**4} + 4*m^{**3} + 6*m^{** \\
& 2 + 4*m + 1) + 9*a*b^{**2}*d^{**m}*m*n*x*x^{**m}*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{** \\
& 2 + 4*m + 1) - 12*a*b^{**2}*d^{**m}*m*n*x*x^{**m}*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} \\
& + 4*m + 1) + 6*a*b^{**2}*d^{**m}*m*n*x*x^{**m}/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) \\
& + 18*a*b^{**2}*d^{**m}*m*n*x*x^{**m}*log(c)*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + \\
& 1) - 12*a*b^{**2}*d^{**m}*m*n*x*x^{**m}*log(c)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + \\
& 9*a*b^{**2}*d^{**m}*m*x*x^{**m}*log(c)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*a*b \\
& **2*d^{**m}*n*x*x^{**m}*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 6*a*b^{** \\
& 2*d^{**m}*n*x*x^{**m}*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 6*a*b^{**2}*d^{** \\
& m*n*x*x^{**m}/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 6*a*b^{**2}*d^{**m}*n*x*x^{**m}*l \\
& og(c)*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 6*a*b^{**2}*d^{**m}*n*x*x^{**m}*lo \\
& g(c)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*a*b^{**2}*d^{**m}*x*x^{**m}*log(c)**2/(m \\
& **4 + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + b^{**3}*d^{**m}*m^{**3}*n*x*x^{**m}*log(x)**3/(m \\
& *4 + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*b^{**3}*d^{**m}*m^{**3}*n*x*x^{**m}*log(c)*log(x) \\
& )**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*b^{**3}*d^{**m}*m^{**3}*n*x*x^{**m}*log(c)* \\
& **2*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + b^{**3}*d^{**m}*m^{**3}*x*x^{**m}*log(c) \\
& **3/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*b^{**3}*d^{**m}*m^{**2}*n*x*x^{**m}*log(x) \\
& )**3/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 3*b^{**3}*d^{**m}*m^{**2}*n*x*x^{**m}*log(c) \\
& **2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 9*b^{**3}*d^{**m}*m^{**2}*n*x*x^{**m}*log \\
& (c)*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 6*b^{**3}*d^{**m}*m^{**2}*n*x \\
& x^{**m}*log(c)*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 9*b^{**3}*d^{**m}*m^{**2}*n \\
& x*x^{**m}*log(c)**2*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 3*b^{**3}*d^{**m}*m \\
& **2*n*x*x^{**m}*log(c)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*b^{**3}*d^{**m}*m^{**2} \\
& *x*x^{**m}*log(c)**3/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*b^{**3}*d^{**m}*m*n*x \\
& *x^{**m}*log(x)**3/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 6*b^{**3}*d^{**m}*m*n*x*x \\
& **m*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 6*b^{**3}*d^{**m}*m*n*x*x^{** \\
& m*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 9*b^{**3}*d^{**m}*m*n*x*x^{**m}*log \\
& (c)*log(x)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 12*b^{**3}*d^{**m}*m*n*x*x \\
& **m*log(c)*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 6*b^{**3}*d^{**m}*m*n*x \\
& x^{**m}*log(c)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 9*b^{**3}*d^{**m}*m*n*x*x^{**m}*log \\
& (c)**2*log(x)/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 6*b^{**3}*d^{**m}*m*n*x*x^{**m}*l \\
& og(c)**2/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 3*b^{**3}*d^{**m}*m*x*x^{**m}*log(c)** \\
& 3/(m^{**4} + 4*m^{**3} + 6*m^{**2} + 4*m + 1) + b^{**3}*d^{**m}*n*x*x^{**m}*log(x)**3/(m \\
& 4 + 4*m^{**3} + 6*m^{**2} + 4*m + 1) - 3*b^{**3}*d^{**m}*n*x*x^{**m}*log(x)**2/(m^{**4} + \\
& 4*m^{**3} + 6*m^{**2} + 4*m + 1) + 6*b^{**3}*d^{**m}*n*x*x^{**m}*log(x)/(m^{**4} + 4*m^{**3}
\end{aligned}$$

```

+ 6*m**2 + 4*m + 1) - 6*b**3*d**m*n**3*x*x**m/(m**4 + 4*m**3 + 6*m**2 + 4*m
+ 1) + 3*b**3*d**m*n**2*x*x**m*log(c)*log(x)**2/(m**4 + 4*m**3 + 6*m**2 +
4*m + 1) - 6*b**3*d**m*n**2*x*x**m*log(c)*log(x)/(m**4 + 4*m**3 + 6*m**2 +
4*m + 1) + 6*b**3*d**m*n**2*x*x**m*log(c)/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1
) + 3*b**3*d**m*n*x*x**m*log(c)**2*log(x)/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1
) - 3*b**3*d**m*n*x*x**m*log(c)**2/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + b**
3*d**m*x*x**m*log(c)**3/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1), Ne(m, -1)), (Pi
ecewise(((a**3*log(c*x**n) + 3*a**2*b*log(c*x**n)**2/2 + a*b**2*log(c*x**n)
**3 + b**3*log(c*x**n)**4/4)/n, Ne(n, 0)), ((a**3 + 3*a**2*b*log(c) + 3*a*b
**2*log(c)**2 + b**3*log(c)**3)*log(x), True))/d, True))

```

### 3.151 $\int (dx)^m \left( a + b \log(cx^n) \right)^2 dx$

**Optimal.** Leaf size=81

$$\frac{(dx)^{m+1} \left( a + b \log(cx^n) \right)^2}{d(m+1)} - \frac{2bn(dx)^{m+1} \left( a + b \log(cx^n) \right)}{d(m+1)^2} + \frac{2b^2n^2(dx)^{m+1}}{d(m+1)^3}$$

[Out]  $2*b^2*n^2*(d*x)^{(1+m)}/d/(1+m)^3 - 2*b*n*(d*x)^{(1+m)*(a+b*\ln(c*x^n))/d/(1+m)^2 + (d*x)^{(1+m)*(a+b*\ln(c*x^n))^2/d/(1+m)}$

**Rubi [A]** time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2305, 2304}

$$\frac{(dx)^{m+1} \left( a + b \log(cx^n) \right)^2}{d(m+1)} - \frac{2bn(dx)^{m+1} \left( a + b \log(cx^n) \right)}{d(m+1)^2} + \frac{2b^2n^2(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(2*b^2*n^2*(d*x)^{(1+m)})/(d*(1+m)^3) - (2*b*n*(d*x)^{(1+m)*(a+b*Log[c*x^n]))/(d*(1+m)^2) + ((d*x)^{(1+m)*(a+b*Log[c*x^n])^2)/(d*(1+m))$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\int (dx)^m (a + b \log(cx^n))^2 dx = \frac{(dx)^{1+m} (a + b \log(cx^n))^2}{d(1+m)} - \frac{(2bn) \int (dx)^m (a + b \log(cx^n)) dx}{1+m}$$

$$= \frac{2b^2 n^2 (dx)^{1+m}}{d(1+m)^3} - \frac{2bn(dx)^{1+m} (a + b \log(cx^n))}{d(1+m)^2} + \frac{(dx)^{1+m} (a + b \log(cx^n))^2}{d(1+m)}$$

**Mathematica [A]** time = 0.04, size = 76, normalized size = 0.94

$$\frac{x(dx)^m (a^2(m+1)^2 + 2b(m+1)(am + a - bn) \log(cx^n) - 2ab(m+1)n + b^2(m+1)^2 \log^2(cx^n) + 2b^2 n^2)}{(m+1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x^n])^2,x]

[Out] (x\*(d\*x)^m\*(a^2\*(1+m)^2 - 2\*a\*b\*(1+m)\*n + 2\*b^2\*n^2 + 2\*b\*(1+m)\*(a + a\*m - b\*n)\*Log[c\*x^n] + b^2\*(1+m)^2\*Log[c\*x^n]^2))/(1+m)^3

**fricas [B]** time = 0.45, size = 208, normalized size = 2.57

$$\frac{((b^2 m^2 + 2 b^2 m + b^2) n^2 x \log(x)^2 + (b^2 m^2 + 2 b^2 m + b^2) x \log(c)^2 + 2 (ab m^2 + 2 ab m + ab - (b^2 m + b^2) n) x \log(c) \log(x) + (a^2 m^2 + 2 a b m^2 + 2 a^2 m + a^2 - 2 (a b m + a b) n) x + 2 ((b^2 m^2 + 2 b^2 m + b^2) n x \log(c) - ((b^2 m + b^2) n^2 - (a b m^2 + 2 a b m + a b) n) x) \log(x) * e^{(m \log(d) + m \log(x))}}{(m^3 + 3 m^2 + 3 m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] ((b^2\*m^2 + 2\*b^2\*m + b^2)\*n^2\*x\*log(x)^2 + (b^2\*m^2 + 2\*b^2\*m + b^2)\*x\*log(c)^2 + 2\*(a\*b\*m^2 + 2\*a\*b\*m + a\*b - (b^2\*m + b^2)\*n)\*x\*log(c) + (a^2\*m^2 + 2\*b^2\*n^2 + 2\*a^2\*m + a^2 - 2\*(a\*b\*m + a\*b)\*n)\*x + 2\*((b^2\*m^2 + 2\*b^2\*m + b^2)\*n\*x\*log(c) - ((b^2\*m + b^2)\*n^2 - (a\*b\*m^2 + 2\*a\*b\*m + a\*b)\*n)\*x)\*log(x)\*e^(m\*log(d) + m\*log(x))/(m^3 + 3\*m^2 + 3\*m + 1)

**giac [B]** time = 0.41, size = 402, normalized size = 4.96

$$\frac{b^2 d^m m^2 n^2 x x^m \log(x)^2}{m^3 + 3 m^2 + 3 m + 1} + \frac{2 b^2 d^m m n^2 x x^m \log(x)^2}{m^3 + 3 m^2 + 3 m + 1} - \frac{2 b^2 d^m m n^2 x x^m \log(x)}{m^3 + 3 m^2 + 3 m + 1} + \frac{2 b^2 d^m m n x x^m \log(c) \log(x)}{m^2 + 2 m + 1} + \frac{b^2 d^m n^2 x x^m \log(c)^2}{m^3 + 3 m^2 + 3 m + 1} + \frac{2 (a b m^2 + 2 a b m + a b - (b^2 m + b^2) n) x \log(c) \log(x)}{m^3 + 3 m^2 + 3 m + 1} + \frac{(a^2 m^2 + 2 a b m^2 + 2 a^2 m + a^2 - 2 (a b m + a b) n) x}{m^3 + 3 m^2 + 3 m + 1} + \frac{2 ((b^2 m^2 + 2 b^2 m + b^2) n x \log(c) - ((b^2 m + b^2) n^2 - (a b m^2 + 2 a b m + a b) n) x) \log(x)}{m^3 + 3 m^2 + 3 m + 1} * e^{(m \log(d) + m \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

```
[Out] b^2*d^m*m^2*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + 2*b^2*d^m*m*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 2*b^2*d^m*m*n^2*x*x^m*log(x)/(m^3 + 3*m^2 + 3*m + 1) + 2*b^2*d^m*m*n*x*x^m*log(c)*log(x)/(m^2 + 2*m + 1) + b^2*d^m*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + 2*a*b*d^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - 2*b^2*d^m*n^2*x*x^m*log(x)/(m^3 + 3*m^2 + 3*m + 1) + 2*b^2*d^m*n*x*x^m*log(c)*log(x)/(m^2 + 2*m + 1) + 2*b^2*d^m*n^2*x*x^m/(m^3 + 3*m^2 + 3*m + 1) - 2*b^2*d^m*n*x*x^m*log(c)/(m^2 + 2*m + 1) + 2*a*b*d^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - 2*a*b*d^m*n*x*x^m/(m^2 + 2*m + 1) + (d*x)^m*b^2*x*log(c)^2/(m + 1) + 2*(d*x)^m*a*b*x*log(c)/(m + 1) + (d*x)^m*a^2*x/(m + 1)
```

**maple [C]** time = 0.25, size = 2126, normalized size = 26.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*ln(c*x^n)+a)^2,x)
```

```
[Out] b^2/(m+1)*x*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)*ln(x^n)^2-b*(-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*m+I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)*m+I*Pi*b*csgn(I*c*x^n)^3*m-I*Pi*b*csgn(I*c*x^n)^2*csgn(I*c)*m-I*csgn(I*c*x^n)^2*csgn(I*x^n)*b*Pi+I*csgn(I*c)*csgn(I*c*x^n)*csgn(I*x^n)*b*Pi+I*csgn(I*c*x^n)^3*b*Pi-I*csgn(I*c)*csgn(I*c*x^n)^2*b*Pi-2*b*ln(c)*m-2*b*ln(c)-2*a*m+2*b*n-2*a)/(m+1)^2*x*exp(1/2*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*Pi*csgn(I*d)*csgn(I*d*x)^2+I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*d*x)^3+2*ln(d)+2*ln(x))*m)*ln(x^n)+1/4*(4*ln(c)^2*b^2*m^2+8*ln(c)^2*b^2*m-Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-4*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+2*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+2*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-4*I*Pi*a*b*csgn(I*c*x^n)^3-4*I*Pi*b^2*csgn(I*c*x^n)^3*ln(c)+4*a^2+8*b^2*n^2-8*a*b*m*n+8*a*b*ln(c)-8*b^2*n*ln(c)+4*b^2*ln(c)^2+4*a^2*m^2+8*a^2*m-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-8*a*b*n-Pi^2*b^2*csgn(I*c*x^n)^6+2*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5-Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-8*I*Pi*ln(c)*b^2*m*csgn(I*c*x^n)^3-4*I*Pi*a*b*m^2*csgn(I*c*x^n)^3+4*I*Pi*b^2*m*n*csgn(I*c*x^n)^3-8*I*Pi*a*b*m*csgn(I*c*x^n)^3+2*Pi^2*b^2*m^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)+4*Pi^2*b^2*m*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-2*Pi^2*b^2*m*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^2*m^2*csgn(I*x^n)*csgn(I*c*x^n)^5+4*Pi^2*b^2*m*csgn(I*x^n)*csgn(I*c*x^n)^5+2*Pi^2*b^2*m^2*csgn(I*c*x^n)^5*csgn(I*c)+4*Pi^2*b^2*m*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^2*m^2*csgn(I*c*x^n)^4*csgn(I*c)^2+4*I*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2+4*I*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(c)+4*I*Pi*b^2*csgn(I*c)*csgn(I*c*x^n)^2*ln(c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+8*I*Pi*a*b*m*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*b^2*n*csgn(I*c*x^n)^3-Pi^2*b^2*m^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-4*I*Pi*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln
```

(c)+8\*ln(c)\*a\*b\*m^2+16\*ln(c)\*a\*b\*m-8\*ln(c)\*b^2\*m\*n+8\*I\*Pi\*a\*b\*m\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+4\*I\*Pi\*ln(c)\*b^2\*m^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2+4\*I\*Pi\*ln(c)\*b^2\*m^2\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+8\*I\*Pi\*ln(c)\*b^2\*m\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-2\*Pi^2\*b^2\*m\*csgn(I\*c\*x^n)^4\*csgn(I\*c)^2-4\*I\*Pi\*b^2\*m\*n\*csgn(I\*c\*x^n)^2\*csgn(I\*c)-4\*I\*Pi\*b^2\*n\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-4\*I\*Pi\*b^2\*n\*csgn(I\*c\*x^n)^2\*csgn(I\*c)-4\*I\*Pi\*ln(c)\*b^2\*m^2\*csgn(I\*c\*x^n)^3-Pi^2\*b^2\*m^2\*csgn(I\*x^n)^2\*csgn(I\*c\*x^n)^2\*csgn(I\*c)^2-2\*Pi^2\*b^2\*m\*csgn(I\*x^n)^2\*csgn(I\*c\*x^n)^2\*csgn(I\*c)^2-4\*Pi^2\*b^2\*m^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^4\*csgn(I\*c)-8\*Pi^2\*b^2\*m\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^4\*csgn(I\*c)+2\*Pi^2\*b^2\*m^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^3\*csgn(I\*c)^2+4\*Pi^2\*b^2\*m\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^3\*csgn(I\*c)^2-4\*I\*Pi\*a\*b\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)-4\*I\*Pi\*ln(c)\*b^2\*m^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-Pi^2\*b^2\*m^2\*csgn(I\*c\*x^n)^6-2\*Pi^2\*b^2\*m\*csgn(I\*c\*x^n)^6-8\*I\*Pi\*ln(c)\*b^2\*m\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-4\*I\*Pi\*a\*b\*m^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)+4\*I\*Pi\*b^2\*m\*n\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-8\*I\*Pi\*a\*b\*m\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)+4\*I\*Pi\*b^2\*n\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)+8\*I\*Pi\*ln(c)\*b^2\*m\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+4\*I\*Pi\*a\*b\*m^2\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2+4\*I\*Pi\*a\*b\*m^2\*csgn(I\*c\*x^n)^2\*csgn(I\*c)-4\*I\*Pi\*b^2\*m\*n\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2)/(m+1)^3\*x\*exp(1/2\*(-I\*Pi\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)+I\*Pi\*csgn(I\*d)\*csgn(I\*d\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*d\*x)^2-I\*Pi\*csgn(I\*d\*x)^3+2\*ln(d)+2\*ln(x))\*m)

**maxima** [A] time = 0.71, size = 132, normalized size = 1.63

$$-\frac{2abd^m nxx^m}{(m+1)^2} - 2 \left( \frac{d^m nxx^m \log(cx^n)}{(m+1)^2} - \frac{d^m n^2 xx^m}{(m+1)^3} \right) b^2 + \frac{(dx)^{m+1} b^2 \log(cx^n)^2}{d(m+1)} + \frac{2(dx)^{m+1} ab \log(cx^n)}{d(m+1)} + \frac{(dx)^{m+1} a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -2\*a\*b\*d^m\*n\*x\*x^m/(m+1)^2 - 2\*(d^m\*n\*x\*x^m\*log(c\*x^n)/(m+1)^2 - d^m\*n^2\*x\*x^m/(m+1)^3)\*b^2 + (d\*x)^(m+1)\*b^2\*log(c\*x^n)^2/(d\*(m+1)) + 2\*(d\*x)^(m+1)\*a\*b\*log(c\*x^n)/(d\*(m+1)) + (d\*x)^(m+1)\*a^2/(d\*(m+1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^m (a + b \ln(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*log(c\*x^n))^2,x)

[Out] int((d\*x)^m\*(a + b\*log(c\*x^n))^2, x)

sympy [A] time = 27.73, size = 891, normalized size = 11.00

$$\left\{ \begin{array}{l} \frac{a^2 d^m m^2 x x^m}{m^3 + 3m^2 + 3m + 1} + \frac{2a^2 d^m m x x^m}{m^3 + 3m^2 + 3m + 1} + \frac{a^2 d^m x x^m}{m^3 + 3m^2 + 3m + 1} + \frac{2abd^m m^2 n x x^m \log(x)}{m^3 + 3m^2 + 3m + 1} + \frac{2abd^m m^2 x x^m \log(c)}{m^3 + 3m^2 + 3m + 1} + \frac{4abd^m m n x x^m \log(x)}{m^3 + 3m^2 + 3m + 1} - \frac{2abd^m m}{m^3 + 3m^2 + 3m + 1} \\ \frac{a^2 \log(cx^n) + ab \log(cx^n)^2 + \frac{b^2 \log(cx^n)^3}{3}}{n} \quad \text{for } n \neq 0 \\ \frac{(a^2 + 2ab \log(c) + b^2 \log(c)^2) \log(x)}{d} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Piecewise((a\*\*2\*d\*\*m\*m\*\*2\*x\*x\*\*m/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*a\*\*2\*d\*\*m\*m\*x\*x\*\*m/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + a\*\*2\*d\*\*m\*x\*x\*\*m/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*a\*b\*d\*\*m\*m\*\*2\*n\*x\*x\*\*m\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*a\*b\*d\*\*m\*m\*\*2\*x\*x\*\*m\*log(c)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 4\*a\*b\*d\*\*m\*m\*n\*x\*x\*\*m\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) - 2\*a\*b\*d\*\*m\*m\*n\*x\*x\*\*m/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 4\*a\*b\*d\*\*m\*m\*x\*x\*\*m\*log(c)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*a\*b\*d\*\*m\*n\*x\*x\*\*m\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) - 2\*a\*b\*d\*\*m\*n\*x\*x\*\*m/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*a\*b\*d\*\*m\*x\*x\*\*m\*log(c)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + b\*\*2\*d\*\*m\*m\*\*2\*n\*\*2\*x\*x\*\*m\*log(x)\*\*2/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*b\*\*2\*d\*\*m\*m\*\*2\*n\*x\*x\*\*m\*log(c)\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + b\*\*2\*d\*\*m\*m\*\*2\*x\*x\*\*m\*log(c)\*\*2/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*b\*\*2\*d\*\*m\*m\*n\*\*2\*x\*x\*\*m\*log(x)\*\*2/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) - 2\*b\*\*2\*d\*\*m\*m\*n\*\*2\*x\*x\*\*m\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 4\*b\*\*2\*d\*\*m\*m\*n\*x\*x\*\*m\*log(c)\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) - 2\*b\*\*2\*d\*\*m\*m\*n\*x\*x\*\*m\*log(c)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*b\*\*2\*d\*\*m\*m\*x\*x\*\*m\*log(c)\*\*2/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + b\*\*2\*d\*\*m\*n\*\*2\*x\*x\*\*m\*log(x)\*\*2/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) - 2\*b\*\*2\*d\*\*m\*n\*\*2\*x\*x\*\*m\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + 2\*b\*\*2\*d\*\*m\*n\*x\*x\*\*m\*log(c)\*log(x)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) - 2\*b\*\*2\*d\*\*m\*n\*x\*x\*\*m\*log(c)/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1) + b\*\*2\*d\*\*m\*x\*x\*\*m\*log(c)\*\*2/(m\*\*3 + 3\*m\*\*2 + 3\*m + 1)), Ne(m, -1)), (Piecewise(((a\*\*2\*log(c\*x\*\*n) + a\*b\*log(c\*x\*\*n))\*\*2 + b\*\*2\*log(c\*x\*\*n)\*\*3/3)/n, Ne(n, 0)), ((a\*\*2 + 2\*a\*b\*log(c) + b\*\*2\*log(c)\*\*2)\*log(x), True))/d, True))

### 3.152 $\int (dx)^m (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=46

$$\frac{(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1}}{d(m+1)^2}$$

[Out]  $-b*n*(d*x)^{(1+m)}/d/(1+m)^2+(d*x)^{(1+m)*(a+b*\ln(c*x^n))/d/(1+m)$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {2304}

$$\frac{(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n]),x]

[Out]  $-((b*n*(d*x)^{(1+m)})/(d*(1+m)^2)) + ((d*x)^{(1+m)*(a + b*Log[c*x^n])})/(d*(1+m))$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int (dx)^m (a + b \log(cx^n)) dx = -\frac{bn(dx)^{1+m}}{d(1+m)^2} + \frac{(dx)^{1+m} (a + b \log(cx^n))}{d(1+m)}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 0.70

$$\frac{x(dx)^m (am + a + b(m+1) \log(cx^n) - bn)}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x^n]),x]



[Out]  $(x*(d*x)^m*(a + a*m - b*n + b*(1 + m)*\text{Log}[c*x^n]))/(1 + m)^2$

**fricas** [A] time = 0.51, size = 52, normalized size = 1.13

$$\frac{((bm + b)nx \log(x) + (bm + b)x \log(c) + (am - bn + a)x)e^{(m \log(d) + m \log(x))}}{m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $((b*m + b)*n*x*\log(x) + (b*m + b)*x*\log(c) + (a*m - b*n + a)*x)*e^{(m*\log(d) + m*\log(x))}/(m^2 + 2*m + 1)$

**giac** [B] time = 0.32, size = 95, normalized size = 2.07

$$\frac{bd^m m n x x^m \log(x)}{m^2 + 2m + 1} + \frac{bd^m n x x^m \log(x)}{m^2 + 2m + 1} - \frac{bd^m n x x^m}{m^2 + 2m + 1} + \frac{(dx)^m b x \log(c)}{m + 1} + \frac{(dx)^m a x}{m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="giac")`

[Out]  $b*d^m*m*n*x*x^m*\log(x)/(m^2 + 2*m + 1) + b*d^m*n*x*x^m*\log(x)/(m^2 + 2*m + 1) - b*d^m*n*x*x^m/(m^2 + 2*m + 1) + (d*x)^m*b*x*\log(c)/(m + 1) + (d*x)^m*a*x/(m + 1)$

**maple** [C] time = 0.17, size = 371, normalized size = 8.07

$$\frac{bx e^{\frac{(-i\pi \operatorname{csgn}(id)\operatorname{csgn}(ix)\operatorname{csgn}(idx) + i\pi \operatorname{csgn}(id)\operatorname{csgn}(idx)^2 + i\pi \operatorname{csgn}(ix)\operatorname{csgn}(idx)^2 - i\pi \operatorname{csgn}(idx)^3 + 2\ln(d) + 2\ln(x))m}}{\ln(x^n)}}{m + 1} \left( i\pi b m \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ix) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(b*ln(c*x^n)+a),x)`

[Out]  $b/(m+1)*x*\exp(1/2*(-I*\text{Pi}*c\text{sgn}(I*d)*c\text{sgn}(I*x)*c\text{sgn}(I*d*x)+I*\text{Pi}*c\text{sgn}(I*d)*c\text{sgn}(I*d*x)^2+I*\text{Pi}*c\text{sgn}(I*x)*c\text{sgn}(I*d*x)^2-I*\text{Pi}*c\text{sgn}(I*d*x)^3+2*\ln(d)+2*\ln(x))*m)*\ln(x^n)-1/2*(I*\text{Pi}*b*m*c\text{sgn}(I*c)*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)-I*\text{Pi}*b*m*c\text{sgn}(I*c)*c\text{sgn}(I*c*x^n)^2-I*\text{Pi}*b*m*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2+I*\text{Pi}*b*m*c\text{sgn}(I*c*x^n)^3+I*\text{Pi}*b*c\text{sgn}(I*c)*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)-I*\text{Pi}*b*c\text{sgn}(I*c)*c\text{sgn}(I*c*x^n)^2-I*\text{Pi}*b*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2+I*\text{Pi}*b*c\text{sgn}(I*c*x^n)^3-2*b*m*\ln(c)-2*a*m+2*b*n-2*b*\ln(c)-2*a)/(m+1)^2*x*\exp(1/2*(-I*\text{Pi}*c\text{sgn}(I*d)*c\text{sgn}(I*x)*c\text{sgn}(I*d*x)+I*\text{Pi}*c\text{sgn}(I*d)*c\text{sgn}(I*d*x)^2+I*\text{Pi}*c\text{sgn}(I*x)*c\text{sgn}(I*d*x)^2-I*\text{Pi}*c\text{sgn}(I*d*x)^3+2*\ln(d)+2*\ln(x))*m)$

**maxima** [A] time = 0.57, size = 57, normalized size = 1.24

$$-\frac{bd^m n x x^m}{(m+1)^2} + \frac{(dx)^{m+1} b \log(cx^n)}{d(m+1)} + \frac{(dx)^{m+1} a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -b\*d^m\*n\*x\*x^m/(m + 1)^2 + (d\*x)^(m + 1)\*b\*log(c\*x^n)/(d\*(m + 1)) + (d\*x)^(m + 1)\*a/(d\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (dx)^m (a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*log(c\*x^n)),x)

[Out] int((d\*x)^m\*(a + b\*log(c\*x^n)), x)

**sympy** [A] time = 10.45, size = 192, normalized size = 4.17

$$\left\{ \begin{array}{l} \frac{ad^m m x x^m}{m^2+2m+1} + \frac{ad^m x x^m}{m^2+2m+1} + \frac{bd^m m n x x^m \log(x)}{m^2+2m+1} + \frac{bd^m m x x^m \log(c)}{m^2+2m+1} + \frac{bd^m n x x^m \log(x)}{m^2+2m+1} - \frac{bd^m n x x^m}{m^2+2m+1} + \frac{bd^m x x^m \log(c)}{m^2+2m+1} \quad \text{for } m \neq -1 \\ \left\{ \begin{array}{l} a \log(x) \quad \text{for } b = 0 \\ -(-a - b \log(c)) \log(x) \quad \text{for } n = 0 \\ \frac{(-a - b \log(cx^n))^2}{2bn} \quad \text{otherwise} \end{array} \right. \\ \frac{\quad}{d} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x\*\*n)),x)

[Out] Piecewise((a\*d\*\*m\*x\*x\*\*m/(m\*\*2 + 2\*m + 1) + a\*d\*\*m\*x\*x\*\*m/(m\*\*2 + 2\*m + 1) + b\*d\*\*m\*n\*x\*x\*\*m\*log(x)/(m\*\*2 + 2\*m + 1) + b\*d\*\*m\*x\*x\*\*m\*log(c)/(m\*\*2 + 2\*m + 1) + b\*d\*\*m\*n\*x\*x\*\*m\*log(x)/(m\*\*2 + 2\*m + 1) - b\*d\*\*m\*n\*x\*x\*\*m/(m\*\*2 + 2\*m + 1) + b\*d\*\*m\*x\*x\*\*m\*log(c)/(m\*\*2 + 2\*m + 1), Ne(m, -1)), (Piecewise((a\*log(x), Eq(b, 0)), (-(-a - b\*log(c))\*log(x), Eq(n, 0)), ((-a - b\*log(c\*x\*\*n))\*\*2/(2\*b\*n), True))/d, True))

$$3.153 \quad \int \frac{(dx)^m}{a+b \log(cx^n)} dx$$

Optimal. Leaf size=66

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{bdn}$$

[Out]  $(d*x)^{(1+m)}*Ei((1+m)*(a+b*\ln(c*x^n))/b/n)/b/d/\exp(a*(1+m)/b/n)/n/((c*x^n)^{(1+m)/n})$

**Rubi [A]** time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2178}

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^m/(a + b*\text{Log}[c*x^n]),x]$

[Out]  $((d*x)^{(1+m)}*\text{ExpIntegralEi}(((1+m)*(a+b*\text{Log}[c*x^n]))/(b*n)))/(b*d*E^{(a*(1+m)/(b*n))*n*(c*x^n)^{(1+m)/n})}$

Rule 2178

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)})*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$  FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2310

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]* (b_.)^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[E^{((m+1)*x)/n}*(a + b*x)^p, x], x, \text{Log}[c*x^n]], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{(dx)^m}{a + b \log(cx^n)} dx = \frac{\left( (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int \frac{e^{\frac{(1+m)x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{dn}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \text{Ei} \left( \frac{(1+m)(a+b \log(cx^n))}{bn} \right)}{bdn}$$

**Mathematica** [A] time = 0.12, size = 67, normalized size = 1.02

$$\frac{x^{-m} (dx)^m \exp \left( -\frac{(m+1)(a+b(\log(cx^n)-n \log(x)))}{bn} \right) \text{Ei} \left( \frac{(m+1)(a+b \log(cx^n))}{bn} \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m/(a + b\*Log[c\*x^n]),x]

[Out] ((d\*x)^m\*ExpIntegralEi[((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^(((1 + m)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n])))/(b\*n))\*n\*x^m)

**fricas** [A] time = 0.44, size = 68, normalized size = 1.03

$$\frac{\text{Ei} \left( \frac{(bm+b)n \log(x)+am+(bm+b) \log(c)+a}{bn} \right) e^{\left( \frac{bmn \log(d)-am-(bm+b) \log(c)-a}{bn} \right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] Ei(((b\*m + b)\*n\*log(x) + a\*m + (b\*m + b)\*log(c) + a)/(b\*n))\*e^((b\*m\*n\*log(d) - a\*m - (b\*m + b)\*log(c) - a)/(b\*n))/(b\*n)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a), x)

**maple** [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \ln(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(b\*ln(c\*x^n)+a), x)

[Out] int((d\*x)^m/(b\*ln(c\*x^n)+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx)^m}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a + b\*log(c\*x^n)), x)

[Out] int((d\*x)^m/(a + b\*log(c\*x^n)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*ln(c\*x\*\*n)), x)

[Out] Integral((d\*x)\*\*m/(a + b\*log(c\*x\*\*n)), x)

$$3.154 \quad \int \frac{(dx)^m}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=100

$$\frac{(m+1)(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{b^2 d n^2} - \frac{(dx)^{m+1}}{bdn(a+b \log(cx^n))}$$

[Out] (1+m)\*(d\*x)^(1+m)\*Ei(((1+m)\*(a+b\*ln(c\*x^n))/b/n)/b^2/d/exp(a\*(1+m)/b/n)/n^2/((c\*x^n)^((1+m)/n))-(d\*x)^(1+m)/b/d/n/(a+b\*ln(c\*x^n))

**Rubi [A]** time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2306, 2310, 2178}

$$\frac{(m+1)(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{b^2 d n^2} - \frac{(dx)^{m+1}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m/(a + b\*Log[c\*x^n])^2,x]

[Out] (((1 + m)\*(d\*x)^(1 + m)\*ExpIntegralEi[(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))]/(b^2\*d\*E^((a\*(1 + m))/(b\*n))\*n^2\*(c\*x^n)^((1 + m)/n)) - (d\*x)^(1 + m)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2178

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[(((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^(p+1))/(b\*d\*n\*(p+1)), x] - Dist[(m+1)/(b\*n\*(p+1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p+1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^((m+1)\*x)

$/n) * (a + b*x)^p, x], x, \text{Log}[c*x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(dx)^m}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{1+m}}{bdn(a + b \log(cx^n))} + \frac{(1+m) \int \frac{(dx)^m}{a+b \log(cx^n)} dx}{bn} \\ &= -\frac{(dx)^{1+m}}{bdn(a + b \log(cx^n))} + \frac{\left( (1+m)(dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int \frac{e^{\frac{(1+m)x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bdn^2} \\ &= \frac{e^{-\frac{a(1+m)}{bn}} (1+m)(dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \text{Ei} \left( \frac{(1+m)(a+b \log(cx^n))}{bn} \right)}{b^2 dn^2} - \frac{(dx)^{1+m}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 89, normalized size = 0.89

$$\frac{(dx)^m \left( (m+1)x^{-m} \exp \left( -\frac{(m+1)(a+b \log(cx^n)-bn \log(x))}{bn} \right) \text{Ei} \left( \frac{(m+1)(a+b \log(cx^n))}{bn} \right) - \frac{bnx}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m/(a + b\*Log[c\*x^n])^2,x]

[Out] ((d\*x)^m\*((1+m)\*ExpIntegralEi[((1+m)\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((1+m)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*x^m - (b\*n\*x)/(a + b\*Log[c\*x^n]))/(b^2\*n^2)

**fricas [A]** time = 0.45, size = 131, normalized size = 1.31

$$\frac{bnxe^{(m \log(d)+m \log(x))} - ((bm + b)n \log(x) + am + (bm + b) \log(c) + a) \text{Ei} \left( \frac{(bm+b)n \log(x)+am+(bm+b) \log(c)+a}{bn} \right) e^{\left( \frac{bmn \log(x)}{bn} \right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -(b\*n\*x\*e^(m\*log(d) + m\*log(x)) - ((b\*m + b)\*n\*log(x) + a\*m + (b\*m + b)\*log(c) + a)\*Ei(((b\*m + b)\*n\*log(x) + a\*m + (b\*m + b)\*log(c) + a)/(b\*n))\*e^((b\*

$m \cdot n \cdot \log(d) - a \cdot m - (b \cdot m + b) \cdot \log(c) - a / (b \cdot n) / (b^3 \cdot n^3 \cdot \log(x) + b^3 \cdot n^2 \cdot \log(c) + a \cdot b^2 \cdot n^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a)^2, x)

**maple** [F] time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b \ln(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(b\*ln(c\*x^n)+a)^2,x)

[Out] int((d\*x)^m/(b\*ln(c\*x^n)+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$d^m(m+1) \int \frac{x^m}{b^2 n \log(c) + b^2 n \log(x^n) + abn} dx - \frac{d^m x x^m}{b^2 n \log(c) + b^2 n \log(x^n) + abn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] d^m\*(m+1)\*integrate(x^m/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n), x) - d^m\*x\*x^m/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx)^m}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a + b\*log(c\*x^n))^2,x)

[Out] int((d\*x)^m/(a + b\*log(c\*x^n))^2, x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral((d\*x)\*\*m/(a + b\*log(c\*x\*\*n))\*\*2, x)

$$3.155 \quad \int \frac{(dx)^m}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=142

$$\frac{(m+1)^2(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(m+1)(dx)^{m+1}}{2b^2dn^2(a+b \log(cx^n))} - \frac{(dx)^{m+1}}{2bdn(a+b \log(cx^n))^2}$$

[Out] 1/2\*(1+m)^2\*(d\*x)^(1+m)\*Ei((1+m)\*(a+b\*ln(c\*x^n))/b/n)/b^3/d/exp(a\*(1+m)/b/n)/n^3/((c\*x^n)^((1+m)/n))-1/2\*(d\*x)^(1+m)/b/d/n/(a+b\*ln(c\*x^n))^2-1/2\*(1+m)\*(d\*x)^(1+m)/b^2/d/n^2/(a+b\*ln(c\*x^n))

**Rubi [A]** time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2306, 2310, 2178}

$$\frac{(m+1)^2(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(m+1)(dx)^{m+1}}{2b^2dn^2(a+b \log(cx^n))} - \frac{(dx)^{m+1}}{2bdn(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m/(a + b\*Log[c\*x^n])^3,x]

[Out] ((1 + m)^2\*(d\*x)^(1 + m)\*ExpIntegralEi[(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))]/(2\*b^3\*d\*E^((a\*(1 + m))/(b\*n))\*n^3\*(c\*x^n)^((1 + m)/n)) - (d\*x)^(1 + m)/(2\*b\*d\*n\*(a + b\*Log[c\*x^n])^2) - ((1 + m)\*(d\*x)^(1 + m))/(2\*b^2\*d\*n^2\*(a + b\*Log[c\*x^n]))

**Rule 2178**

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))/((c\_) + (d\_)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

**Rule 2306**

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[(((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

**Rule 2310**

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(dx)^m}{(a + b \log(cx^n))^3} dx &= -\frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} + \frac{(1+m) \int \frac{(dx)^m}{(a+b \log(cx^n))^2} dx}{2bn} \\ &= -\frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} - \frac{(1+m)(dx)^{1+m}}{2b^2dn^2(a + b \log(cx^n))} + \frac{(1+m)^2 \int \frac{(dx)^m}{a+b \log(cx^n)} dx}{2b^2n^2} \\ &= -\frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} - \frac{(1+m)(dx)^{1+m}}{2b^2dn^2(a + b \log(cx^n))} + \frac{\left( (1+m)^2(dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) S}{2b} \\ &= \frac{e^{-\frac{a(1+m)}{bn}} (1+m)^2(dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \operatorname{Ei}\left(\frac{(1+m)(a+b \log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} \end{aligned}$$

**Mathematica [A]** time = 0.37, size = 113, normalized size = 0.80

$$\frac{(dx)^m \left( (m+1)^2 x^{-m} \exp\left(-\frac{(m+1)(a+b \log(cx^n)-bn \log(x))}{bn}\right) \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right) - \frac{bnx(am+a+b(m+1) \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m/(a + b\*Log[c\*x^n])^3,x]

[Out] ((d\*x)^m\*(((1+m)^2\*ExpIntegralEi[(((1+m)\*(a + b\*Log[c\*x^n]))/(b\*n))]/(E^(((1+m)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*x^m) - (b\*n\*x\*(a + a\*m + b\*n + b\*(1+m)\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2))/(2\*b^3\*n^3)

**fricas [B]** time = 0.47, size = 322, normalized size = 2.27

$$\frac{\left( (b^2m^2 + 2b^2m + b^2)n^2 \log(x)^2 + a^2m^2 + 2a^2m + (b^2m^2 + 2b^2m + b^2) \log(c)^2 + a^2 + 2(abm^2 + 2abm + ab) \right) S}{2b^3n^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(((b^2\*m^2 + 2\*b^2\*m + b^2)\*n^2\*log(x)^2 + a^2\*m^2 + 2\*a^2\*m + (b^2\*m^2 + 2\*b^2\*m + b^2)\*log(c)^2 + a^2 + 2\*(a\*b\*m^2 + 2\*a\*b\*m + a\*b)\*log(c) + 2\*(b^2\*m^2 + 2\*b^2\*m + b^2)\*n\*log(c) + (a\*b\*m^2 + 2\*a\*b\*m + a\*b)\*n\*log(x))\*Ei(((b\*m + b)\*n\*log(x) + a\*m + (b\*m + b)\*log(c) + a)/(b\*n))\*e^((b\*m\*n\*log(d) - a\*m - (b\*m + b)\*log(c) - a)/(b\*n)) - ((b^2\*m + b^2)\*n^2\*x\*log(x) + (b^2\*m + b^2)\*n\*x\*log(c) + (b^2\*n^2 + (a\*b\*m + a\*b)\*n)\*x)\*e^(m\*log(d) + m\*log(x)))/(b^5\*n^5\*log(x)^2 + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3 + 2\*(b^5\*n^4\*log(c) + a\*b^4\*n^4)\*log(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b \log(cx^n) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a)^3, x)

**maple** [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b \ln(cx^n) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(b\*ln(c\*x^n)+a)^3,x)

[Out] int((d\*x)^m/(b\*ln(c\*x^n)+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(m^2 + 2m + 1)d^m \int \frac{x^m}{2(b^3n^2 \log(c) + b^3n^2 \log(x^n) + ab^2n^2)} dx - \frac{bd^m(m+1)xx^m \log(x^n) + (ad^m(m+1) + 1)d^m}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) \log(x^n) + ab^3n^2 \log(x^n)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] (m^2 + 2\*m + 1)\*d^m\*integrate(1/2\*x^m/(b^3\*n^2\*log(c) + b^3\*n^2\*log(x^n) + a\*b^2\*n^2), x) - 1/2\*(b\*d^m\*(m + 1)\*x\*x^m\*log(x^n) + (a\*d^m\*(m + 1) + (d^m\*

$(m + 1) \cdot \log(c) + d^{m \cdot n} \cdot b) \cdot x \cdot x^m / (b^4 \cdot n^2 \cdot \log(c)^2 + b^4 \cdot n^2 \cdot \log(x^n)^2 + 2 \cdot a \cdot b^3 \cdot n^2 \cdot \log(c) + a^2 \cdot b^2 \cdot n^2 + 2 \cdot (b^4 \cdot n^2 \cdot \log(c) + a \cdot b^3 \cdot n^2) \cdot \log(x^n))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx)^m}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a + b\*log(c\*x^n))^3,x)

[Out] int((d\*x)^m/(a + b\*log(c\*x^n))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral((d\*x)\*\*m/(a + b\*log(c\*x\*\*n))\*\*3, x)

### 3.156 $\int (dx)^{-1+n} \log^3(cx^n) dx$

Optimal. Leaf size=74

$$\frac{(dx)^n \log^3(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{6(dx)^n}{dn}$$

[Out]  $-6*(d*x)^n/d/n+6*(d*x)^n*\ln(c*x^n)/d/n-3*(d*x)^n*\ln(c*x^n)^2/d/n+(d*x)^n*\ln(c*x^n)^3/d/n$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{(dx)^n \log^3(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{6(dx)^n}{dn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{-1+n}*\text{Log}[c*x^n]^3, x]$

[Out]  $(-6*(d*x)^n)/(d*n) + (6*(d*x)^n*\text{Log}[c*x^n])/d/n - (3*(d*x)^n*\text{Log}[c*x^n]^2)/d/n + ((d*x)^n*\text{Log}[c*x^n]^3)/d/n$

#### Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])/d*(m+1), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/d*(m+1)^2, x] \text{ /; } \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 2305

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^p/d*(m+1), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{GtQ}[p, 0]$

#### Rubi steps

$$\begin{aligned}
\int (dx)^{-1+n} \log^3(cx^n) dx &= \frac{(dx)^n \log^3(cx^n)}{dn} - 3 \int (dx)^{-1+n} \log^2(cx^n) dx \\
&= -\frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn} + 6 \int (dx)^{-1+n} \log(cx^n) dx \\
&= -\frac{6(dx)^n}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.54

$$\frac{(dx)^n (\log^3(cx^n) - 3 \log^2(cx^n) + 6 \log(cx^n) - 6)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)\*Log[c\*x^n]^3,x]

[Out] ((d\*x)^n\*(-6 + 6\*Log[c\*x^n] - 3\*Log[c\*x^n]^2 + Log[c\*x^n]^3))/(d\*n)

**fricas [A]** time = 0.44, size = 73, normalized size = 0.99

$$\frac{(n^3 \log(x)^3 + \log(c)^3 + 3(n^2 \log(c) - n^2) \log(x)^2 - 3 \log(c)^2 + 3(n \log(c)^2 - 2n \log(c) + 2n) \log(x) + 6 \log(c) - 6) d^{n-1} x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^3,x, algorithm="fricas")

[Out] (n^3\*log(x)^3 + log(c)^3 + 3\*(n^2\*log(c) - n^2)\*log(x)^2 - 3\*log(c)^2 + 3\*(n\*log(c)^2 - 2\*n\*log(c) + 2\*n)\*log(x) + 6\*log(c) - 6)\*d^(n - 1)\*x^n/n

**giac [B]** time = 0.53, size = 170, normalized size = 2.30

$$\frac{d^n n^2 x^n \log(x)^3}{d} + \frac{3 d^n n x^n \log(c) \log(x)^2}{d} + \frac{\frac{1}{d} x^n |d|^{2n} \log(c)^3}{dn} + \frac{3 d^n x^n \log(c)^2 \log(x)}{d} - \frac{3 d^n n x^n \log(x)^2}{d} - \frac{6 d^n x^n \log(c) - 6}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^3,x, algorithm="giac")

[Out] d^n\*n^2\*x^n\*log(x)^3/d + 3\*d^n\*n\*x^n\*log(c)\*log(x)^2/d + (1/d)^n\*x^n\*abs(d)^(2\*n)\*log(c)^3/(d\*n) + 3\*d^n\*x^n\*log(c)^2\*log(x)/d - 3\*d^n\*n\*x^n\*log(x)^2/d - 6\*d^n\*x^n\*log(c)\*log(x)/d - 3\*d^n\*n\*x^n\*log(c)^2/(d\*n) + 6\*d^n\*x^n\*log(x)/d + 6\*d^n\*x^n\*log(c)/(d\*n) - 6\*d^n\*x^n/(d\*n)

maple [C] time = 0.30, size = 2008, normalized size = 27.14

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (d*x)^{(n-1)} * \ln(c*x^n)^3, x$

[Out]  $\frac{1}{n*x} \exp\left(\frac{1}{2}(n-1)\left(-i\pi \operatorname{csgn}(I*d) \operatorname{csgn}(I*x) \operatorname{csgn}(I*d*x) + i\pi \operatorname{csgn}(I*d) \operatorname{csgn}(I*d*x)^2 + i\pi \operatorname{csgn}(I*x) \operatorname{csgn}(I*d*x)^2 - i\pi \operatorname{csgn}(I*d*x)^3 + 2\ln(d) + 2\ln(x)\right)\right) \ln(x^n)^3 + \frac{3}{2} \left(i\pi \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^2 - i\pi \operatorname{csgn}(I*c) \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n) - i\pi \operatorname{csgn}(I*c*x^n)^3 + i\pi \operatorname{csgn}(I*c) \operatorname{csgn}(I*c*x^n)^2 + 2\ln(c) - 2\right) / n*x \exp\left(\frac{1}{2}(n-1)\left(-i\pi \operatorname{csgn}(I*d) \operatorname{csgn}(I*x) \operatorname{csgn}(I*d*x) + i\pi \operatorname{csgn}(I*d) \operatorname{csgn}(I*d*x)^2 + i\pi \operatorname{csgn}(I*x) \operatorname{csgn}(I*d*x)^2 - i\pi \operatorname{csgn}(I*d*x)^3 + 2\ln(d) + 2\ln(x)\right)\right) \ln(x^n)^2 + \frac{3}{4} \left(-\pi^2 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^4 + 2\pi^2 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^3 \operatorname{csgn}(I*c) - \pi^2 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^2 \operatorname{csgn}(I*c)^2 + 2\pi^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^5 - 4\pi^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c) + 2\pi^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^3 \operatorname{csgn}(I*c)^2 - \pi^2 \operatorname{csgn}(I*c*x^n)^6 + 2\pi^2 \operatorname{csgn}(I*c*x^n)^5 \operatorname{csgn}(I*c) - \pi^2 \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c)^2 + 4i\pi \operatorname{csgn}(I*c*x^n)^3 - 4i\pi \ln(c) \operatorname{csgn}(I*c*x^n)^3 + 4i\pi \ln(c) \operatorname{csgn}(I*c*x^n)^2 \operatorname{csgn}(I*c) - 4i\pi \operatorname{csgn}(I*c*x^n)^2 \operatorname{csgn}(I*c) + 4i\pi \ln(c) \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^2 + 4i\pi \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n) \operatorname{csgn}(I*c) - 4i\pi \ln(c) \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n) \operatorname{csgn}(I*c) - 4i\pi \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^2 + 4\ln(c)^2 - 8\ln(c) + 8\right) / n*x \exp\left(\frac{1}{2}(n-1)\left(-i\pi \operatorname{csgn}(I*d) \operatorname{csgn}(I*x) \operatorname{csgn}(I*d*x) + i\pi \operatorname{csgn}(I*d) \operatorname{csgn}(I*d*x)^2 + i\pi \operatorname{csgn}(I*x) \operatorname{csgn}(I*d*x)^2 - i\pi \operatorname{csgn}(I*d*x)^3 + 2\ln(d) + 2\ln(x)\right)\right) \ln(x^n) + \frac{1}{8} \left(-48 - 12\pi^2 \operatorname{csgn}(I*c*x^n)^5 \operatorname{csgn}(I*c) + 6\pi^2 \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c)^2 - 12\pi^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^5 + 6\pi^2 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^4 + 6\pi^2 \operatorname{csgn}(I*c*x^n)^6 + i\pi^3 \operatorname{csgn}(I*c*x^n)^9 - 24i\pi \operatorname{csgn}(I*c*x^n)^3 - 6\ln(c) \operatorname{csgn}(I*c*x^n)^6 + 48\ln(c) - 24\ln(c)^2 + 24i\pi \operatorname{csgn}(I*c*x^n)^2 \operatorname{csgn}(I*c) + 24i\pi \ln(c) \operatorname{csgn}(I*c*x^n)^3 - 12i\pi \ln(c)^2 \operatorname{csgn}(I*c*x^n)^3 - i\pi^3 \operatorname{csgn}(I*x^n)^3 \operatorname{csgn}(I*c*x^n)^6 + 3i\pi^3 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^7 - 3i\pi^3 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^8 - 6\ln(c) \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^4 + 12\ln(c) \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^5 + 12\ln(c) \operatorname{csgn}(I*c*x^n)^5 \operatorname{csgn}(I*c) - 6\ln(c) \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c)^2 + 24i\pi \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^2 - 12\pi^2 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^3 \operatorname{csgn}(I*c) + 6\pi^2 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^2 \operatorname{csgn}(I*c)^2 + 24\pi^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c) - 12\pi^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^3 \operatorname{csgn}(I*c)^2 + 8\ln(c)^3 - 3i\pi^3 \operatorname{csgn}(I*c*x^n)^8 \operatorname{csgn}(I*c) + 3i\pi^3 \operatorname{csgn}(I*c*x^n)^7 \operatorname{csgn}(I*c)^2 - i\pi^3 \operatorname{csgn}(I*c*x^n)^6 \operatorname{csgn}(I*c)^3 - 3i\pi^3 \operatorname{csgn}(I*x^n)^3 \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c)^2 - 9i\pi^3 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^6 \operatorname{csgn}(I*c) + 9i\pi^3 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^5 \operatorname{csgn}(I*c)^2 - 3i\pi^3 \operatorname{csgn}(I*x^n)^2 \operatorname{csgn}(I*c*x^n)^4 \operatorname{csgn}(I*c)^3 + 24i\pi \ln(c) \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n) \operatorname{csgn}(I*c) - 12i\pi \ln(c)^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n) \operatorname{csgn}(I*c) + 9i\pi^3 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^7 \operatorname{csgn}(I*c) - 9i\pi^3 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^6 \operatorname{csgn}(I*c)^2 + 3i\pi^3 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^5 \operatorname{csgn}(I*c)^3 + 12i\pi \ln(c)^2 \operatorname{csgn}(I*x^n) \operatorname{csgn}(I*c*x^n)^2 - 24i$



$I \cdot \ln(c) \cdot \text{Pi} \cdot \text{csgn}(I \cdot x^n) \cdot \text{csgn}(I \cdot c \cdot x^n)^2 - 24 \cdot I \cdot \ln(c) \cdot \text{Pi} \cdot \text{csgn}(I \cdot c \cdot x^n)^2 \cdot \text{csgn}(I \cdot c) + 12 \cdot I \cdot \ln(c)^2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot c \cdot x^n)^2 \cdot \text{csgn}(I \cdot c) + 3 \cdot I \cdot \text{Pi}^3 \cdot \text{csgn}(I \cdot x^n)^3 \cdot \text{csgn}(I \cdot c \cdot x^n)^5 \cdot \text{csgn}(I \cdot c) + I \cdot \text{Pi}^3 \cdot \text{csgn}(I \cdot x^n)^3 \cdot \text{csgn}(I \cdot c \cdot x^n)^3 \cdot \text{csgn}(I \cdot c)^3 + 12 \cdot \ln(c) \cdot \text{Pi}^2 \cdot \text{csgn}(I \cdot x^n)^2 \cdot \text{csgn}(I \cdot c \cdot x^n)^3 \cdot \text{csgn}(I \cdot c) - 6 \cdot \ln(c) \cdot \text{Pi}^2 \cdot \text{csgn}(I \cdot x^n)^2 \cdot \text{csgn}(I \cdot c \cdot x^n)^2 \cdot \text{csgn}(I \cdot c)^2 - 24 \cdot \ln(c) \cdot \text{Pi}^2 \cdot \text{csgn}(I \cdot x^n) \cdot \text{csgn}(I \cdot c \cdot x^n)^4 \cdot \text{csgn}(I \cdot c) + 12 \cdot \ln(c) \cdot \text{Pi}^2 \cdot \text{csgn}(I \cdot x^n) \cdot \text{csgn}(I \cdot c \cdot x^n)^3 \cdot \text{csgn}(I \cdot c)^2 - 24 \cdot I \cdot \text{Pi} \cdot \text{csgn}(I \cdot x^n) \cdot \text{csgn}(I \cdot c \cdot x^n) \cdot \text{csgn}(I \cdot c) / n \cdot x \cdot \exp(1/2 \cdot (n-1) \cdot (-I \cdot \text{Pi} \cdot \text{csgn}(I \cdot d) \cdot \text{csgn}(I \cdot x) \cdot \text{csgn}(I \cdot d \cdot x) + I \cdot \text{Pi} \cdot \text{csgn}(I \cdot d) \cdot \text{csgn}(I \cdot d \cdot x)^2 + I \cdot \text{Pi} \cdot \text{csgn}(I \cdot x) \cdot \text{csgn}(I \cdot d \cdot x)^2 - I \cdot \text{Pi} \cdot \text{csgn}(I \cdot d \cdot x)^3 + 2 \cdot \ln(d) + 2 \cdot \ln(x))$

**maxima** [A] time = 0.59, size = 75, normalized size = 1.01

$$-\frac{3d^{n-1}x^n \log(cx^n)^2}{n} + \frac{(dx)^n \log(cx^n)^3}{dn} + \frac{6\left(\frac{d^n x^n \log(cx^n)}{n} - \frac{d^n x^n}{n}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^3,x, algorithm="maxima")

[Out]  $-3 \cdot d^{n-1} \cdot x^n \cdot \log(c \cdot x^n)^2 / n + (d \cdot x)^n \cdot \log(c \cdot x^n)^3 / (d \cdot n) + 6 \cdot (d^n \cdot x^n \cdot \log(c \cdot x^n) / n - d^n \cdot x^n / n) / d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \ln(cx^n)^3 (dx)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x^n)^3\*(d\*x)^(n-1),x)

[Out] int(log(c\*x^n)^3\*(d\*x)^(n-1), x)

**sympy** [A] time = 123.18, size = 292, normalized size = 3.95

$$\left\{ \begin{array}{l} \infty x \log(c)^3 \\ \frac{\log(c)^3 \log(x)}{d} \\ 0^{n-1} \left( n^3 x \log(x)^3 - 3n^3 x \log(x)^2 + 6n^3 x \log(x) - 6n^3 x + 3n^2 x \log(c) \log(x)^2 - 6n^2 x \log(c) \log(x) + 6n^2 x \log(c)^2 \right) \\ \frac{d^n n^2 x^n \log(x)^3}{d} + \frac{3d^n n x^n \log(c) \log(x)^2}{d} - \frac{3d^n n x^n \log(x)^2}{d} + \frac{3d^n x^n \log(c)^2 \log(x)}{d} - \frac{6d^n x^n \log(c) \log(x)}{d} + \frac{6d^n x^n \log(x)}{d} + \frac{d^n x^n \log(c)^3}{dn} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(-1+n)\*ln(c\*x\*\*n)\*\*3,x)

```
[Out] Piecewise((zoo*x*log(c)**3, Eq(d, 0) & Eq(n, 0)), (log(c)**3*log(x)/d, Eq(n, 0)), (0**(n - 1)*(n**3*x*log(x)**3 - 3*n**3*x*log(x)**2 + 6*n**3*x*log(x) - 6*n**3*x + 3*n**2*x*log(c)*log(x)**2 - 6*n**2*x*log(c)*log(x) + 6*n**2*x*log(c) + 3*n*x*log(c)**2*log(x) - 3*n*x*log(c)**2 + x*log(c)**3), Eq(d, 0)), (d**n*n**2*x**n*log(x)**3/d + 3*d**n*n*x**n*log(c)*log(x)**2/d - 3*d**n*n*x**n*log(x)**2/d + 3*d**n*x**n*log(c)**2*log(x)/d - 6*d**n*x**n*log(c)*log(x)/d + 6*d**n*x**n*log(x)/d + d**n*x**n*log(c)**3/(d*n) - 3*d**n*x**n*log(c)**2/(d*n) + 6*d**n*x**n*log(c)/(d*n) - 6*d**n*x**n/(d*n), True))
```

### 3.157 $\int (dx)^{-1+n} \log^2(cx^n) dx$

Optimal. Leaf size=53

$$\frac{(dx)^n \log^2(cx^n)}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{2(dx)^n}{dn}$$

[Out]  $2*(d*x)^n/d/n - 2*(d*x)^n*\ln(c*x^n)/d/n + (d*x)^n*\ln(c*x^n)^2/d/n$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{(dx)^n \log^2(cx^n)}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{2(dx)^n}{dn}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(-1 + n)\*Log[c\*x^n]^2,x]

[Out]  $(2*(d*x)^n)/(d*n) - (2*(d*x)^n*\text{Log}[c*x^n])/(d*n) + ((d*x)^n*\text{Log}[c*x^n]^2)/(d*n)$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int (dx)^{-1+n} \log^2(cx^n) dx &= \frac{(dx)^n \log^2(cx^n)}{dn} - 2 \int (dx)^{-1+n} \log(cx^n) dx \\ &= \frac{2(dx)^n}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{(dx)^n \log^2(cx^n)}{dn} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.57

$$\frac{(dx)^n (\log^2(cx^n) - 2 \log(cx^n) + 2)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)\*Log[c\*x^n]^2,x]

[Out] ((d\*x)^n\*(2 - 2\*Log[c\*x^n] + Log[c\*x^n]^2))/(d\*n)

**fricas** [A] time = 0.45, size = 42, normalized size = 0.79

$$\frac{(n^2 \log(x)^2 + \log(c)^2 + 2(n \log(c) - n) \log(x) - 2 \log(c) + 2)d^{n-1}x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^2,x, algorithm="fricas")

[Out] (n^2\*log(x)^2 + log(c)^2 + 2\*(n\*log(c) - n)\*log(x) - 2\*log(c) + 2)\*d^(n - 1)\*x^n/n

**giac** [A] time = 0.35, size = 99, normalized size = 1.87

$$\frac{d^n n x^n \log(x)^2}{d} + \frac{\frac{1}{d} x^n |d|^{2n} \log(c)^2}{dn} + \frac{2 d^n x^n \log(c) \log(x)}{d} - \frac{2 d^n x^n \log(x)}{d} - \frac{2 d^n x^n \log(c)}{dn} + \frac{2 d^n x^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^2,x, algorithm="giac")

[Out] d^n\*n\*x^n\*log(x)^2/d + (1/d)^n\*x^n\*abs(d)^(2\*n)\*log(c)^2/(d\*n) + 2\*d^n\*x^n\*log(c)\*log(x)/d - 2\*d^n\*x^n\*log(x)/d - 2\*d^n\*x^n\*log(c)/(d\*n) + 2\*d^n\*x^n/(d\*n)

**maple** [C] time = 0.18, size = 750, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(n-1)\*ln(c\*x^n)^2,x)

[Out] 1/n\*x\*exp(1/2\*(n-1)\*(-I\*Pi\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)+I\*Pi\*csgn(I\*d)\*csgn(I\*d\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*d\*x)^2-I\*Pi\*csgn(I\*d\*x)^3+2\*ln(d)+2\*ln(x)))\*ln(x^n)^2+(-I\*Pi\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)+I\*Pi\*csgn(I\*c)\*csgn

$$\begin{aligned} & n(I*c*x^n)^2 + I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2 - I*Pi*csgn(I*c*x^n)^3 + 2*\ln(c) - \\ & 2)/n*x*\exp(1/2*(n-1)*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x) + I*Pi*csgn(I*d)* \\ & csgn(I*d*x)^2 + I*Pi*csgn(I*x)*csgn(I*d*x)^2 - I*Pi*csgn(I*d*x)^3 + 2*\ln(d) + 2*\ln(x)) \\ & * \ln(x^n) + 1/4*(-Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2 + 2*Pi^2*csgn(I*c)^2 \\ & *csgn(I*x^n)*csgn(I*c*x^n)^3 - Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4 + 2*Pi^2*csgn(I*c) \\ & *csgn(I*x^n)^2*csgn(I*c*x^n)^3 - 4*Pi^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4 + 2*Pi^2 \\ & *csgn(I*c)*csgn(I*c*x^n)^5 - Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4 + 2*Pi^2*csgn(I*x^n) \\ & *csgn(I*c*x^n)^5 - Pi^2*csgn(I*c*x^n)^6 - 4*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n) \\ & * \ln(c) + 4*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2*\ln(c) + 4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2 \\ & *\ln(c) - 4*I*Pi*csgn(I*c*x^n)^3*\ln(c) + 4*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n) \\ & - 4*I*Pi*csgn(I*c)*csgn(I*c*x^n)^2 - 4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2 + 4*I*Pi*csgn(I*c*x^n)^3 \\ & + 4*\ln(c)^2 - 8*\ln(c) + 8)/n*x*\exp(1/2*(n-1)*(-I*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x) + I*Pi*csgn(I*d) \\ & *csgn(I*d*x)^2 + I*Pi*csgn(I*x)*csgn(I*d*x)^2 - I*Pi*csgn(I*d*x)^3 + 2*\ln(d) + 2*\ln(x)) \end{aligned}$$

**maxima** [A] time = 0.74, size = 53, normalized size = 1.00

$$-\frac{2d^{n-1}x^n \log(cx^n)}{n} + \frac{2d^{n-1}x^n}{n} + \frac{(dx)^n \log(cx^n)^2}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^2,x, algorithm="maxima")

[Out] -2\*d^(n - 1)\*x^n\*log(c\*x^n)/n + 2\*d^(n - 1)\*x^n/n + (d\*x)^n\*log(c\*x^n)^2/(d\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \ln(cx^n)^2 (dx)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(c\*x^n)^2\*(d\*x)^(n - 1), x)

[Out] int(log(c\*x^n)^2\*(d\*x)^(n - 1), x)

**sympy** [A] time = 43.86, size = 163, normalized size = 3.08

$$\left\{ \begin{array}{ll} \infty x \log(c)^2 & \text{for } d = 0 \wedge n = 0 \\ \frac{\log(c)^2 \log(x)}{d} & \text{for } n = 0 \\ 0^{n-1} \left( n^2 x \log(x)^2 - 2n^2 x \log(x) + 2n^2 x + 2nx \log(c) \log(x) - 2nx \log(c) + x \log(c)^2 \right) & \text{for } d = 0 \\ \frac{d^n n x^n \log(x)^2}{d} + \frac{2d^n x^n \log(c) \log(x)}{d} - \frac{2d^n x^n \log(x)}{d} + \frac{d^n x^n \log(c)^2}{dn} - \frac{2d^n x^n \log(c)}{dn} + \frac{2d^n x^n}{dn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n)**2,x)
```

```
[Out] Piecewise((zoo*x*log(c)**2, Eq(d, 0) & Eq(n, 0)), (log(c)**2*log(x)/d, Eq(n, 0)), (0**(n - 1)*(n**2*x*log(x)**2 - 2*n**2*x*log(x) + 2*n**2*x + 2*n*x*log(c)*log(x) - 2*n*x*log(c) + x*log(c)**2), Eq(d, 0)), (d**n*n*x**n*log(x)**2/d + 2*d**n*x**n*log(c)*log(x)/d - 2*d**n*x**n*log(x)/d + d**n*x**n*log(c)**2/(d*n) - 2*d**n*x**n*log(c)/(d*n) + 2*d**n*x**n/(d*n), True))
```

### 3.158 $\int (dx)^{-1+n} \log(cx^n) dx$

Optimal. Leaf size=32

$$\frac{(dx)^n \log(cx^n)}{dn} - \frac{(dx)^n}{dn}$$

[Out]  $-(d*x)^n/d/n+(d*x)^n*\ln(c*x^n)/d/n$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$\frac{(dx)^n \log(cx^n)}{dn} - \frac{(dx)^n}{dn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{-1+n}*\text{Log}[c*x^n], x]$

[Out]  $-((d*x)^n/(d*n)) + ((d*x)^n*\text{Log}[c*x^n])/(d*n)$

Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\int (dx)^{-1+n} \log(cx^n) dx = -\frac{(dx)^n}{dn} + \frac{(dx)^n \log(cx^n)}{dn}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.62

$$\frac{(dx)^n (\log(cx^n) - 1)}{dn}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d*x)^{-1+n}*\text{Log}[c*x^n], x]$

[Out]  $((d*x)^n*(-1 + \text{Log}[c*x^n]))/(d*n)$

fricas [A] time = 0.48, size = 20, normalized size = 0.62

$$\frac{(n \log(x) + \log(c) - 1)d^{n-1}x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n),x, algorithm="fricas")

[Out] (n\*log(x) + log(c) - 1)\*d^(n - 1)\*x^n/n

**giac** [A] time = 0.36, size = 50, normalized size = 1.56

$$\frac{\frac{1^n}{d} x^n |d|^{2n} \log(c)}{dn} + \frac{d^n x^n \log(x)}{d} - \frac{d^n x^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n),x, algorithm="giac")

[Out] (1/d)^n\*x^n\*abs(d)^(2\*n)\*log(c)/(d\*n) + d^n\*x^n\*log(x)/d - d^n\*x^n/(d\*n)

**maple** [C] time = 0.14, size = 263, normalized size = 8.22

$$\frac{x e^{\frac{(n-1)(-i\pi \operatorname{csgn}(id)\operatorname{csgn}(ix)\operatorname{csgn}(idx)+i\pi \operatorname{csgn}(id)\operatorname{csgn}(idx)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(idx)^2-i\pi \operatorname{csgn}(idx)^3+2\ln(d)+2\ln(x))}{2}} \ln(x^n)}{n} + \frac{(-i\pi \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ix^n))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(n-1)\*ln(c\*x^n),x)

[Out] 1/n\*x\*exp(1/2\*(n-1)\*(-I\*Pi\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)+I\*Pi\*csgn(I\*d)\*csgn(I\*d\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*d\*x)^2-I\*Pi\*csgn(I\*d\*x)^3+2\*ln(d)+2\*ln(x)))\*ln(x^n)+1/2\*(-I\*Pi\*csgn(I\*c)\*csgn(I\*x^n)\*csgn(I\*c\*x^n)+I\*Pi\*csgn(I\*c)\*csgn(I\*c\*x^n)^2+I\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*csgn(I\*c\*x^n)^3+2\*ln(c)-2)/n\*x\*exp(1/2\*(n-1)\*(-I\*Pi\*csgn(I\*d)\*csgn(I\*x)\*csgn(I\*d\*x)+I\*Pi\*csgn(I\*d)\*csgn(I\*d\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*d\*x)^2-I\*Pi\*csgn(I\*d\*x)^3+2\*ln(d)+2\*ln(x)))

**maxima** [A] time = 0.67, size = 32, normalized size = 1.00

$$-\frac{d^{n-1}x^n}{n} + \frac{(dx)^n \log(cx^n)}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n),x, algorithm="maxima")

[Out] -d^(n - 1)\*x^n/n + (d\*x)^n\*log(c\*x^n)/(d\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \ln(cx^n) (dx)^{n-1} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)*(d*x)^(n - 1), x)
```

```
[Out] int(log(c*x^n)*(d*x)^(n - 1), x)
```

sympy [A] time = 13.72, size = 68, normalized size = 2.12

$$\left\{ \begin{array}{ll} \infty x \log(c) & \text{for } d = 0 \wedge n = 0 \\ \frac{\log(c) \log(x)}{d} & \text{for } n = 0 \\ 0^{n-1} (nx \log(x) - nx + x \log(c)) & \text{for } d = 0 \\ \frac{d^n x^n \log(x)}{d} + \frac{d^n x^n \log(c)}{dn} - \frac{d^n x^n}{dn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n), x)
```

```
[Out] Piecewise((zoo*x*log(c), Eq(d, 0) & Eq(n, 0)), (log(c)*log(x)/d, Eq(n, 0)),
(0**(n - 1)*(n*x*log(x) - n*x + x*log(c)), Eq(d, 0)), (d**n*x**n*log(x)/d
+ d**n*x**n*log(c)/(d*n) - d**n*x**n/(d*n), True))
```

$$3.159 \quad \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx$$

Optimal. Leaf size=27

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn}$$

[Out]  $x^{(1-n)}*(d*x)^{(-1+n)}*Li(c*x^n)/c/n$

**Rubi [A]** time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2308, 2307, 2298}

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] `Int[(d*x)^(-1 + n)/Log[c*x^n], x]`

[Out] `(x^(1 - n)*(d*x)^(-1 + n)*LogIntegral[c*x^n])/(c*n)`

Rule 2298

`Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :=> Simp[LogIntegral[c*x]/c, x] /; FreeQ[c, x]`

Rule 2307

`Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :=> Dist[1/n, Subst[Int[1/Log[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]`

Rule 2308

`Int[((d_)*(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :=> Dist[(d*x)^m/x^m, Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]`

Rubi steps

$$\begin{aligned} \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx &= (x^{1-n}(dx)^{-1+n}) \int \frac{x^{-1+n}}{\log(cx^n)} dx \\ &= \frac{(x^{1-n}(dx)^{-1+n}) \operatorname{Subst}\left(\int \frac{1}{\log(cx)} dx, x, x^n\right)}{n} \\ &= \frac{x^{1-n}(dx)^{-1+n} \operatorname{li}(cx^n)}{cn} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^{1-n}(dx)^{n-1} \operatorname{li}(cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)/Log[c\*x^n], x]

[Out] (x^(1 - n)\*(d\*x)^(-1 + n)\*LogIntegral[c\*x^n])/(c\*n)

**fricas** [A] time = 0.46, size = 20, normalized size = 0.74

$$\frac{d^{n-1} \operatorname{Ei}(n \log(x) + \log(c))}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n), x, algorithm="fricas")

[Out] d^(n - 1)\*Ei(n\*log(x) + log(c))/(c\*n)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n), x, algorithm="giac")

[Out] integrate((d\*x)^(n - 1)/log(c\*x^n), x)

**maple** [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(n-1)/ln(c*x^n),x)`

[Out] `int((d*x)^(n-1)/ln(c*x^n),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(-1+n)/log(c*x^n),x, algorithm="maxima")`

[Out] `integrate((d*x)^(n - 1)/log(c*x^n), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(dx)^{n-1}}{\ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(n - 1)/log(c*x^n),x)`

[Out] `int((d*x)^(n - 1)/log(c*x^n), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(-1+n)/ln(c*x**n),x)`

[Out] `Integral((d*x)**(n - 1)/log(c*x**n), x)`

$$3.160 \quad \int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx$$

**Optimal.** Leaf size=49

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn} - \frac{(dx)^n}{dn \log(cx^n)}$$

[Out]  $x^{(1-n)}*(d*x)^{(-1+n)}*Li(c*x^n)/c/n-(d*x)^n/d/n/\ln(c*x^n)$

**Rubi [A]** time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2306, 2308, 2307, 2298}

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn} - \frac{(dx)^n}{dn \log(cx^n)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(-1 + n)/Log[c\*x^n]^2,x]

[Out]  $-((d*x)^n/(d*n*\text{Log}[c*x^n])) + (x^{(1-n)}*(d*x)^{(-1+n)}*\text{LogIntegral}[c*x^n])/(c*n)$

**Rule 2298**

Int[Log[(c\_.)\*(x\_)^(-1)], x\_Symbol] := Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

**Rule 2306**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^(p+1))/(b\*d\*n\*(p+1)), x] - Dist[(m+1)/(b\*n\*(p+1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p+1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

**Rule 2307**

Int[(x\_)^(m\_.)/Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[1/n, Subst[Int[1/Log[c\*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]

**Rule 2308**

Int[((d\_.)\*(x\_)^(m\_.)/Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[(d\*x)^m/x^m, Int[x^m/Log[c\*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx &= -\frac{(dx)^n}{dn \log(cx^n)} + \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx \\
&= -\frac{(dx)^n}{dn \log(cx^n)} + (x^{1-n}(dx)^{-1+n}) \int \frac{x^{-1+n}}{\log(cx^n)} dx \\
&= -\frac{(dx)^n}{dn \log(cx^n)} + \frac{(x^{1-n}(dx)^{-1+n}) \text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, x^n\right)}{n} \\
&= -\frac{(dx)^n}{dn \log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n} \text{li}(cx^n)}{cn}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 1.00

$$\frac{x^{1-n}(dx)^{n-1} \text{li}(cx^n)}{cn} - \frac{x(dx)^{n-1}}{n \log(cx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)/Log[c\*x^n]^2,x]

[Out] -((x\*(d\*x)^(-1 + n))/(n\*Log[c\*x^n])) + (x^(1 - n)\*(d\*x)^(-1 + n)\*LogIntegral[c\*x^n])/(c\*n)

**fricas [A]** time = 0.44, size = 50, normalized size = 1.02

$$\frac{d^{n-1}x^n - \frac{(n \log(x) + \log(c))d^{n-1} \text{Ei}(n \log(x) + \log(c))}{c}}{n^2 \log(x) + n \log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^2,x, algorithm="fricas")

[Out] -(d^(n - 1)\*x^n - (n\*log(x) + log(c))\*d^(n - 1)\*Ei(n\*log(x) + log(c))/c)/(n^2\*log(x) + n\*log(c))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^2,x, algorithm="giac")

[Out] integrate((d\*x)^(n - 1)/log(c\*x^n)^2, x)

**maple** [F] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\ln(cx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(n-1)/ln(c\*x^n)^2,x)

[Out] int((d\*x)^(n-1)/ln(c\*x^n)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$d^n \int \frac{x^n}{dx \log(c) + dx \log(x^n)} dx - \frac{d^n x^n}{dn \log(c) + dn \log(x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^2,x, algorithm="maxima")

[Out] d^n\*integrate(x^n/(d\*x\*log(c) + d\*x\*log(x^n)), x) - d^n\*x^n/(d\*n\*log(c) + d\*n\*log(x^n))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx)^{n-1}}{\ln(cx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(n - 1)/log(c\*x^n)^2,x)

[Out] int((d\*x)^(n - 1)/log(c\*x^n)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(-1+n)/ln(c\*x\*\*n)\*\*2,x)

[Out] Integral((d\*x)\*\*(n - 1)/log(c\*x\*\*n)\*\*2, x)

$$3.161 \quad \int \frac{(dx)^{-1+n}}{\log^3(cx^n)} dx$$

**Optimal.** Leaf size=77

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{2cn} - \frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)}$$

[Out]  $1/2*x^{(1-n)}*(d*x)^{(-1+n)}*Li(c*x^n)/c/n-1/2*(d*x)^n/d/n/\ln(c*x^n)^2-1/2*(d*x)^n/d/n/\ln(c*x^n)$

**Rubi [A]** time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2306, 2308, 2307, 2298}

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{2cn} - \frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(-1 + n)/Log[c\*x^n]^3,x]

[Out]  $-(d*x)^n/(2*d*n*Log[c*x^n]^2) - (d*x)^n/(2*d*n*Log[c*x^n]) + (x^{(1 - n)}*(d*x)^{-1 + n}*LogIntegral[c*x^n])/(2*c*n)$

Rule 2298

Int[Log[(c\_.)\*(x\_)]^(-1), x\_Symbol] := Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

Rule 2307

Int[(x\_)^(m\_.)/Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Dist[1/n, Subst[Int[1/Log[c\*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]

Rule 2308



`Int[((d_)*(x_))^(m_)/Log[(c_)*(x_)^(n_)], x_Symbol] :> Dist[(d*x)^m/x^m,  
Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]`

### Rubi steps

$$\begin{aligned}
 \int \frac{(dx)^{-1+n}}{\log^3(cx^n)} dx &= -\frac{(dx)^n}{2dn \log^2(cx^n)} + \frac{1}{2} \int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx \\
 &= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{1}{2} \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx \\
 &= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{1}{2} (x^{1-n}(dx)^{-1+n}) \int \frac{x^{-1+n}}{\log(cx^n)} dx \\
 &= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{(x^{1-n}(dx)^{-1+n}) \text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, x^n\right)}{2n} \\
 &= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n} \text{li}(cx^n)}{2cn}
 \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 61, normalized size = 0.79

$$\frac{x^{-n}(dx)^n (\text{li}(cx^n) \log^2(cx^n) - cx^n (\log(cx^n) + 1))}{2cdn \log^2(cx^n)}$$

Antiderivative was successfully verified.

[In] `Integrate[(d*x)^(-1 + n)/Log[c*x^n]^3,x]`

[Out] `((d*x)^n*(-(c*x^n*(1 + Log[c*x^n])) + Log[c*x^n]^2*LogIntegral[c*x^n]))/(2*c*d*n*x^n*Log[c*x^n]^2)`

**fricas** [A] time = 0.44, size = 84, normalized size = 1.09

$$\frac{(n \log(x) + \log(c) + 1)d^{n-1}x^n - \frac{(n^2 \log(x)^2 + 2n \log(c) \log(x) + \log(c)^2)d^{n-1}\text{Ei}(n \log(x) + \log(c))}{c}}{2(n^3 \log(x)^2 + 2n^2 \log(c) \log(x) + n \log(c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(-1+n)/log(c*x^n)^3,x, algorithm="fricas")`

[Out] `-1/2*((n*log(x) + log(c) + 1)*d^(n - 1)*x^n - (n^2*log(x)^2 + 2*n*log(c)*log(x) + log(c)^2)*d^(n - 1)*Ei(n*log(x) + log(c))/c)/(n^3*log(x)^2 + 2*n^2*log(c)*log(x) + n*log(c)^2)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^3,x, algorithm="giac")

[Out] integrate((d\*x)^(n - 1)/log(c\*x^n)^3, x)

**maple** [F] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\ln(cx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(n-1)/ln(c\*x^n)^3,x)

[Out] int((d\*x)^(n-1)/ln(c\*x^n)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$d^n \int \frac{x^n}{2(dx \log(c) + dx \log(x^n))} dx - \frac{d^n x^n \log(x^n) + (d^n \log(c) + d^n)x^n}{2(dn \log(c)^2 + 2dn \log(c) \log(x^n) + dn \log(x^n)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^3,x, algorithm="maxima")

[Out] d^n\*integrate(1/2\*x^n/(d\*x\*log(c) + d\*x\*log(x^n)), x) - 1/2\*(d^n\*x^n\*log(x^n) + (d^n\*log(c) + d^n)\*x^n)/(d^n\*log(c)^2 + 2\*d^n\*log(c)\*log(x^n) + d^n\*log(x^n)^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx)^{n-1}}{\ln(cx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(n - 1)/log(c\*x^n)^3,x)

[Out] int((d\*x)^(n - 1)/log(c\*x^n)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(-1+n)/ln(c\*x\*\*n)\*\*3, x)

[Out] Integral((d\*x)\*\*(n - 1)/log(c\*x\*\*n)\*\*3, x)

### 3.162 $\int x^m \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=111

$$\frac{3\sqrt{\pi} n^{3/2} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(m+1)^{5/2}} + \frac{x^{m+1} \log^{\frac{3}{2}}(ax^n)}{m+1} - \frac{3nx^{m+1} \sqrt{\log(ax^n)}}{2(m+1)^2}$$

[Out]  $x^{(1+m)} \ln(a*x^n)^{(3/2)} / (1+m) + 3/4 * n^{(3/2)} * x^{(1+m)} * \operatorname{erfi}((1+m)^{(1/2)} * \ln(a*x^n)^{(1/2)} / n^{(1/2)}) * \operatorname{Pi}^{(1/2)} / (1+m)^{(5/2)} / ((a*x^n)^{((1+m)/n)}) - 3/2 * n * x^{(1+m)} * \ln(a*x^n)^{(1/2)} / (1+m)^2$

**Rubi [A]** time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{3\sqrt{\pi} n^{3/2} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(m+1)^{5/2}} + \frac{x^{m+1} \log^{\frac{3}{2}}(ax^n)}{m+1} - \frac{3nx^{m+1} \sqrt{\log(ax^n)}}{2(m+1)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^m * \operatorname{Log}[a*x^n]^{(3/2)}, x]$

[Out]  $(3*n^{(3/2)} * \operatorname{Sqrt}[\operatorname{Pi}] * x^{(1+m)} * \operatorname{Erfi}[(\operatorname{Sqrt}[1+m] * \operatorname{Sqrt}[\operatorname{Log}[a*x^n]]) / \operatorname{Sqrt}[n]]) / (4*(1+m)^{(5/2)} * (a*x^n)^{((1+m)/n)}) - (3*n*x^{(1+m)} * \operatorname{Sqrt}[\operatorname{Log}[a*x^n]]) / (2*(1+m)^2) + (x^{(1+m)} * \operatorname{Log}[a*x^n]^{(3/2)}) / (1+m)$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_)))} / \operatorname{Sqrt}[(c_.) + (d_.) * (x_)], x\_Symbol] :$   
 $> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.) * ((c_.) + (d_.) * (x_))^{2})}, x\_Symbol] := \operatorname{Simp}[(F^{a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]])} / (2*d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

#### Rule 2305

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.) * (x_)^{(n_.)}] * (b_.)^{(p_.)} * ((d_.) * (x_))^{(m_.)}, x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)} * (a + b * \operatorname{Log}[c*x^n])^p / (d*(m+1)), x] - \operatorname{Dist}[(b*n * p) / (m+1), \operatorname{Int}[(d*x)^m * (a + b * \operatorname{Log}[c*x^n])^{(p-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b,$

$c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

### Rule 2310

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $] \rightarrow \text{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[E^{((m+1)*x)/n}*(a+b*x)^p, x], x, \text{Log}[c*x^n]], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

### Rubi steps

$$\begin{aligned} \int x^m \log^{\frac{3}{2}}(ax^n) dx &= \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} - \frac{(3n) \int x^m \sqrt{\log(ax^n)} dx}{2(1+m)} \\ &= -\frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3n^2) \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{4(1+m)^2} \\ &= -\frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3nx^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{4(1+m)^2} \\ &= -\frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3nx^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{2(1+m)^2} \\ &= \frac{3n^{3/2} \sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \text{erfi}\left(\frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(1+m)^{5/2}} - \frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} \end{aligned}$$

**Mathematica [A]** time = 0.19, size = 101, normalized size = 0.91

$$\frac{x^{m+1} \left( 3\sqrt{\pi} n^{3/2} (ax^n)^{-\frac{m+1}{n}} \text{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) + 2\sqrt{m+1} \sqrt{\log(ax^n)} (2(m+1) \log(ax^n) - 3n) \right)}{4(m+1)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Log[a\*x^n]^(3/2), x]

[Out] (x^(1+m)\*((3\*n^(3/2)\*Sqrt[Pi]\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])]/Sqrt[n])/(a\*x^n)^((1+m)/n) + 2\*Sqrt[1+m]\*Sqrt[Log[a\*x^n]]\*(-3\*n + 2\*(1+m)\*Log[a\*x^n]))/(4\*(1+m)^(5/2))

**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \log(ax^n)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] integral(x^m\*log(a\*x^n)^(3/2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^m\*log(a\*x^n)^(3/2), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int x^m \ln(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*ln(a\*x^n)^(3/2),x)

[Out] int(x^m\*ln(a\*x^n)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^m\*log(a\*x^n)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \ln(ax^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*log(a*x^n)^(3/2),x)`

[Out] `int(x^m*log(a*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*ln(a*x**n)**(3/2),x)`

[Out] `Integral(x**m*log(a*x**n)**(3/2), x)`

### 3.163 $\int x^m \sqrt{\log(ax^n)} dx$

Optimal. Leaf size=86

$$\frac{x^{m+1} \sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi} \sqrt{n} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

[Out]  $-1/2*x^{(1+m)}*\operatorname{erfi}((1+m)^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*n^{(1/2)}*\pi^{(1/2)}/(1+m)^{(3/2)}/((a*x^n)^{((1+m)/n)}+x^{(1+m)}*\ln(a*x^n)^{(1/2)}/(1+m))$

**Rubi [A]** time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{x^{m+1} \sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi} \sqrt{n} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[x^m*Sqrt[Log[a*x^n]],x]`

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi]*x^{(1+m)}*\operatorname{Erfi}[(\operatorname{Sqrt}[1+m]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(2*(1+m)^{(3/2)}*(a*x^n)^{((1+m)/n)} + (x^{(1+m)}*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/(1+m)$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))/Sqrt[(c_.)+(d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e-(c*f)/d)+(f*g*x^2)/d), x], x, Sqrt[c+d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.)+(b_.)*((c_.)+(d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c+d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2305

`Int[((a_.)+Log[(c_.)*(x_)^(n_.)]*(b_.))^((p_.)*((d_.)*(x_)^(m_.)), x_Symbol] :> Simp[((d*x)^(m+1)*(a+b*Log[c*x^n])^p)/(d*(m+1)), x] - Dist[(b*n*p)/(m+1), Int[(d*x)^m*(a+b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]`



Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :-> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x)
/n]*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int x^m \sqrt{\log(ax^n)} dx &= \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} - \frac{n \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{2(1+m)} \\ &= \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} - \frac{\left(x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{2(1+m)} \\ &= \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} - \frac{\left(x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{1+m} \\ &= -\frac{\sqrt{n} \sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(1+m)^{3/2}} + \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 1.00

$$\frac{x^{m+1} \sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi} \sqrt{n} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[Log[a*x^n]], x]
```

```
[Out] -1/2*(Sqrt[n]*Sqrt[Pi]*x^(1+m)*Erfi[(Sqrt[1+m]*Sqrt[Log[a*x^n]])/Sqrt[n]])/((1+m)^(3/2)*(a*x^n)^((1+m)/n)) + (x^(1+m)*Sqrt[Log[a*x^n]])/(1+m)
```

**fricas [F]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \sqrt{\log(ax^n)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(a*x^n)^(1/2),x, algorithm="fricas")
```

[Out] `integral(x^m*sqrt(log(a*x^n)), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*log(a*x^n)^(1/2),x, algorithm="giac")`

[Out] `integrate(x^m*sqrt(log(a*x^n)), x)`

**maple** [F] time = 0.32, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*ln(a*x^n)^(1/2),x)`

[Out] `int(x^m*ln(a*x^n)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*log(a*x^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^m*sqrt(log(a*x^n)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*log(a*x^n)^(1/2),x)`

[Out] `int(x^m*log(a*x^n)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(log(a*x**n)), x)
```

$$3.164 \quad \int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Optimal. Leaf size=61

$$\frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1} \sqrt{n}}$$

[Out]  $x^{(1+m)} \operatorname{erfi}\left(\frac{(1+m)^{(1/2)} \ln(a x^n)^{(1/2)} / n^{(1/2)}}{\sqrt{n}}\right) \pi^{(1/2)} / ((a x^n)^{((1+m)/n)}) / (1+m)^{(1/2)} / n^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1} \sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi]\*x^(1+m)\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[1+m]\*Sqrt[n]\*(a\*x^n)^((1+m)/n))

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^((p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^(((m+1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\sqrt{\log(ax^n)}} dx &= \frac{\left(x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(2x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\
&= \frac{\sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{1+m} \sqrt{n}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 61, normalized size = 1.00

$$\frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1} \sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi]\*x^(1+m)\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[1+m]\*Sqrt[n]\*(a\*x^n)^((1+m)/n))

**fricas [F]** time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^m}{\sqrt{\log(ax^n)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(1/2), x, algorithm="fricas")

[Out] integral(x^m/sqrt(log(a\*x^n)), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x^m/sqrt(log(a\*x^n)), x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/ln(a\*x^n)^(1/2),x)

[Out] int(x^m/ln(a\*x^n)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^m/sqrt(log(a\*x^n)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/log(a\*x^n)^(1/2),x)

[Out] int(x^m/log(a\*x^n)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(x\*\*m/sqrt(log(a\*x\*\*n)), x)

$$3.165 \quad \int \frac{x^m}{\log^2(ax^n)} dx$$

**Optimal.** Leaf size=83

$$\frac{2\sqrt{\pi} \sqrt{m+1} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

[Out]  $2*x^{(1+m)}*erfi((1+m)^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*(1+m)^{(1/2)}*Pi^{(1/2)}/n^{(3/2)}/((a*x^n)^{((1+m)/n)}-2*x^{(1+m)}/n/\ln(a*x^n)^{(1/2)})$

**Rubi [A]** time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{2\sqrt{\pi} \sqrt{m+1} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^m/\operatorname{Log}[a*x^n]^{(3/2)}, x]$

[Out]  $(2*\operatorname{Sqrt}[1+m]*\operatorname{Sqrt}[Pi]*x^{(1+m)}*\operatorname{Erfi}[(\operatorname{Sqrt}[1+m]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])]/\operatorname{Sqrt}[n])/(n^{(3/2)}*(a*x^n)^{((1+m)/n)}) - (2*x^{(1+m)})/(n*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

**Rule 2180**

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \&\& !\$UseGamma == True$

**Rule 2204**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(2)}), x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \&\& \operatorname{PosQ}[b]$

**Rule 2306**

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)]^{(n_.)}*(b_)]^{(p_)}*((d_.)*(x_)]^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x]$

/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned} \int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} + \frac{(2(1+m)) \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{n} \\ &= -\frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} + \frac{\left(2(1+m)x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\ &= -\frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} + \frac{\left(4(1+m)x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\ &= \frac{2\sqrt{1+m}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 86, normalized size = 1.04

$$\frac{2\sqrt{\pi}\sqrt{m+1}e^{-\frac{(m+1)(\log(ax^n)-n\log(x))}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Log[a\*x^n]^(3/2), x]

[Out] (2\*Sqrt[1 + m]\*Sqrt[Pi]\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(E^((1 + m)\*(-(n\*Log[x]) + Log[a\*x^n]))/n)\*n^(3/2) - (2\*x^(1 + m))/(n\*Sqrt[Log[a\*x^n]])



**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\log(ax^n)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] integral(x^m/log(a\*x^n)^(3/2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^m/log(a\*x^n)^(3/2), x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\ln(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/ln(a\*x^n)^(3/2),x)

[Out] int(x^m/ln(a\*x^n)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^m/log(a\*x^n)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\ln(ax^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/log(a*x^n)^(3/2),x)`

[Out] `int(x^m/log(a*x^n)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/ln(a*x**n)**(3/2),x)`

[Out] `Integral(x**m/log(a*x**n)**(3/2), x)`

$$3.166 \quad \int \frac{x^m}{5 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=112

$$\frac{4\sqrt{\pi} (m+1)^{3/2} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4(m+1)x^{m+1}}{3n^2 \sqrt{\log(ax^n)}} - \frac{2x^{m+1}}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/3*x^{(1+m)}/n/\ln(a*x^n)^{(3/2)}+4/3*(1+m)^{(3/2)}*x^{(1+m)}*\operatorname{erfi}((1+m)^{(1/2)}*\ln(a*x^n)^{(1/2)}/n^{(1/2)})*\operatorname{Pi}^{(1/2)}/n^{(5/2)}/((a*x^n)^{((1+m)/n)})-4/3*(1+m)*x^{(1+m)}/n^2/\ln(a*x^n)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{4\sqrt{\pi} (m+1)^{3/2} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4(m+1)x^{m+1}}{3n^2 \sqrt{\log(ax^n)}} - \frac{2x^{m+1}}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x^m/Log[a\*x^n]^(5/2), x]

[Out]  $(4*(1+m)^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*x^{(1+m)}*\operatorname{Erfi}[(\operatorname{Sqrt}[1+m]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(3*n^{(5/2)}*(a*x^n)^{((1+m)/n)}) - (2*x^{(1+m)})/(3*n*\operatorname{Log}[a*x^n]^{(3/2)}) - (4*(1+m)*x^{(1+m)})/(3*n^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2180

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))/Sqrt[(c\_) + (d\_)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_) + (b\_)\*((c\_) + (d\_)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2306

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^(p+1))/(b\*d\*n\*(p+1)), x] -

Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^(m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{(2(1+m)) \int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
 &= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}} + \frac{(4(1+m)^2) \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
 &= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}} + \frac{(4(1+m)^2 x^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3} \\
 &= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}} + \frac{(8(1+m)^2 x^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
 &= \frac{4(1+m)^{3/2} \sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}}
 \end{aligned}$$

**Mathematica** [A] time = 0.42, size = 103, normalized size = 0.92

$$\frac{2 \left( \frac{2\sqrt{\pi} (m+1)^{3/2} e^{\frac{(m+1)(n \log(x) - \log(ax^n))}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}} - \frac{x^{m+1} (2(m+1) \log(ax^n) + n)}{\log^{\frac{3}{2}}(ax^n)} \right)}{3n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Log[a\*x^n]^(5/2), x]

[Out]  $(2*((2*E^{((1+m)*(n*\text{Log}[x] - \text{Log}[a*x^n]))/n})*(1+m)^{3/2}*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(\text{Sqrt}[1+m]*\text{Sqrt}[\text{Log}[a*x^n]])/\text{Sqrt}[n]])/\text{Sqrt}[n] - (x^{(1+m)*(n+2*(1+m)*\text{Log}[a*x^n])}/\text{Log}[a*x^n]^{3/2}))/((3*n^2))$

**fricas** [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\log(ax^n)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/log(a*x^n)^(5/2),x, algorithm="fricas")`

[Out] `integral(x^m/log(a*x^n)^(5/2), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/log(a*x^n)^(5/2),x, algorithm="giac")`

[Out] `integrate(x^m/log(a*x^n)^(5/2), x)`

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\ln(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/ln(a*x^n)^(5/2),x)`

[Out] `int(x^m/ln(a*x^n)^(5/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/log(a*x^n)^(5/2),x, algorithm="maxima")`

[Out] integrate(x<sup>m</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\ln(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>m</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

[Out] int(x<sup>m</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\log(ax^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/ln(a\*x\*\*n)\*\*(5/2), x)

[Out] Integral(x\*\*m/log(a\*x\*\*n)\*\*(5/2), x)

$$3.167 \quad \int (dx)^m \left( a + b \log(cx^n) \right)^p dx$$

**Optimal.** Leaf size=106

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \left( a + b \log(cx^n) \right)^p \left( -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)^{-p} \Gamma\left( p+1, -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)}{d(m+1)}$$

[Out]  $(d*x)^{(1+m)*\text{GAMMA}(1+p, -(1+m)*(a+b*\ln(c*x^n))/b/n)*(a+b*\ln(c*x^n))^p/d/\exp(a*(1+m)/b/n)/(1+m)/((c*x^n)^{((1+m)/n)})/((-1+m)*(a+b*\ln(c*x^n))/b/n)^p$

**Rubi [A]** time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \left( a + b \log(cx^n) \right)^p \left( -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)^{-p} \text{Gamma}\left( p+1, -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^p,x]

[Out]  $((d*x)^{(1+m)*\text{Gamma}[1+p, -(((1+m)*(a+b*\text{Log}[c*x^n]))/(b*n))]}*(a+b*\text{Log}[c*x^n])^p)/(d*\text{E}^{((a*(1+m))/(b*n))*(1+m)*(c*x^n)^{((1+m)/n)*(-(((1+m)*(a+b*\text{Log}[c*x^n]))/(b*n)))})})^p$

**Rule 2181**

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

**Rule 2310**

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^p\*((d\_)\*(x\_))^(m\_), x\_Symbol]  
 ] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

**Rubi steps**

$$\int (dx)^m (a + b \log(cx^n))^p dx = \frac{\left( (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int e^{\frac{(1+m)x}{n}} (a + bx)^p dx, x, \log(cx^n) \right)}{dn}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}} (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma \left( 1 + p, -\frac{(1+m)(a+b \log(cx^n))}{bn} \right) (a + b \log(cx^n))^p \left( -\frac{(1+m)}{bn} \right)}{d(1+m)}$$

**Mathematica** [A] time = 0.14, size = 107, normalized size = 1.01

$$\frac{x^{-m} (dx)^m (a + b \log(cx^n))^p \exp \left( -\frac{(m+1)(a+b \log(cx^n)) - bn \log(x)}{bn} \right) \left( -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)^{-p} \Gamma \left( p + 1, -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)}{m + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x^n])^p,x]

[Out] ((d\*x)^m\*Gamma[1 + p, -(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(((1 + m)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*(1 + m)\*x^m\*(-(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left( (dx)^m (b \log(cx^n) + a)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((d\*x)^m\*(b\*log(c\*x^n) + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="giac")

[Out] integrate((d\*x)^m\*(b\*log(c\*x^n) + a)^p, x)

**maple** [F] time = 1.68, size = 0, normalized size = 0.00

$$\int (dx)^m (b \ln(cx^n) + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(b*ln(c*x^n)+a)^p,x)`

[Out] `int((d*x)^m*(b*ln(c*x^n)+a)^p,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*log(c*x^n))^p,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^m (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a + b*log(c*x^n))^p,x)`

[Out] `int((d*x)^m*(a + b*log(c*x^n))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(a+b*ln(c*x**n))**p,x)`

[Out] `Integral((d*x)**m*(a + b*log(c*x**n))**p, x)`

### 3.168 $\int x^2 (a + b \log(cx^n))^p dx$

Optimal. Leaf size=89

$$3^{-p-1} x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma \left( p + 1, -\frac{3(a + b \log(cx^n))}{bn} \right)$$

[Out]  $3^{(-1-p)} * x^3 * \text{GAMMA}(1+p, -3*(a+b*\ln(c*x^n))/b/n) * (a+b*\ln(c*x^n))^p / \exp(3*a/b/n) / ((c*x^n)^{(3/n)}) / (((-a-b*\ln(c*x^n))/b/n)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$3^{-p-1} x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{3(a + b \log(cx^n))}{bn} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{Log}[c*x^n])^p, x]$

[Out]  $(3^{(-1-p)} * x^3 * \text{Gamma}[1+p, (-3*(a + b*\text{Log}[c*x^n]))/(b*n)] * (a + b*\text{Log}[c*x^n])^p) / (E^{((3*a)/(b*n))} * (c*x^n)^{(3/n)} * (-((a + b*\text{Log}[c*x^n])/(b*n))))^p)$

#### Rule 2181

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2310

```
Int[((a_) + Log[(c_)*(x_)^(n_)])*(b_)^(p_)*((d_)*(x_))^(m_), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rubi steps

$$\int x^2 (a + b \log(cx^n))^p dx = \frac{(x^3 (cx^n)^{-3/n}) \text{Subst}\left(\int e^{\frac{3x}{n}} (a + bx)^p dx, x, \log(cx^n)\right)}{n}$$

$$= 3^{-1-p} e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \Gamma\left(1 + p, -\frac{3(a + b \log(cx^n))}{bn}\right) (a + b \log(cx^n))^p \left(-\frac{a + b \log(cx^n)}{bn}\right)$$

**Mathematica** [A] time = 0.10, size = 89, normalized size = 1.00

$$3^{-p-1} x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} (a + b \log(cx^n))^p \left(-\frac{a + b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, -\frac{3(a + b \log(cx^n))}{bn}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x^n])^p,x]

[Out] (3^(-1 - p)\*x^3\*Gamma[1 + p, (-3\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*(-(a + b\*Log[c\*x^n])/(b\*n)))^p)

**fricas** [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(cx^n) + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx^n) + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p\*x^2, x)

**maple** [F] time = 1.37, size = 0, normalized size = 0.00

$$\int x^2 (b \ln(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*ln(c*x^n)+a)^p,x)`

[Out] `int(x^2*(b*ln(c*x^n)+a)^p,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*log(c*x^n))^p,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*log(c*x^n))^p,x)`

[Out] `int(x^2*(a + b*log(c*x^n))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*ln(c*x**n))**p,x)`

[Out] `Integral(x**2*(a + b*log(c*x**n))**p, x)`

$$3.169 \quad \int x \left( a + b \log(cx^n) \right)^p dx$$

Optimal. Leaf size=89

$$2^{-p-1} x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \left( a + b \log(cx^n) \right)^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma \left( p + 1, -\frac{2(a + b \log(cx^n))}{bn} \right)$$

[Out]  $2^{(-1-p)} * x^2 * \text{GAMMA}(1+p, -2*(a+b*\ln(c*x^n))/b/n) * (a+b*\ln(c*x^n))^{p/\exp(2*a/b/n)} / ((c*x^n)^{(2/n)}) / (((-a-b*\ln(c*x^n))/b/n)^p)$

**Rubi [A]** time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2310, 2181}

$$2^{-p-1} x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \left( a + b \log(cx^n) \right)^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{2(a + b \log(cx^n))}{bn} \right)$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x^n])^p,x]

[Out]  $(2^{(-1-p)} * x^2 * \text{Gamma}[1+p, (-2*(a + b*\text{Log}[c*x^n]))/(b*n)]) * (a + b*\text{Log}[c*x^n])^p / (E^{((2*a)/(b*n))} * (c*x^n)^{(2/n)} * (-((a + b*\text{Log}[c*x^n]))/(b*n)))^p)$

Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol]  
 ] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int x (a + b \log(cx^n))^p dx = \frac{(x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int e^{\frac{2x}{n}} (a + bx)^p dx, x, \log(cx^n) \right)}{n}$$

$$= 2^{-1-p} e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \Gamma \left( 1 + p, -\frac{2(a + b \log(cx^n))}{bn} \right) (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)$$

**Mathematica** [A] time = 0.08, size = 89, normalized size = 1.00

$$2^{-p-1} x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma \left( p + 1, -\frac{2(a + b \log(cx^n))}{bn} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n])^p,x]

[Out] (2^(-1 - p)\*x^2\*Gamma[1 + p, (-2\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*(-(a + b\*Log[c\*x^n])/(b\*n)))^p)

**fricas** [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral} \left( (b \log(cx^n) + a)^p x, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx^n) + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p\*x, x)

**maple** [F] time = 1.27, size = 0, normalized size = 0.00

$$\int x (b \ln(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*ln(c*x^n)+a)^p,x)`

[Out] `int(x*(b*ln(c*x^n)+a)^p,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^n))^p,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x (a + b \ln(c x^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*log(c*x^n))^p,x)`

[Out] `int(x*(a + b*log(c*x^n))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \log(c x^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*ln(c*x**n))**p,x)`

[Out] `Integral(x*(a + b*log(c*x**n))**p, x)`

### 3.170 $\int (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=80

$$xe^{-\frac{a}{bn}} (cx^n)^{-1/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma\left(p + 1, -\frac{a + b \log(cx^n)}{bn}\right)$$

[Out] x\*GAMMA(1+p, (-a-b\*ln(c\*x^n))/b/n)\*(a+b\*ln(c\*x^n))^p/exp(a/b/n)/((c\*x^n)^(1/n))/((-a-b\*ln(c\*x^n))/b/n)^p

**Rubi [A]** time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2300, 2181}

$$xe^{-\frac{a}{bn}} (cx^n)^{-1/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma}\left(p + 1, -\frac{a + b \log(cx^n)}{bn}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p, x]

[Out] (x\*Gamma[1 + p, -((a + b\*Log[c\*x^n])/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*(-((a + b\*Log[c\*x^n])/(b\*n)))^p)

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p, x_Symbol] :> Dist[x/(n*(c*x^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]
```

#### Rubi steps



$$\int (a + b \log(cx^n))^p dx = \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int e^{\frac{x}{n}}(a + bx)^p dx, x, \log(cx^n)\right)}{n}$$

$$= e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \Gamma\left(1 + p, -\frac{a + b \log(cx^n)}{bn}\right) (a + b \log(cx^n))^p \left(-\frac{a + b \log(cx^n)}{bn}\right)^{-p}$$

**Mathematica [A]** time = 0.07, size = 80, normalized size = 1.00

$$x e^{-\frac{a}{bn}} (cx^n)^{-1/n} (a + b \log(cx^n))^p \left(-\frac{a + b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, -\frac{a + b \log(cx^n)}{bn}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p, x]

[Out] (x\*Gamma[1 + p, -((a + b\*Log[c\*x^n])/(b\*n))])\*(a + b\*Log[c\*x^n])^p/(E^(a/(b\*n)))\*(c\*x^n)^n^(-1)\*(-((a + b\*Log[c\*x^n])/(b\*n)))^p)

**fricas [A]** time = 0.45, size = 52, normalized size = 0.65

$$e^{\left(\frac{bnp \log\left(-\frac{1}{bn}\right) + b \log(c) + a}{bn}\right)} \Gamma\left(p + 1, -\frac{bn \log(x) + b \log(c) + a}{bn}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p, x, algorithm="fricas")

[Out] e^(-(b\*n\*p\*log(-1/(b\*n)) + b\*log(c) + a)/(b\*n))\*gamma(p + 1, -(b\*n\*log(x) + b\*log(c) + a)/(b\*n))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p, x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p, x)

**maple [F]** time = 0.93, size = 0, normalized size = 0.00

$$\int (b \ln(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^n)+a)^p,x)`

[Out] `int((b*ln(c*x^n)+a)^p,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^p,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))^p,x)`

[Out] `int((a + b*log(c*x^n))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**p,x)`

[Out] `Integral((a + b*log(c*x**n))**p, x)`

$$3.171 \quad \int \frac{(a+b \log(cx^n))^p}{x} dx$$

Optimal. Leaf size=26

$$\frac{(a + b \log(cx^n))^{p+1}}{bn(p+1)}$$

[Out] (a+b\*ln(c\*x^n))^(1+p)/b/n/(1+p)

**Rubi [A]** time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$\frac{(a + b \log(cx^n))^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x, x]

[Out] (a + b\*Log[c\*x^n])^(1 + p)/(b\*n\*(1 + p))

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^p/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(cx^n))^p}{x} dx &= \frac{\text{Subst}\left(\int x^p dx, x, a + b \log(cx^n)\right)}{bn} \\ &= \frac{(a + b \log(cx^n))^{1+p}}{bn(1+p)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{(a + b \log(cx^n))^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x, x]

[Out] (a + b\*Log[c\*x^n])^(1 + p)/(b\*n\*(1 + p))

**fricas** [A] time = 0.42, size = 35, normalized size = 1.35

$$\frac{(bn \log(x) + b \log(c) + a)(bn \log(x) + b \log(c) + a)^p}{bnp + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x,x, algorithm="fricas")

[Out] (b\*n\*log(x) + b\*log(c) + a)\*(b\*n\*log(x) + b\*log(c) + a)^p/(b\*n\*p + b\*n)

**giac** [A] time = 0.25, size = 27, normalized size = 1.04

$$\frac{(bn \log(x) + b \log(c) + a)^{p+1}}{bn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x,x, algorithm="giac")

[Out] (b\*n\*log(x) + b\*log(c) + a)^(p + 1)/(b\*n\*(p + 1))

**maple** [A] time = 0.03, size = 27, normalized size = 1.04

$$\frac{(b \ln(cx^n) + a)^{p+1}}{(p+1)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^n)+a)^p/x, x)

[Out] (b\*ln(c\*x^n)+a)^(p+1)/b/n/(p+1)

**maxima** [A] time = 0.71, size = 26, normalized size = 1.00

$$\frac{(b \log(cx^n) + a)^{p+1}}{bn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x,x, algorithm="maxima")

[Out] (b\*log(c\*x^n) + a)^(p + 1)/(b\*n\*(p + 1))

**mupad** [B] time = 3.67, size = 26, normalized size = 1.00

$$\frac{(a + b \ln(cx^n))^{p+1}}{bn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^p/x,x)

[Out] (a + b\*log(c\*x^n))^p/(b\*n\*(p + 1))

**sympy** [A] time = 1.86, size = 56, normalized size = 2.15

$$- \begin{cases} -a^p \log(x) & \text{for } b = 0 \\ -(a + b \log(c))^p \log(x) & \text{for } n = 0 \\ \left\{ \begin{array}{l} \frac{(a+b \log(cx^n))^{p+1}}{p+1} \\ \log(a + b \log(cx^n)) \end{array} \right. & \text{for } p \neq -1 \\ \frac{\log(a + b \log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x,x)

[Out] -Piecewise((-a\*\*p\*log(x), Eq(b, 0)), (-(a + b\*log(c))\*\*p\*log(x), Eq(n, 0)), (-Piecewise(((a + b\*log(c\*x\*\*n))\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*log(c\*x\*\*n)), True)))/(b\*n), True))

$$3.172 \quad \int \frac{(a+b \log(cx^n))^p}{x^2} dx$$

**Optimal.** Leaf size=78

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{a+b \log(cx^n)}{bn}\right)}{x}$$

[Out]  $-\exp(a/b/n) * (c*x^n)^{(1/n)} * \text{GAMMA}(1+p, (a+b*\ln(c*x^n))/b/n) * (a+b*\ln(c*x^n))^p / x / ((a+b*\ln(c*x^n))/b/n)^p$

**Rubi [A]** time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \text{Gamma}\left(p+1, \frac{a+b \log(cx^n)}{bn}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x^2, x]

[Out]  $-((E^{(a/(b*n))} * (c*x^n)^n)^{-1} * \text{Gamma}[1 + p, (a + b*\text{Log}[c*x^n])/(b*n)]) * (a + b * \text{Log}[c*x^n])^p / (x * ((a + b*\text{Log}[c*x^n])/(b*n))^p)$

**Rule 2181**

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

**Rule 2310**

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol]  
 := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

**Rubi steps**

$$\int \frac{(a + b \log(cx^n))^p}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int e^{-\frac{x}{n}}(a + bx)^p dx, x, \log(cx^n)\right)}{nx}$$

$$= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \Gamma\left(1 + p, \frac{a+b \log(cx^n)}{bn}\right) (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p}}{x}$$

**Mathematica** [A] time = 0.08, size = 78, normalized size = 1.00

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, \frac{a+b \log(cx^n)}{bn}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x^2,x]

[Out] -((E^(a/(b\*n)))\*(c\*x^n)^n^(-1)\*Gamma[1 + p, (a + b\*Log[c\*x^n])/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(x\*((a + b\*Log[c\*x^n])/(b\*n))^p)

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \log(cx^n) + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^2,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(cx^n) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^2,x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p/x^2, x)

**maple** [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(cx^n) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^n)+a)^p/x^2,x)`

[Out] `int((b*ln(c*x^n)+a)^p/x^2,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^p/x^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(cx^n))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^n))^p/x^2,x)`

[Out] `int((a + b*log(c*x^n))^p/x^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(cx^n))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**p/x**2,x)`

[Out] `Integral((a + b*log(c*x**n))**p/x**2, x)`



$$3.173 \quad \int \frac{(a+b \log(cx^n))^p}{x^3} dx$$

**Optimal.** Leaf size=89

$$\frac{2^{-p-1} e^{\frac{2a}{bn}} (cx^n)^{2/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{2(a+b \log(cx^n))}{bn}\right)}{x^2}$$

[Out]  $-2^{-(1+p)} \exp(2a/b/n) (c*x^n)^{(2/n)} * \text{GAMMA}(1+p, 2*(a+b*\ln(c*x^n))/b/n) * (a+b*\ln(c*x^n))^p / x^2 / (((a+b*\ln(c*x^n))/b/n)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{2^{-p-1} e^{\frac{2a}{bn}} (cx^n)^{2/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \text{Gamma}\left(p+1, \frac{2(a+b \log(cx^n))}{bn}\right)}{x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x^3, x]

[Out]  $-((2^{-(1+p)} * E^{((2*a)/(b*n))} * (c*x^n)^{(2/n)} * \text{Gamma}[1+p, (2*(a+b*\text{Log}[c*x^n]))/(b*n)] * (a+b*\text{Log}[c*x^n])^p) / (x^2 * ((a+b*\text{Log}[c*x^n])/(b*n))^p)$

**Rule 2181**

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d)\*(c + d\*x))]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

**Rule 2310**

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^p\*((d\_)\*(x\_))^(m\_), x\_Symbol]  
 :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

**Rubi steps**

$$\int \frac{(a + b \log(cx^n))^p}{x^3} dx = \frac{(cx^n)^{2/n} \text{Subst}\left(\int e^{-\frac{2x}{n}} (a + bx)^p dx, x, \log(cx^n)\right)}{nx^2}$$

$$= \frac{2^{-1-p} e^{\frac{2a}{bn}} (cx^n)^{2/n} \Gamma\left(1 + p, \frac{2(a+b \log(cx^n))}{bn}\right) (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p}}{x^2}$$

**Mathematica** [A] time = 0.09, size = 89, normalized size = 1.00

$$\frac{2^{-p-1} e^{\frac{2a}{bn}} (cx^n)^{2/n} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, \frac{2(a+b \log(cx^n))}{bn}\right)}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x^3, x]

[Out] -((2^(-1 - p)\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*Gamma[1 + p, (2\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(x^2\*((a + b\*Log[c\*x^n])/(b\*n))^p))

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \log(cx^n) + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^3,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(cx^n) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^3,x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p/x^3, x)

**maple** [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(c x^n) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^n)+a)^p/x^3,x)

[Out] int((b\*ln(c\*x^n)+a)^p/x^3,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(c x^n))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^p/x^3,x)

[Out] int((a + b\*log(c\*x^n))^p/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(c x^n))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x\*\*3,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*p/x\*\*3, x)

$$3.174 \quad \int \frac{(a+b \log(cx^n))^p}{x^4} dx$$

**Optimal.** Leaf size=89

$$\frac{3^{-p-1} e^{\frac{3a}{bn}} (cx^n)^{3/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{3(a+b \log(cx^n))}{bn}\right)}{x^3}$$

[Out]  $-3^{-(1-p)} \exp(3a/b/n) * (c*x^n)^{(3/n)} * \text{GAMMA}(1+p, 3*(a+b*\ln(c*x^n))/b/n) * (a+b*\ln(c*x^n))^p / x^3 / (((a+b*\ln(c*x^n))/b/n)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{3^{-p-1} e^{\frac{3a}{bn}} (cx^n)^{3/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \text{Gamma}\left(p+1, \frac{3(a+b \log(cx^n))}{bn}\right)}{x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x^4, x]

[Out]  $-((3^{-(1-p)} * E^{((3*a)/(b*n))} * (c*x^n)^{(3/n)} * \text{Gamma}[1+p, (3*(a+b*\text{Log}[c*x^n]))/(b*n)]) * (a+b*\text{Log}[c*x^n])^p) / (x^3 * ((a+b*\text{Log}[c*x^n])/(b*n))^p)$

#### Rule 2181

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)) \* ((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol]
 := -Simp[(F^(g\*(e - (c\*f)/d)) \* (c + d\*x)^FracPart[m]\*Gamma[m + 1, -(f\*g\*Log[F])/d]) \* (c + d\*x)] / (d \* (-(f\*g\*Log[F])/d)^(IntPart[m] + 1) \* (-(f\*g\*Log[F]) \* (c + d\*x) / d) ^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol]
 := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rubi steps

$$\int \frac{(a + b \log(cx^n))^p}{x^4} dx = \frac{(cx^n)^{3/n} \text{Subst}\left(\int e^{-\frac{3x}{n}} (a + bx)^p dx, x, \log(cx^n)\right)}{nx^3}$$

$$= \frac{3^{-1-p} e^{\frac{3a}{bn}} (cx^n)^{3/n} \Gamma\left(1 + p, \frac{3(a+b \log(cx^n))}{bn}\right) (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p}}{x^3}$$

**Mathematica [A]** time = 0.09, size = 89, normalized size = 1.00

$$\frac{3^{-p-1} e^{\frac{3a}{bn}} (cx^n)^{3/n} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, \frac{3(a+b \log(cx^n))}{bn}\right)}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x^4, x]

[Out] -((3^(-1 - p)\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*Gamma[1 + p, (3\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(x^3\*((a + b\*Log[c\*x^n])/(b\*n))^p))

**fricas [F]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \log(cx^n) + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^4, x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p/x^4, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(cx^n) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^4, x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p/x^4, x)

**maple** [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(c x^n) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^n)+a)^p/x^4,x)

[Out] int((b\*ln(c\*x^n)+a)^p/x^4,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(c x^n))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^n))^p/x^4,x)

[Out] int((a + b\*log(c\*x^n))^p/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(cx^n))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x\*\*4,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*p/x\*\*4, x)

### 3.175 $\int (dx)^m (a + b \log(cx))^p dx$

**Optimal.** Leaf size=86

$$\frac{e^{-\frac{a(m+1)}{b}} (cx)^{-m-1} (dx)^{m+1} (a + b \log(cx))^p \left(-\frac{(m+1)(a+b \log(cx))}{b}\right)^{-p} \Gamma\left(p+1, -\frac{(m+1)(a+b \log(cx))}{b}\right)}{d(m+1)}$$

[Out]  $(c*x)^{-1-m}*(d*x)^{1+m}*GAMMA(1+p, -(1+m)*(a+b*\ln(c*x))/b)*(a+b*\ln(c*x))^p/d/\exp(a*(1+m)/b)/(1+m)/((-1+m)*(a+b*\ln(c*x))/b)^p$

**Rubi [A]** time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{e^{-\frac{a(m+1)}{b}} (cx)^{-m-1} (dx)^{m+1} (a + b \log(cx))^p \left(-\frac{(m+1)(a+b \log(cx))}{b}\right)^{-p} \text{Gamma}\left(p+1, -\frac{(m+1)(a+b \log(cx))}{b}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x])^p, x]

[Out]  $((c*x)^{-1-m}*(d*x)^{1+m}*Gamma[1+p, -(((1+m)*(a+b*Log[c*x]))/b)]*(a+b*Log[c*x])^p)/(d*E^{(a*(1+m))/b}*(1+m)*(-(((1+m)*(a+b*Log[c*x]))/b))^p)$

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.)+(f_.)*(x_)))*((c_.)+(d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e-(c*f)/d))*(c+d*x)^FracPart[m]*Gamma[m+1, (-((f*g*Log[F])/d))*(c+d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m]+1)*(-((f*g*Log[F])*(c+d*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2310

```
Int[((a_.)+Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol]
:> Dist[(d*x)^(m+1)/(d*n*(c*x^n)^((m+1)/n)), Subst[Int[E^((m+1)*x)/n]*(a+b*x)^p, x], x, Log[c*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rubi steps

$$\int (dx)^m (a + b \log(cx))^p dx = \frac{\left( (cx)^{-1-m} (dx)^{1+m} \right) \text{Subst} \left( \int e^{(1+m)x} (a + bx)^p dx, x, \log(cx) \right)}{d}$$

$$= \frac{e^{-\frac{a(1+m)}{b}} (cx)^{-1-m} (dx)^{1+m} \Gamma \left( 1 + p, -\frac{(1+m)(a+b \log(cx))}{b} \right) (a + b \log(cx))^p \left( -\frac{(1+m)(a+b \log(cx))}{b} \right)}{d(1+m)}$$

**Mathematica** [A] time = 0.12, size = 82, normalized size = 0.95

$$\frac{e^{-\frac{a(m+1)}{b}} (cx)^{-m} (dx)^m (a + b \log(cx))^p \left( -\frac{(m+1)(a+b \log(cx))}{b} \right)^{-p} \Gamma \left( p + 1, -\frac{(m+1)(a+b \log(cx))}{b} \right)}{c(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x])^p,x]

[Out] ((d\*x)^m\*Gamma[1 + p, -(((1 + m)\*(a + b\*Log[c\*x])))/b])\*(a + b\*Log[c\*x])^p)/(c\*E^((a\*(1 + m))/b)\*(1 + m)\*(c\*x)^m\*(-(((1 + m)\*(a + b\*Log[c\*x])))/b))^p)

**fricas** [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left( (dx)^m (b \log(cx) + a)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] integral((d\*x)^m\*(b\*log(c\*x) + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b \log(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x))^p,x, algorithm="giac")

[Out] integrate((d\*x)^m\*(b\*log(c\*x) + a)^p, x)

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (dx)^m (b \ln(cx) + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a+b*ln(c*x))^p,x)`

[Out] `int((d*x)^m*(a+b*ln(c*x))^p,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b \log(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*log(c*x))^p,x, algorithm="maxima")`

[Out] `integrate((d*x)^m*(b*log(c*x) + a)^p, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \ln(cx))^p (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x))^p*(d*x)^m,x)`

[Out] `int((a + b*log(c*x))^p*(d*x)^m, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(a+b*ln(c*x))**p,x)`

[Out] `Integral((d*x)**m*(a + b*log(c*x))**p, x)`

### 3.176 $\int x^2(a + b \log(cx))^p dx$

Optimal. Leaf size=63

$$\frac{3^{-p-1} e^{-\frac{3a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{3(a+b \log(cx))}{b}\right)}{c^3}$$

[Out]  $3^{(-1-p)} * \text{GAMMA}(1+p, -3*(a+b*\ln(c*x))/b) * (a+b*\ln(c*x))^p / c^3 / \exp(3*a/b) / (((-a-b*\ln(c*x))/b)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$\frac{3^{-p-1} e^{-\frac{3a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \text{Gamma}\left(p+1, -\frac{3(a+b \log(cx))}{b}\right)}{c^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{Log}[c*x])^p, x]$

[Out]  $(3^{(-1-p)} * \text{Gamma}[1+p, (-3*(a + b*\text{Log}[c*x]))/b] * (a + b*\text{Log}[c*x])^p) / (c^3 * E^{((3*a)/b)} * (-((a + b*\text{Log}[c*x])/b))^p)$

#### Rule 2181

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x]))/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol]
:> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]
```

#### Rubi steps

$$\int x^2(a + b \log(cx))^p dx = \frac{\text{Subst}\left(\int e^{3x}(a + bx)^p dx, x, \log(cx)\right)}{c^3}$$

$$= \frac{3^{-1-p} e^{-\frac{3a}{b}} \Gamma\left(1 + p, -\frac{3(a+b \log(cx))}{b}\right) (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p}}{c^3}$$

**Mathematica** [A] time = 0.07, size = 63, normalized size = 1.00

$$\frac{3^{-p-1} e^{-\frac{3a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{3(a+b \log(cx))}{b}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x])^p,x]

[Out] (3^(-1 - p)\*Gamma[1 + p, (-3\*(a + b\*Log[c\*x]))/b]\*(a + b\*Log[c\*x])^p)/(c^3\*E^((3\*a)/b)\*(-(a + b\*Log[c\*x])/b))^p

**fricas** [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(cx) + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx) + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p\*x^2, x)

**maple** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^2 (b \ln(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*ln(c*x)+a)^p,x)`

[Out] `int(x^2*(b*ln(c*x)+a)^p,x)`

**maxima** [A] time = 0.83, size = 44, normalized size = 0.70

$$\frac{(b \log(cx) + a)^{p+1} e^{\left(-\frac{3a}{b}\right)} E_{-p}\left(-\frac{3(b \log(cx) + a)}{b}\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*log(c*x))^p,x, algorithm="maxima")`

[Out] `-(b*log(c*x) + a)^(p + 1)*e^(-3*a/b)*exp_integral_e(-p, -3*(b*log(c*x) + a)/b)/(b*c^3)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 (a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*log(c*x))^p,x)`

[Out] `int(x^2*(a + b*log(c*x))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*ln(c*x))**p,x)`

[Out] `Integral(x**2*(a + b*log(c*x))**p, x)`

### 3.177 $\int x(a + b \log(cx))^p dx$

Optimal. Leaf size=63

$$\frac{2^{-p-1} e^{-\frac{2a}{b}} (a + b \log(cx))^p \left( -\frac{a+b \log(cx)}{b} \right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(cx))}{b}\right)}{c^2}$$

[Out]  $2^{(-1-p)} \text{GAMMA}(1+p, -2*(a+b*\ln(c*x))/b) * (a+b*\ln(c*x))^p / c^2 / \exp(2*a/b) / (((-a-b*\ln(c*x))/b)^p)$

**Rubi [A]** time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2309, 2181}

$$\frac{2^{-p-1} e^{-\frac{2a}{b}} (a + b \log(cx))^p \left( -\frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma}\left(p+1, -\frac{2(a+b \log(cx))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{Log}[c*x])^p, x]$

[Out]  $(2^{(-1-p)} * \text{Gamma}[1+p, (-2*(a + b*\text{Log}[c*x]))/b] * (a + b*\text{Log}[c*x])^p) / (c^2 * E^{((2*a)/b)} * (-((a + b*\text{Log}[c*x])/b))^p)$

#### Rule 2181

$\text{Int}[(F_)^((g_.) * ((e_.) + (f_.) * (x_))) * ((c_.) + (d_.) * (x_))^{(m_)}, x\_Symbol]$   
 $:\> -\text{Simp}[(F^{(g*(e - (c*f)/d)}) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m+1, (-((f*g*\text{Log}[F])/d)) * (c + d*x)]) / (d * (-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m]+1} * (-((f*g*\text{Log}[F]) * (c + d*x))/d))^{\text{FracPart}[m]}], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_)] * (b_.)^{(p_)} * (x_)^{(m_.)}, x\_Symbol] :\> \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x} * (a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$  FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rubi steps

$$\int x(a + b \log(cx))^p dx = \frac{\text{Subst}\left(\int e^{2x}(a + bx)^p dx, x, \log(cx)\right)}{c^2}$$

$$= \frac{2^{-1-p} e^{-\frac{2a}{b}} \Gamma\left(1 + p, -\frac{2(a+b \log(cx))}{b}\right) (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p}}{c^2}$$

**Mathematica** [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{2^{-p-1} e^{-\frac{2a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{2(a+b \log(cx))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x])^p,x]

[Out] (2^(-1 - p)\*Gamma[1 + p, (-2\*(a + b\*Log[c\*x]))/b]\*(a + b\*Log[c\*x])^p)/(c^2\*E^((2\*a)/b)\*(-(a + b\*Log[c\*x])/b))^p

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(cx) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx) + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p\*x, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x(b \ln(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*ln(c*x)+a)^p,x)`

[Out] `int(x*(b*ln(c*x)+a)^p,x)`

**maxima** [A] time = 0.81, size = 44, normalized size = 0.70

$$\frac{(b \log(cx) + a)^{p+1} e^{\left(-\frac{2a}{b}\right)} E_{-p}\left(-\frac{2(b \log(cx) + a)}{b}\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x))^p,x, algorithm="maxima")`

[Out] `-(b*log(c*x) + a)^(p + 1)*e^(-2*a/b)*exp_integral_e(-p, -2*(b*log(c*x) + a)/b)/(b*c^2)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x (a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*log(c*x))^p,x)`

[Out] `int(x*(a + b*log(c*x))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*ln(c*x))**p,x)`

[Out] `Integral(x*(a + b*log(c*x))**p, x)`

### 3.178 $\int (a + b \log(cx))^p dx$

Optimal. Leaf size=56

$$\frac{e^{-\frac{a}{b}}(a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{a+b \log(cx)}{b}\right)}{c}$$

[Out] GAMMA(1+p, (-a-b\*ln(c\*x))/b)\*(a+b\*ln(c\*x))^p/c/exp(a/b)/(((a-b\*ln(c\*x))/b)^p)

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2299, 2181}

$$\frac{e^{-\frac{a}{b}}(a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{a+b \log(cx)}{b}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p, x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x])/b)]\*(a + b\*Log[c\*x])^p)/(c\*E^(a/b)\*(-((a + b\*Log[c\*x])/b))^p)

#### Rule 2181

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[1/n]
```

#### Rubi steps



$$\int (a + b \log(cx))^p dx = \frac{\text{Subst}\left(\int e^x (a + bx)^p dx, x, \log(cx)\right)}{c}$$

$$= \frac{e^{-\frac{a}{b}} \Gamma\left(1 + p, -\frac{a+b \log(cx)}{b}\right) (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p}}{c}$$

**Mathematica [A]** time = 0.03, size = 56, normalized size = 1.00

$$\frac{e^{-\frac{a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{a+b \log(cx)}{b}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p,x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x])/b)]\*(a + b\*Log[c\*x])^p)/(c\*E^(a/b)\*(-(a + b\*Log[c\*x])/b))^p

**fricas [A]** time = 0.42, size = 38, normalized size = 0.68

$$\frac{e^{\left(-\frac{bp \log\left(-\frac{1}{b}\right) + a}{b}\right)} \Gamma\left(p + 1, -\frac{b \log(cx) + a}{b}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] e^(-(b\*p\*log(-1/b) + a)/b)\*gamma(p + 1, -(b\*log(c\*x) + a)/b)/c

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p, x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (b \ln(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x)+a)^p,x)`

[Out] `int((b*ln(c*x)+a)^p,x)`

**maxima** [A] time = 0.84, size = 44, normalized size = 0.79

$$\frac{(b \log(cx) + a)^{p+1} e^{\left(-\frac{a}{b}\right)} E_{-p}\left(-\frac{b \log(cx) + a}{b}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p,x, algorithm="maxima")`

[Out] `-(b*log(c*x) + a)^(p + 1)*e^(-a/b)*exp_integral_e(-p, -(b*log(c*x) + a)/b)/(b*c)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x))^p,x)`

[Out] `int((a + b*log(c*x))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x))**p,x)`

[Out] `Integral((a + b*log(c*x))**p, x)`

$$3.179 \quad \int \frac{(a+b \log(cx))^p}{x} dx$$

Optimal. Leaf size=21

$$\frac{(a + b \log(cx))^{p+1}}{b(p+1)}$$

[Out] (a+b\*ln(c\*x))^(1+p)/b/(1+p)

**Rubi** [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{(a + b \log(cx))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p/x, x]

[Out] (a + b\*Log[c\*x])^(1 + p)/(b\*(1 + p))

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(cx))^p}{x} dx &= \frac{\text{Subst}\left(\int x^p dx, x, a + b \log(cx)\right)}{b} \\ &= \frac{(a + b \log(cx))^{1+p}}{b(1+p)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{(a + b \log(cx))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x,x]

[Out] (a + b\*Log[c\*x])^(1 + p)/(b\*(1 + p))

**fricas** [A] time = 0.42, size = 26, normalized size = 1.24

$$\frac{(b \log(cx) + a)(b \log(cx) + a)^p}{bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x,x, algorithm="fricas")

[Out] (b\*log(c\*x) + a)\*(b\*log(c\*x) + a)^p/(b\*p + b)

**giac** [A] time = 0.39, size = 21, normalized size = 1.00

$$\frac{(b \log(cx) + a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x,x, algorithm="giac")

[Out] (b\*log(c\*x) + a)^(p + 1)/(b\*(p + 1))

**maple** [A] time = 0.03, size = 22, normalized size = 1.05

$$\frac{(b \ln(cx) + a)^{p+1}}{(p+1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x)+a)^p/x,x)

[Out] (b\*ln(c\*x)+a)^(p+1)/b/(p+1)

**maxima** [A] time = 0.53, size = 21, normalized size = 1.00

$$\frac{(b \log(cx) + a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x,x, algorithm="maxima")

[Out] (b\*log(c\*x) + a)^(p + 1)/(b\*(p + 1))

**mupad [B]** time = 3.70, size = 21, normalized size = 1.00

$$\frac{(a + b \ln(cx))^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x))^p/x,x)

[Out] (a + b\*log(c\*x))^(p + 1)/(b\*(p + 1))

**sympy [A]** time = 1.31, size = 39, normalized size = 1.86

$$- \begin{cases} -a^p \log(x) & \text{for } b = 0 \\ \left\{ \begin{array}{l} \frac{(a+b \log(cx))^{p+1}}{p+1} \\ \log(a + b \log(cx)) \end{array} \right. & \text{for } p \neq -1 \\ \frac{\log(a + b \log(cx))}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x))\*\*p/x,x)

[Out] -Piecewise((-a\*\*p\*log(x), Eq(b, 0)), (-Piecewise(((a + b\*log(c\*x))\*\*(p + 1))/(p + 1), Ne(p, -1)), (log(a + b\*log(c\*x)), True))/b, True))

$$3.180 \quad \int \frac{(a+b \log(cx))^p}{x^2} dx$$

Optimal. Leaf size=52

$$-ce^{a/b}(a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, \frac{a+b \log(cx)}{b}\right)$$

[Out]  $-c*\exp(a/b)*\text{GAMMA}(1+p, (a+b*\ln(c*x))/b)*(a+b*\ln(c*x))^p/(((a+b*\ln(c*x))/b)^p)$

**Rubi [A]** time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$-ce^{a/b}(a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \text{Gamma}\left(p+1, \frac{a+b \log(cx)}{b}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[c*x])^p/x^2, x]$

[Out]  $-((c*E^{(a/b)*\text{Gamma}[1 + p, (a + b*\text{Log}[c*x])/b]}*(a + b*\text{Log}[c*x])^p)/((a + b*\text{Log}[c*x])/b)^p)$

Rule 2181

$\text{Int}[(F\_)^{((g\_)*(e\_)+(f\_)*(x\_))}*((c\_)+(d\_)*(x\_))^m], x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x)])/(d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x))/d))^{\text{FracPart}[m]}], x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m, x\}$  &&  $!\text{IntegerQ}[m]$

Rule 2309

$\text{Int}[(a + \text{Log}[c*x]*b)^p*(x^m), x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$   $\text{FreeQ}\{a, b, c, p, x\}$  &&  $\text{IntegerQ}[m]$

Rubi steps

$$\int \frac{(a + b \log(cx))^p}{x^2} dx = c \operatorname{Subst} \left( \int e^{-x} (a + bx)^p dx, x, \log(cx) \right) \\ = -ce^{a/b} \Gamma \left( 1 + p, \frac{a + b \log(cx)}{b} \right) (a + b \log(cx))^p \left( \frac{a + b \log(cx)}{b} \right)^{-p}$$

**Mathematica** [A] time = 0.04, size = 48, normalized size = 0.92

$$-ce^{a/b} \left( \frac{a}{b} + \log(cx) \right)^{-p} (a + b \log(cx))^p \Gamma \left( p + 1, \frac{a}{b} + \log(cx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x^2,x]

[Out] -((c\*E^(a/b)\*Gamma[1 + p, a/b + Log[c\*x]])\*(a + b\*Log[c\*x])^p)/(a/b + Log[c\*x])^p)

**fricas** [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(b \log(cx) + a)^p}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^2,x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(cx) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^2,x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p/x^2, x)

**maple** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(cx) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x)+a)^p/x^2,x)`

[Out] `int((b*ln(c*x)+a)^p/x^2,x)`

**maxima** [A] time = 0.63, size = 40, normalized size = 0.77

$$\frac{(b \log(cx) + a)^{p+1} c e^{\frac{a}{b}} E_{-p}\left(\frac{b \log(cx) + a}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^2,x, algorithm="maxima")`

[Out] `-(b*log(c*x) + a)^(p + 1)*c*e^(a/b)*exp_integral_e(-p, (b*log(c*x) + a)/b)/b`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b \ln(cx))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x))^p/x^2,x)`

[Out] `int((a + b*log(c*x))^p/x^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(cx))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x))**p/x**2,x)`

[Out] `Integral((a + b*log(c*x))**p/x**2, x)`



$$3.181 \quad \int \frac{(a+b \log(cx))^p}{x^3} dx$$

Optimal. Leaf size=63

$$c^2 \left(-2^{-p-1}\right) e^{\frac{2a}{b}} (a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, \frac{2(a+b \log(cx))}{b}\right)$$

[Out]  $-2^{(-1-p)} * c^2 * \exp(2*a/b) * \text{GAMMA}(1+p, 2*(a+b*\ln(c*x))/b) * (a+b*\ln(c*x))^{-p} / ((a+b*\ln(c*x))/b)^p$

**Rubi [A]** time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$c^2 \left(-2^{-p-1}\right) e^{\frac{2a}{b}} (a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \text{Gamma}\left(p+1, \frac{2(a+b \log(cx))}{b}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p/x^3, x]

[Out]  $-((2^{(-1-p)} * c^2 * E^{((2*a)/b)} * \text{Gamma}[1+p, (2*(a+b*\text{Log}[c*x]))/b]) * (a+b*\text{Log}[c*x])^p) / ((a+b*\text{Log}[c*x])/b)^p$

Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 2309

Int[((a\_) + Log[(c\_)\*(x\_)])\*(b\_)^(p\_)\*(x\_)^(m\_), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx))^p}{x^3} dx &= c^2 \text{Subst}\left(\int e^{-2x}(a+bx)^p dx, x, \log(cx)\right) \\ &= -2^{-1-p} c^2 e^{\frac{2a}{b}} \Gamma\left(1+p, \frac{2(a+b \log(cx))}{b}\right) (a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 63, normalized size = 1.00

$$c^2 \left(-2^{-p-1}\right) e^{\frac{2a}{b}} (a + b \log(cx))^p \left(\frac{a + b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, \frac{2(a + b \log(cx))}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x^3, x]

[Out] -((2^(-1 - p)\*c^2\*E^((2\*a)/b)\*Gamma[1 + p, (2\*(a + b\*Log[c\*x]))/b])\*(a + b\*Log[c\*x])^p)/((a + b\*Log[c\*x])/b)^p

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \log(cx) + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^3, x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(cx) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^3, x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p/x^3, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(cx) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x)+a)^p/x^3, x)

[Out] int((b\*ln(c\*x)+a)^p/x^3, x)

**maxima** [A] time = 0.70, size = 44, normalized size = 0.70

$$\frac{(b \log(cx) + a)^{p+1} c^2 e^{\left(\frac{2a}{b}\right)} E_{-p}\left(\frac{2(b \log(cx) + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^3,x, algorithm="maxima")

[Out] -(b\*log(c\*x) + a)^(p + 1)\*c^2\*e^(2\*a/b)\*exp\_integral\_e(-p, 2\*(b\*log(c\*x) + a)/b)/b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b \ln(cx))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x))^p/x^3,x)

[Out] int((a + b\*log(c\*x))^p/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(cx))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x))\*\*p/x\*\*3,x)

[Out] Integral((a + b\*log(c\*x))\*\*p/x\*\*3, x)

$$3.182 \quad \int \frac{(a+b \log(cx))^p}{x^4} dx$$

Optimal. Leaf size=63

$$c^3 (-3^{-p-1}) e^{\frac{3a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \Gamma \left( p+1, \frac{3(a+b \log(cx))}{b} \right)$$

[Out]  $-3^{-(1-p)} * c^3 * \exp(3*a/b) * \text{GAMMA}(1+p, 3*(a+b*\ln(c*x))/b) * (a+b*\ln(c*x))^p / ((a+b*\ln(c*x))/b)^p$

**Rubi [A]** time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$c^3 (-3^{-p-1}) e^{\frac{3a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{3(a+b \log(cx))}{b} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[c*x])^p/x^4, x]$

[Out]  $-((3^{-(1-p)} * c^3 * E^{(3*a)/b} * \text{Gamma}[1 + p, (3*(a + b*\text{Log}[c*x]))/b]) * (a + b*\text{Log}[c*x])^p) / ((a + b*\text{Log}[c*x])/b)^p$

Rule 2181

$\text{Int}[(F_)^m * ((g_) * ((e_) + (f_) * (x_))) * ((c_) + (d_) * (x_))^{m_}], x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m + 1, (-(f*g*\text{Log}[F])/d) * (c + d*x)]}) / (d * (-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-(f*g*\text{Log}[F] * (c + d*x)/d))^{\text{FracPart}[m]}], x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m, x\} \ \&\amp; \ !\text{IntegerQ}[m]$

Rule 2309

$\text{Int}[(a_) + \text{Log}[(c_) * (x_)] * (b_)]^{p_} * (x_)^{m_}], x\_Symbol]$   $\rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x} * (a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$   $\text{FreeQ}\{a, b, c, p, x\} \ \&\amp; \ \text{IntegerQ}[m]$

Rubi steps

$$\int \frac{(a+b \log(cx))^p}{x^4} dx = c^3 \text{Subst} \left( \int e^{-3x} (a+bx)^p dx, x, \log(cx) \right)$$

$$= -3^{-1-p} c^3 e^{\frac{3a}{b}} \Gamma \left( 1+p, \frac{3(a+b \log(cx))}{b} \right) (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p}$$

**Mathematica** [A] time = 0.04, size = 63, normalized size = 1.00

$$c^3 \left(-3^{-p-1}\right) e^{\frac{3a}{b}} (a + b \log(cx))^p \left(\frac{a + b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, \frac{3(a + b \log(cx))}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x^4, x]

[Out] -((3^(-1 - p)\*c^3\*E^((3\*a)/b)\*Gamma[1 + p, (3\*(a + b\*Log[c\*x]))/b])\*(a + b\*Log[c\*x])^p)/((a + b\*Log[c\*x])/b)^p)

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b \log(cx) + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^4,x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p/x^4, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(cx) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^4,x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p/x^4, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(cx) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x)+a)^p/x^4,x)

[Out] int((b\*ln(c\*x)+a)^p/x^4,x)

**maxima** [A] time = 0.69, size = 44, normalized size = 0.70

$$\frac{(b \log(cx) + a)^{p+1} c^3 e^{\left(\frac{3a}{b}\right)} E_{-p}\left(\frac{3(b \log(cx) + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x^4,x, algorithm="maxima")

[Out] -(b\*log(c\*x) + a)^(p + 1)\*c^3\*e^(3\*a/b)\*exp\_integral\_e(-p, 3\*(b\*log(c\*x) + a)/b)/b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b \ln(cx))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x))^p/x^4,x)

[Out] int((a + b\*log(c\*x))^p/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(cx))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x))\*\*p/x\*\*4,x)

[Out] Integral((a + b\*log(c\*x))\*\*p/x\*\*4, x)

$$3.183 \quad \int (dx)^m \left( a + b \log(c\sqrt{x}) \right)^p dx$$

**Optimal.** Leaf size=107

$$\frac{2^{-p} e^{-\frac{2a(m+1)}{b}} (c\sqrt{x})^{-2(m+1)} (dx)^{m+1} (a + b \log(c\sqrt{x}))^p \left( -\frac{(m+1)(a+b \log(c\sqrt{x}))}{b} \right)^{-p} \Gamma\left(p+1, -\frac{2(m+1)(a+b \log(c\sqrt{x}))}{b}\right)}{d(m+1)}$$

[Out]  $(d*x)^{(1+m)} * \text{GAMMA}(1+p, -2*(1+m)*(a+b*\ln(c*x^{(1/2)}))/b)*(a+b*\ln(c*x^{(1/2)}))^p / (2^p)/d/\exp(2*a*(1+m)/b)/(1+m)/((-1+m)*(a+b*\ln(c*x^{(1/2)}))/b)^p/((c*x^{(1/2)})^{(2+2*m)})$

**Rubi [A]** time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2310, 2181}

$$\frac{2^{-p} e^{-\frac{2a(m+1)}{b}} (c\sqrt{x})^{-2(m+1)} (dx)^{m+1} (a + b \log(c\sqrt{x}))^p \left( -\frac{(m+1)(a+b \log(c\sqrt{x}))}{b} \right)^{-p} \text{Gamma}\left(p+1, -\frac{2(m+1)(a+b \log(c\sqrt{x}))}{b}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^m*(a + b*\text{Log}[c*\text{Sqrt}[x]])^p, x]$

[Out]  $((d*x)^{(1+m)} * \text{Gamma}[1+p, (-2*(1+m)*(a + b*\text{Log}[c*\text{Sqrt}[x]]))/b]*(a + b*\text{Log}[c*\text{Sqrt}[x]])^p) / (2^p*d*\text{E}^{((2*a*(1+m))/b)*(1+m)*(c*\text{Sqrt}[x])^{(2*(1+m))}} * (-(((1+m)*(a + b*\text{Log}[c*\text{Sqrt}[x]]))/b))^p)$

**Rule 2181**

$\text{Int}[(F\_)^{(g\_)*((e\_)+(f\_)*(x\_))}*((c\_)+(d\_)*(x\_))^{(m\_)}, x\_Symbol]$   
 $:\> -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x)]) / (d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x))/d))^{\text{FracPart}[m]}], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

**Rule 2310**

$\text{Int}[(a\_ + \text{Log}[(c\_)*(x\_)^{(n\_)}]*(b\_))^{\text{p}_}*((d\_)*(x\_))^{\text{m}_}], x\_Symbol]$   
 $:\> \text{Dist}[(d*x)^{(m+1)} / (d*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[\text{E}^{((m+1)*x)/n}*(a + b*x)^p, x], x, \text{Log}[c*x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x]

**Rubi steps**

$$\int (dx)^m (a + b \log(c\sqrt{x}))^p dx = \frac{\left(2(c\sqrt{x})^{-2(1+m)} (dx)^{1+m}\right) \text{Subst}\left(\int e^{2(1+m)x} (a + bx)^p dx, x, \log(c\sqrt{x})\right)}{d}$$

$$= \frac{2^{-p} e^{-\frac{2a(1+m)}{b}} (c\sqrt{x})^{-2(1+m)} (dx)^{1+m} \Gamma\left(1 + p, -\frac{2(1+m)(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p}{d(1+m)}$$

**Mathematica [A]** time = 0.19, size = 103, normalized size = 0.96

$$\frac{2^{-p} e^{-\frac{2a(m+1)}{b}} (c\sqrt{x})^{-2m} (dx)^m (a + b \log(c\sqrt{x}))^p \left(-\frac{(m+1)(a+b \log(c\sqrt{x}))}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{2(m+1)(a+b \log(c\sqrt{x}))}{b}\right)}{c^2(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*Sqrt[x]])^p,x]

[Out] ((d\*x)^m\*Gamma[1 + p, (-2\*(1 + m)\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p\*c^2\*E^((2\*a\*(1 + m))/b)\*(1 + m)\*(c\*Sqrt[x])^(2\*m)\*(-((1 + m)\*(a + b\*Log[c\*Sqrt[x]]))/b))^p)

**fricas [F]** time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m (b \log(c\sqrt{x}) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^(1/2)))^p,x, algorithm="fricas")

[Out] integral((d\*x)^m\*(b\*log(c\*sqrt(x)) + a)^p, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b \log(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^(1/2)))^p,x, algorithm="giac")

[Out] integrate((d\*x)^m\*(b\*log(c\*sqrt(x)) + a)^p, x)



**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (dx)^m (b \ln(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*ln(c\*x^(1/2))))^p,x)

[Out] int((d\*x)^m\*(a+b\*ln(c\*x^(1/2))))^p,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (b \log(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="maxima")

[Out] integrate((d\*x)^m\*(b\*log(c\*sqrt(x)) + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^m (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*log(c\*x^(1/2))))^p,x)

[Out] int((d\*x)^m\*(a + b\*log(c\*x^(1/2))))^p, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \log(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x\*\*(1/2))))\*\*p,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*log(c\*sqrt(x))))\*\*p, x)

### 3.184 $\int x^2 \left( a + b \log(c\sqrt{x}) \right)^p dx$

Optimal. Leaf size=80

$$\frac{2^{-p} 3^{-p-1} e^{-\frac{6a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p + 1, -\frac{6(a+b \log(c\sqrt{x}))}{b} \right)}{c^6}$$

[Out]  $3^{(-1-p)} \text{GAMMA}(1+p, -6*(a+b*\ln(c*x^{(1/2)}))/b) * (a+b*\ln(c*x^{(1/2)}))^p / (2^p) / c^6 / \exp(6*a/b) / (((-a-b*\ln(c*x^{(1/2)}))/b)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$\frac{2^{-p} 3^{-p-1} e^{-\frac{6a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{6(a+b \log(c\sqrt{x}))}{b} \right)}{c^6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{Log}[c*\text{Sqrt}[x]])^p, x]$

[Out]  $(3^{(-1-p)}*\text{Gamma}[1+p, (-6*(a + b*\text{Log}[c*\text{Sqrt}[x]]))/b]*(a + b*\text{Log}[c*\text{Sqrt}[x]])^p)/(2^p*c^6*\text{E}^((6*a)/b)*(-((a + b*\text{Log}[c*\text{Sqrt}[x]])/b))^p)$

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x]))/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p)*((d_.)*(x_))^(m_.), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)*x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rubi steps

$$\int x^2 (a + b \log(c\sqrt{x}))^p dx = \frac{2 \text{Subst}\left(\int e^{6x}(a + bx)^p dx, x, \log(c\sqrt{x})\right)}{c^6}$$

$$= \frac{2^{-p} 3^{-1-p} e^{-\frac{6a}{b}} \Gamma\left(1 + p, -\frac{6(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p}}{c^6}$$

**Mathematica [A]** time = 0.11, size = 80, normalized size = 1.00

$$\frac{2^{-p} 3^{-p-1} e^{-\frac{6a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{6(a+b \log(c\sqrt{x}))}{b}\right)}{c^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*Sqrt[x]])^p,x]

[Out] (3^(-1 - p)\*Gamma[1 + p, (-6\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p\*c^6\*E^((6\*a)/b)\*(-(a + b\*Log[c\*Sqrt[x]])/b)^p)

**fricas [F]** time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(c\sqrt{x}) + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p\*x^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(c\sqrt{x}) + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p\*x^2, x)

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int x^2 (b \ln(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*ln(c*x^(1/2))+a)^p,x)`

[Out] `int(x^2*(b*ln(c*x^(1/2))+a)^p,x)`

**maxima** [A] time = 0.64, size = 48, normalized size = 0.60

$$\frac{2(b \log(c\sqrt{x}) + a)^{p+1} e^{\left(-\frac{6a}{b}\right)} E_{-p}\left(-\frac{6(b \log(c\sqrt{x}) + a)}{b}\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")`

[Out] `-2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-6*a/b)*exp_integral_e(-p, -6*(b*log(c*sqrt(x)) + a)/b)/(b*c^6)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*log(c*x^(1/2)))^p,x)`

[Out] `int(x^2*(a + b*log(c*x^(1/2)))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \log(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*ln(c*x**(1/2)))**p,x)`

[Out] `Integral(x**2*(a + b*log(c*sqrt(x)))**p, x)`

### 3.185 $\int x \left( a + b \log(c\sqrt{x}) \right)^p dx$

Optimal. Leaf size=75

$$\frac{2^{-2p-1} e^{-\frac{4a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma\left( p+1, -\frac{4(a+b \log(c\sqrt{x}))}{b} \right)}{c^4}$$

[Out]  $2^{(-1-2*p)} * \text{GAMMA}(1+p, -4*(a+b*\ln(c*x^{(1/2)}))/b) * (a+b*\ln(c*x^{(1/2)}))^p / c^4 / \exp(4*a/b) / (((-a-b*\ln(c*x^{(1/2)}))/b)^p)$

**Rubi [A]** time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{2^{-2p-1} e^{-\frac{4a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma}\left( p+1, -\frac{4(a+b \log(c\sqrt{x}))}{b} \right)}{c^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{Log}[c*\text{Sqrt}[x]])^p, x]$

[Out]  $(2^{(-1 - 2*p)} * \text{Gamma}[1 + p, (-4*(a + b*\text{Log}[c*\text{Sqrt}[x]]))/b] * (a + b*\text{Log}[c*\text{Sqrt}[x]])^p) / (c^4 * E^{(4*a/b)} * (-((a + b*\text{Log}[c*\text{Sqrt}[x]]))/b))^p$

#### Rule 2181

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol]$   
 $:\> -\text{Simp}[(F^{(g*(e - (c*f)/d))} * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d)) * (c + d*x)]) / (d * (-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-((f*g*\text{Log}[F]) * (c + d*x))/d))^{\text{FracPart}[m]}], x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x \&\& !\text{IntegerQ}[m]$

#### Rule 2310

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}] * (b_.)]^{(p_)} * ((d_.)*(x_))^{(m_.)}, x\_Symbol]$   
 $:\> \text{Dist}[(d*x)^{(m+1)} / (d*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[E^{((m+1)*x/n)} * (a + b*x)^p, x], x, \text{Log}[c*x^n], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rubi steps

$$\int x (a + b \log(c\sqrt{x}))^p dx = \frac{2 \text{Subst}\left(\int e^{4x} (a + bx)^p dx, x, \log(c\sqrt{x})\right)}{c^4}$$

$$= \frac{2^{-1-2p} e^{-\frac{4a}{b}} \Gamma\left(1 + p, -\frac{4(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p}}{c^4}$$

**Mathematica** [A] time = 0.07, size = 75, normalized size = 1.00

$$\frac{2^{-2p-1} e^{-\frac{4a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{4(a+b \log(c\sqrt{x}))}{b}\right)}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*Sqrt[x]])^p,x]

[Out] (2^(-1 - 2\*p)\*Gamma[1 + p, (-4\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(c^4\*E^((4\*a)/b)\*(-(a + b\*Log[c\*Sqrt[x]])/b))^p

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(c\sqrt{x}) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(c\sqrt{x}) + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p\*x, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x (b \ln(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*ln(c*x^(1/2))+a)^p,x)`

[Out] `int(x*(b*ln(c*x^(1/2))+a)^p,x)`

**maxima** [A] time = 0.68, size = 48, normalized size = 0.64

$$\frac{2 \left( b \log(c\sqrt{x}) + a \right)^{p+1} e^{\left( -\frac{4a}{b} \right)} E_{-p} \left( -\frac{4(b \log(c\sqrt{x}) + a)}{b} \right)}{bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")`

[Out] `-2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-4*a/b)*exp_integral_e(-p, -4*(b*log(c*sqrt(x)) + a)/b)/(b*c^4)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left( a + b \ln(c\sqrt{x}) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*log(c*x^(1/2)))^p,x)`

[Out] `int(x*(a + b*log(c*x^(1/2)))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left( a + b \log(c\sqrt{x}) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*ln(c*x**(1/2)))**p,x)`

[Out] `Integral(x*(a + b*log(c*sqrt(x)))**p, x)`

### 3.186 $\int (a + b \log(c\sqrt{x}))^p dx$

**Optimal.** Leaf size=73

$$\frac{2^{-p} e^{-\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right)}{c^2}$$

[Out] GAMMA(1+p, -2\*(a+b\*ln(c\*x^(1/2)))/b)\*(a+b\*ln(c\*x^(1/2)))^p/(2^p)/c^2/exp(2\*a/b)/(((a+b\*ln(c\*x^(1/2)))/b)^p)

**Rubi [A]** time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2299, 2181}

$$\frac{2^{-p} e^{-\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \text{Gamma}\left(p+1, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p, x]

[Out] (Gamma[1 + p, (-2\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p \* c^2 \* E^((2\*a)/b) \* (-((a + b\*Log[c\*Sqrt[x]])/b))^p)

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p, x_Symbol] :> Dist[1/(n*c^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[1/n]
```

#### Rubi steps



$$\int (a + b \log(c\sqrt{x}))^p dx = \frac{2 \text{Subst}\left(\int e^{2x}(a + bx)^p dx, x, \log(c\sqrt{x})\right)}{c^2}$$

$$= \frac{2^{-p} e^{-\frac{2a}{b}} \Gamma\left(1 + p, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p}}{c^2}$$

**Mathematica [A]** time = 0.03, size = 73, normalized size = 1.00

$$\frac{2^{-p} e^{-\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p, x]

[Out] (Gamma[1 + p, (-2\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p \* c^2 \* E^((2\*a)/b)\*(-(a + b\*Log[c\*Sqrt[x]])/b))^p

**fricas [F]** time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(c\sqrt{x}) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p, x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p, x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p, x)

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int (b \ln(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^(1/2))+a)^p,x)`

[Out] `int((b*ln(c*x^(1/2))+a)^p,x)`

**maxima** [A] time = 0.80, size = 48, normalized size = 0.66

$$\frac{2(b \log(c\sqrt{x}) + a)^{p+1} e^{\left(-\frac{2a}{b}\right)} E_{-p}\left(-\frac{2(b \log(c\sqrt{x}) + a)}{b}\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")`

[Out] `-2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-2*a/b)*exp_integral_e(-p, -2*(b*log(c*sqrt(x)) + a)/b)/(b*c^2)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^(1/2)))^p,x)`

[Out] `int((a + b*log(c*x^(1/2)))^p, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \log(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**(1/2)))**p,x)`

[Out] `Integral((a + b*log(c*sqrt(x)))**p, x)`

$$3.187 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x} dx$$

Optimal. Leaf size=26

$$\frac{2(a+b \log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

[Out] 2\*(a+b\*ln(c\*x^(1/2)))^(1+p)/b/(1+p)

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2302, 30}

$$\frac{2(a+b \log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x,x]

[Out] (2\*(a + b\*Log[c\*Sqrt[x]])^(1 + p))/(b\*(1 + p))

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(c\sqrt{x}))^p}{x} dx &= \frac{2 \text{Subst}\left(\int x^p dx, x, a+b \log(c\sqrt{x})\right)}{b} \\ &= \frac{2(a+b \log(c\sqrt{x}))^{1+p}}{b(1+p)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{2(a + b \log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x,x]

[Out] (2\*(a + b\*Log[c\*Sqrt[x]])^(1 + p))/(b\*(1 + p))

**fricas** [A] time = 0.43, size = 31, normalized size = 1.19

$$\frac{2(b \log(c\sqrt{x}) + a)(b \log(c\sqrt{x}) + a)^p}{bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x,x, algorithm="fricas")

[Out] 2\*(b\*log(c\*sqrt(x)) + a)\*(b\*log(c\*sqrt(x)) + a)^p/(b\*p + b)

**giac** [A] time = 0.31, size = 25, normalized size = 0.96

$$\frac{2\left(b \log(c) + \frac{1}{2} b \log(x) + a\right)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x,x, algorithm="giac")

[Out] 2\*(b\*log(c) + 1/2\*b\*log(x) + a)^(p + 1)/(b\*(p + 1))

**maple** [A] time = 0.03, size = 25, normalized size = 0.96

$$\frac{2(b \ln(c\sqrt{x}) + a)^{p+1}}{(p+1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*ln(c\*x^(1/2))+a)^p/x,x)

[Out] 2\*(b\*ln(c\*x^(1/2))+a)^(p+1)/b/(p+1)

**maxima** [A] time = 0.56, size = 24, normalized size = 0.92

$$\frac{2(b \log(c\sqrt{x}) + a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x,x, algorithm="maxima")

[Out] 2\*(b\*log(c\*sqrt(x)) + a)^(p + 1)/(b\*(p + 1))

**mupad** [B] time = 3.72, size = 24, normalized size = 0.92

$$\frac{2(a + b \ln(c\sqrt{x}))^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*log(c\*x^(1/2)))^p/x,x)

[Out] (2\*(a + b\*log(c\*x^(1/2)))^(p + 1))/(b\*(p + 1))

**sympy** [A] time = 4.76, size = 48, normalized size = 1.85

$$-\left\{ \begin{array}{ll} -a^p \log(x) & \text{for } b = 0 \\ 2 \left\{ \begin{array}{ll} \frac{(a+b \log(c\sqrt{x}))^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a + b \log(c\sqrt{x})) & \text{otherwise} \end{array} \right. & \text{otherwise} \\ -\frac{\quad}{b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*(1/2)))\*\*p/x,x)

[Out] -Piecewise((-a\*\*p\*log(x), Eq(b, 0)), (-2\*Piecewise(((a + b\*log(c\*sqrt(x)))\*\*  
\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*log(c\*sqrt(x))), True))/b, True))

$$3.188 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x^2} dx$$

Optimal. Leaf size=73

$$c^2 (-2^{-p}) e^{\frac{2a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p+1, \frac{2(a+b \log(c\sqrt{x}))}{b} \right)$$

[Out]  $-c^2 \exp(2*a/b) * \text{GAMMA}(1+p, 2*(a+b*\ln(c*x^{(1/2)}))/b) * (a+b*\ln(c*x^{(1/2)}))^{-p} / (2^{-p} / (((a+b*\ln(c*x^{(1/2)}))/b)^p))$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$c^2 (-2^{-p}) e^{\frac{2a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x^2,x]

[Out]  $-((c^2 * E^{((2*a)/b)} * \text{Gamma}[1 + p, (2*(a + b*\text{Log}[c*\text{Sqrt}[x]))]/b]) * (a + b*\text{Log}[c*\text{Sqrt}[x]])^p) / (2^{-p} * ((a + b*\text{Log}[c*\text{Sqrt}[x]))/b)^p)$

Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]
 := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol]
 := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{(a + b \log(c\sqrt{x}))^p}{x^2} dx = (2c^2) \text{Subst} \left( \int e^{-2x} (a + bx)^p dx, x, \log(c\sqrt{x}) \right) \\ = -2^{-p} c^2 e^{\frac{2a}{b}} \Gamma \left( 1 + p, \frac{2(a + b \log(c\sqrt{x}))}{b} \right) (a + b \log(c\sqrt{x}))^p \left( \frac{a + b \log(c\sqrt{x})}{b} \right)^{-p}$$

**Mathematica** [A] time = 0.06, size = 73, normalized size = 1.00

$$c^2 (-2^{-p}) e^{\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left( \frac{a + b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p + 1, \frac{2(a + b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x^2, x]

[Out] -((c^2\*E^((2\*a)/b)\*Gamma[1 + p, (2\*(a + b\*Log[c\*Sqrt[x]]))]/b)\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p\*((a + b\*Log[c\*Sqrt[x]])/b)^p)

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b \log(c\sqrt{x}) + a)^p}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^2, x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(c\sqrt{x}) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^2, x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p/x^2, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(c\sqrt{x}) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^(1/2))+a)^p/x^2,x)`

[Out] `int((b*ln(c*x^(1/2))+a)^p/x^2,x)`

**maxima** [A] time = 0.97, size = 48, normalized size = 0.66

$$\frac{2 \left( b \log(c\sqrt{x}) + a \right)^{p+1} c^2 e^{\left( \frac{2a}{b} \right)} E_{-p} \left( \frac{2 \left( b \log(c\sqrt{x}) + a \right)}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^(1/2)))^p/x^2,x, algorithm="maxima")`

[Out] `-2*(b*log(c*sqrt(x)) + a)^(p + 1)*c^2*e^(2*a/b)*exp_integral_e(-p, 2*(b*log(c*sqrt(x)) + a)/b)/b`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left( a + b \ln(c\sqrt{x}) \right)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^(1/2)))^p/x^2,x)`

[Out] `int((a + b*log(c*x^(1/2)))^p/x^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left( a + b \log(c\sqrt{x}) \right)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**(1/2)))**p/x**2,x)`

[Out] `Integral((a + b*log(c*sqrt(x)))**p/x**2, x)`



$$3.189 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x^3} dx$$

**Optimal.** Leaf size=75

$$c^4 (-2^{-2p-1}) e^{\frac{4a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p+1, \frac{4(a+b \log(c\sqrt{x}))}{b} \right)$$

[Out]  $-2^{(-1-2*p)} * c^4 * \exp(4*a/b) * \text{GAMMA}(1+p, 4*(a+b*\ln(c*x^{(1/2)}))/b) * (a+b*\ln(c*x^{(1/2)}))^p / (((a+b*\ln(c*x^{(1/2)}))/b)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$c^4 (-2^{-2p-1}) e^{\frac{4a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{4(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x^3,x]

[Out]  $-((2^{(-1-2*p)} * c^4 * E^{(4*a)/b} * \text{Gamma}[1+p, (4*(a+b*\text{Log}[c*\text{Sqrt}[x]))]/b]) * (a+b*\text{Log}[c*\text{Sqrt}[x]])^p) / ((a+b*\text{Log}[c*\text{Sqrt}[x]])/b)^p)$

**Rule 2181**

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

**Rule 2310**

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol]  
 ] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

**Rubi steps**

$$\int \frac{(a + b \log(c\sqrt{x}))^p}{x^3} dx = (2c^4) \text{Subst} \left( \int e^{-4x} (a + bx)^p dx, x, \log(c\sqrt{x}) \right) \\ = -2^{-1-2p} c^4 e^{\frac{4a}{b}} \Gamma \left( 1 + p, \frac{4(a + b \log(c\sqrt{x}))}{b} \right) (a + b \log(c\sqrt{x}))^p \left( \frac{a + b \log(c\sqrt{x})}{b} \right)^{-p}$$

**Mathematica** [A] time = 0.05, size = 75, normalized size = 1.00

$$c^4 (-2^{-2p-1}) e^{\frac{4a}{b}} (a + b \log(c\sqrt{x}))^p \left( \frac{a + b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p + 1, \frac{4(a + b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x^3,x]

[Out] -((2^(-1 - 2\*p))\*c^4\*E^((4\*a)/b)\*Gamma[1 + p, (4\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/((a + b\*Log[c\*Sqrt[x]])/b)^p)

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b \log(c\sqrt{x}) + a)^p}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^3,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(c\sqrt{x}) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^3,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p/x^3, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(c\sqrt{x}) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^(1/2))+a)^p/x^3,x)`

[Out] `int((b*ln(c*x^(1/2))+a)^p/x^3,x)`

**maxima** [A] time = 0.78, size = 48, normalized size = 0.64

$$\frac{2 \left( b \log(c\sqrt{x}) + a \right)^{p+1} c^4 e^{\left( \frac{4a}{b} \right)} E_{-p} \left( \frac{4 \left( b \log(c\sqrt{x}) + a \right)}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^(1/2)))^p/x^3,x, algorithm="maxima")`

[Out] `-2*(b*log(c*sqrt(x)) + a)^(p + 1)*c^4*e^(4*a/b)*exp_integral_e(-p, 4*(b*log(c*sqrt(x)) + a)/b)/b`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(c\sqrt{x}))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^(1/2)))^p/x^3,x)`

[Out] `int((a + b*log(c*x^(1/2)))^p/x^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \log(c\sqrt{x}))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**(1/2)))**p/x**3,x)`

[Out] `Integral((a + b*log(c*sqrt(x)))**p/x**3, x)`

$$3.190 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x^4} dx$$

Optimal. Leaf size=80

$$c^6 (-2^{-p}) 3^{-p-1} e^{\frac{6a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p+1, \frac{6(a+b \log(c\sqrt{x}))}{b} \right)$$

[Out]  $-3^{(-1-p)} * c^6 * \exp(6*a/b) * \text{GAMMA}(1+p, 6*(a+b*\ln(c*x^{(1/2)}))/b) * (a+b*\ln(c*x^{(1/2)}))^p / (2^p) / (((a+b*\ln(c*x^{(1/2)}))/b)^p)$

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$c^6 (-2^{-p}) 3^{-p-1} e^{\frac{6a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{6(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x^4, x]

[Out]  $-((3^{(-1-p)} * c^6 * E^{((6*a)/b)} * \text{Gamma}[1+p, (6*(a+b*\text{Log}[c*\text{Sqrt}[x]))])/b) * (a+b*\text{Log}[c*\text{Sqrt}[x]])^p) / (2^p * ((a+b*\text{Log}[c*\text{Sqrt}[x]))/b)^p)$

Rule 2181

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_))^(m\_), x\_Symbol]  
 := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\int \frac{(a + b \log(c\sqrt{x}))^p}{x^4} dx = (2c^6) \text{Subst} \left( \int e^{-6x} (a + bx)^p dx, x, \log(c\sqrt{x}) \right) \\ = -2^{-p} 3^{-1-p} c^6 e^{\frac{6a}{b}} \Gamma \left( 1 + p, \frac{6(a + b \log(c\sqrt{x}))}{b} \right) (a + b \log(c\sqrt{x}))^p \left( \frac{a + b \log(c\sqrt{x})}{b} \right)$$

**Mathematica** [A] time = 0.05, size = 80, normalized size = 1.00

$$c^6 (-2^{-p}) 3^{-p-1} e^{\frac{6a}{b}} (a + b \log(c\sqrt{x}))^p \left( \frac{a + b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma \left( p + 1, \frac{6(a + b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x^4, x]

[Out] -((3^(-1 - p)\*c^6\*E^((6\*a)/b)\*Gamma[1 + p, (6\*(a + b\*Log[c\*Sqrt[x]]))]/b)\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p\*((a + b\*Log[c\*Sqrt[x]])/b)^p)

**fricas** [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b \log(c\sqrt{x}) + a)^p}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^4, x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p/x^4, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \log(c\sqrt{x}) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^4, x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p/x^4, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b \ln(c\sqrt{x}) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*ln(c*x^(1/2))+a)^p/x^4,x)`

[Out] `int((b*ln(c*x^(1/2))+a)^p/x^4,x)`

**maxima** [A] time = 0.77, size = 48, normalized size = 0.60

$$\frac{2 \left( b \log(c\sqrt{x}) + a \right)^{p+1} c^6 e^{\left( \frac{6a}{b} \right)} E_{-p} \left( \frac{6(b \log(c\sqrt{x}) + a)}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^(1/2)))^p/x^4,x, algorithm="maxima")`

[Out] `-2*(b*log(c*sqrt(x)) + a)^(p + 1)*c^6*e^(6*a/b)*exp_integral_e(-p, 6*(b*log(c*sqrt(x)) + a)/b)/b`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \ln(c\sqrt{x}))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*log(c*x^(1/2)))^p/x^4,x)`

[Out] `int((a + b*log(c*x^(1/2)))^p/x^4, x)`

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**(1/2)))**p/x**4,x)`

[Out] Exception raised: HeuristicGCDFailed

$$3.191 \quad \int x^{-1+n} \left( a + b \log(cx^n) \right)^p dx$$

Optimal. Leaf size=65

$$\frac{e^{-\frac{a}{b}} \left( a + b \log(cx^n) \right)^p \left( -\frac{a+b \log(cx^n)}{b} \right)^{-p} \Gamma \left( p + 1, -\frac{a+b \log(cx^n)}{b} \right)}{cn}$$

[Out] GAMMA(1+p, (-a-b\*ln(c\*x^n))/b)\*(a+b\*ln(c\*x^n))^p/c/exp(a/b)/n/(((a+b\*ln(c\*x^n))/b)^p)

**Rubi [A]** time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$\frac{e^{-\frac{a}{b}} \left( a + b \log(cx^n) \right)^p \left( -\frac{a+b \log(cx^n)}{b} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{a+b \log(cx^n)}{b} \right)}{cn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)\*(a + b\*Log[c\*x^n])^p, x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x^n])/b)]\*(a + b\*Log[c\*x^n])^p)/(c\*E^(a/b)\*n\*(-((a + b\*Log[c\*x^n])/b))^p)

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -((f*g*Log[F])/d)*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p)*((d_.)*(x_))^(m_.), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x)/n*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\int x^{-1+n} (a + b \log(cx^n))^p dx = \frac{\text{Subst}\left(\int e^x (a + bx)^p dx, x, \log(cx^n)\right)}{cn}$$

$$= \frac{e^{-\frac{a}{b}} \Gamma\left(1 + p, -\frac{a+b \log(cx^n)}{b}\right) (a + b \log(cx^n))^p \left(-\frac{a+b \log(cx^n)}{b}\right)^{-p}}{cn}$$

**Mathematica** [A] time = 0.05, size = 65, normalized size = 1.00

$$\frac{e^{-\frac{a}{b}} (a + b \log(cx^n))^p \left(-\frac{a+b \log(cx^n)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{a+b \log(cx^n)}{b}\right)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(-1 + n)</sup>\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>, x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x<sup>n</sup>])/b)]\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>/(c\*E<sup>(a/b)</sup>\*n\*(-((a + b\*Log[c\*x<sup>n</sup>])/b))<sup>p</sup>)

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \log(cx^n) + a\right)^p x^{n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*(a+b\*log(c\*x<sup>n</sup>))<sup>p</sup>, x, algorithm="fricas")

[Out] integral((b\*log(c\*x<sup>n</sup>) + a)<sup>p</sup>\*x<sup>(n - 1)</sup>, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \log(cx^n) + a)^p x^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*(a+b\*log(c\*x<sup>n</sup>))<sup>p</sup>, x, algorithm="giac")

[Out] integrate((b\*log(c\*x<sup>n</sup>) + a)<sup>p</sup>\*x<sup>(n - 1)</sup>, x)

**maple** [F] time = 4.26, size = 0, normalized size = 0.00

$$\int x^{n-1} (b \ln(cx^n) + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*ln(c*x^n)+a)^p,x)
```

```
[Out] int(x^(n-1)*(b*ln(c*x^n)+a)^p,x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of the first argument is 0 which is not of the expected type LIST
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{n-1} (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*log(c*x^n))^p,x)
```

```
[Out] int(x^(n - 1)*(a + b*log(c*x^n))^p, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{n-1} (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x**(n - 1)*(a + b*log(c*x**n))**p, x)
```

### 3.192 $\int (dx^q)^m (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=114

$$\frac{x(dx^q)^m e^{-\frac{amq+a}{bn}} (cx^n)^{-\frac{mq+1}{n}} (a + b \log(cx^n))^p \left(-\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)^{-p} \Gamma\left(p+1, -\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)}{mq+1}$$

[Out]  $x*(d*x^q)^m*\text{GAMMA}(1+p, -(m*q+1)*(a+b*\ln(c*x^n))/b/n)*(a+b*\ln(c*x^n))^p/\exp((a*m*q+a)/b/n)/(m*q+1)/((c*x^n)^((m*q+1)/n))/((-m*q+1)*(a+b*\ln(c*x^n))/b/n)^p$

**Rubi [A]** time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {15, 2310, 2181}

$$\frac{x(dx^q)^m e^{-\frac{amq+a}{bn}} (cx^n)^{-\frac{mq+1}{n}} (a + b \log(cx^n))^p \left(-\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)^{-p} \text{Gamma}\left(p+1, -\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)}{mq+1}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x^q)^m*(a + b*\text{Log}[c*x^n])^p, x]$

[Out]  $(x*(d*x^q)^m*\text{Gamma}[1 + p, -(((1 + m*q)*(a + b*\text{Log}[c*x^n]))/(b*n))]*(a + b*\text{Log}[c*x^n])^p)/(E^((a + a*m*q)/(b*n))*(1 + m*q)*(c*x^n)^((1 + m*q)/n)*(-(((1 + m*q)*(a + b*\text{Log}[c*x^n]))/(b*n))))^p$

#### Rule 15

$\text{Int}[(u_.)*((a_.)*(x_)^(n_))^(m_), x\_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]}*(a*x^n)^{\text{FracPart}[m]})/x^{n*\text{FracPart}[m]}, \text{Int}[u*x^{(m*n)}, x], x] /; \text{FreeQ}\{a, m, n, x\} \&\amp; \text{!IntegerQ}[m]$

#### Rule 2181

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x)])/(d*(-((f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)*(-((f*g*\text{Log}[F])*(c + d*x))/d)})^{\text{FracPart}[m]}], x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x \&\amp; \text{!IntegerQ}[m]$

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int (dx^q)^m (a + b \log(cx^n))^p dx &= (x^{-mq} (dx^q)^m) \int x^{mq} (a + b \log(cx^n))^p dx \\ &= \frac{\left( x (cx^n)^{-\frac{1+mq}{n}} (dx^q)^m \right) \text{Subst} \left( \int e^{\frac{(1+mq)x}{n}} (a + bx)^p dx, x, \log(cx^n) \right)}{n} \\ &= \frac{e^{-\frac{a+amq}{bn}} x (cx^n)^{-\frac{1+mq}{n}} (dx^q)^m \Gamma \left( 1 + p, -\frac{(1+mq)(a+b \log(cx^n))}{bn} \right) (a + b \log(cx^n))^p}{1 + mq} \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 118, normalized size = 1.04

$$\frac{x^{-mq} (dx^q)^m (a + b \log(cx^n))^p \exp \left( -\frac{(mq+1)(a+b \log(cx^n)-bn \log(x))}{bn} \right) \left( -\frac{(mq+1)(a+b \log(cx^n))}{bn} \right)^{-p} \Gamma \left( p + 1, -\frac{(mq+1)(a+b \log(cx^n))}{bn} \right)}{mq + 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^q)^m*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((d*x^q)^m*Gamma[1 + p, -(((1 + m*q)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log
[c*x^n])^p)/(E^(((1 + m*q)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m*q
)*x^(m*q)*(-(((1 + m*q)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( (dx^q)^m (b \log(cx^n) + a)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^q)^m*(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((d*x^q)^m*(b*log(c*x^n) + a)^p, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^q)^m (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^q)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="giac")

[Out] integrate((d\*x^q)^m\*(b\*log(c\*x^n) + a)^p, x)

maple [F] time = 2.13, size = 0, normalized size = 0.00

$$\int (dx^q)^m (b \ln(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^q)^m\*(b\*ln(c\*x^n)+a)^p,x)

[Out] int((d\*x^q)^m\*(b\*ln(c\*x^n)+a)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^q)^m (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^q)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate((d\*x^q)^m\*(b\*log(c\*x^n) + a)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx^q)^m (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^q)^m\*(a + b\*log(c\*x^n))^p,x)

[Out] int((d\*x^q)^m\*(a + b\*log(c\*x^n))^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^q)^m (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*q)\*\*m\*(a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Integral((d\*x\*\*q)\*\*m\*(a + b\*log(c\*x\*\*n))\*\*p, x)

$$3.193 \quad \int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p dx$$

**Optimal.** Leaf size=136

$$\frac{x (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p e^{-\frac{a(m_1 q_1 + m_2 q_2 + 1)}{bn}} (cx^n)^{-\frac{m_1 q_1 + m_2 q_2 + 1}{n}} \left( -\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n))}{bn} \right)^{-p} \Gamma\left(p + \frac{m_1 q_1 + m_2 q_2 + 1}{n}\right)}{m_1 q_1 + m_2 q_2 + 1}$$

[Out]  $x*(d_1*x^{q_1})^{m_1}*(d_2*x^{q_2})^{m_2}*GAMMA(1+p, -(m_1*q_1+m_2*q_2+1)*(a+b*\ln(c*x^n))/b/n) * (a+b*\ln(c*x^n))^p / \exp(a*(m_1*q_1+m_2*q_2+1)/b/n) / (m_1*q_1+m_2*q_2+1) / ((c*x^n)^{(m_1*q_1+m_2*q_2+1)/n}) / ((-(m_1*q_1+m_2*q_2+1)*(a+b*\ln(c*x^n))/b/n)^p)$

**Rubi [A]** time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {15, 2310, 2181}

$$\frac{x (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p e^{-\frac{a(m_1 q_1 + m_2 q_2 + 1)}{bn}} (cx^n)^{-\frac{m_1 q_1 + m_2 q_2 + 1}{n}} \left( -\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n))}{bn} \right)^{-p} \Gamma\left(p + \frac{m_1 q_1 + m_2 q_2 + 1}{n}\right)}{m_1 q_1 + m_2 q_2 + 1}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d_1*x^{q_1})^{m_1}*(d_2*x^{q_2})^{m_2}*(a + b*\text{Log}[c*x^n])^p, x]$

[Out]  $(x*(d_1*x^{q_1})^{m_1}*(d_2*x^{q_2})^{m_2}*\text{Gamma}[1 + p, -(((1 + m_1*q_1 + m_2*q_2)*(a + b*\text{Log}[c*x^n]))/(b*n))])*(a + b*\text{Log}[c*x^n])^p / (E^{((a*(1 + m_1*q_1 + m_2*q_2))/(b*n))}*(1 + m_1*q_1 + m_2*q_2)*(c*x^n)^{((1 + m_1*q_1 + m_2*q_2)/n)}*(-(((1 + m_1*q_1 + m_2*q_2)*(a + b*\text{Log}[c*x^n]))/(b*n)))^p)$

### Rule 15

$\text{Int}[(u_.)*((a_.)*(x_.)^{(n_.)})^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]}*(a*x^n)^{\text{FracPart}[m]})/x^{(n*\text{FracPart}[m])}, \text{Int}[u*x^{(m*n)}, x], x] /;$  FreeQ[{a, m, n}, x] && !IntegerQ[m]

### Rule 2181

$\text{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))}*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x)]) / (d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x)/d))^{\text{FracPart}[m]}), x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^(m + 1)/n), Subst[Int[E^(((m + 1)*x
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p dx &= \left( x^{-m_1 q_1} (d_1 x^{q_1})^{m_1} \right) \int x^{m_1 q_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p dx \\ &= \left( x^{-m_1 q_1 - m_2 q_2} (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} \right) \int x^{m_1 q_1 + m_2 q_2} (a + b \log(cx^n))^p dx \\ &= \frac{\left( x (cx^n)^{-\frac{1+m_1 q_1 + m_2 q_2}{n}} (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} \right) \text{Subst} \left( \int e^{\frac{(1+m_1 q_1 + m_2 q_2)x}{n}} dx \right)}{n} \\ &= \frac{e^{-\frac{a(1+m_1 q_1 + m_2 q_2)}{bn}} x (cx^n)^{-\frac{1+m_1 q_1 + m_2 q_2}{n}} (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} \Gamma \left( 1 + p \right)}{1 -} \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 142, normalized size = 1.04

$$\frac{(d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} x^{-m_1 q_1 - m_2 q_2} (a + b \log(cx^n))^p \exp \left( -\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n) - bn \log(x))}{bn} \right) \left( -\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n) - bn \log(x))}{bn} \right)^{m_1 q_1 + m_2 q_2 + 1}}{m_1 q_1 + m_2 q_2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a + b\*Log[c\*x^n])^p,x]

[Out] (x^(-(m1\*q1) - m2\*q2)\*(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*Gamma[1 + p, -(((1 + m1\*q1 + m2\*q2)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(((1 + m1\*q1 + m2\*q2)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*(1 + m1\*q1 + m2\*q2)\*(-(((1 + m1\*q1 + m2\*q2)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**fricas [F]** time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left( (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \log(cx^n) + a)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*log(c\*x^n) + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*log(c\*x^n))^p,x, algorithm="giac")

[Out] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*log(c\*x^n) + a)^p, x)

**maple** [F] time = 24.09, size = 0, normalized size = 0.00

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \ln(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*ln(c\*x^n)+a)^p,x)

[Out] int((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*ln(c\*x^n)+a)^p,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*log(c\*x^n) + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a + b\*log(c\*x^n))^p,x)

[Out] int((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a + b\*log(c\*x^n))^p, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x\*\*q1)\*\*m1\*(d2\*x\*\*q2)\*\*m2\*(a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Timed out





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
  If[Head[expn]===Plus || Head[expn]===Times,
```

```
    Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
  If[ElementaryFunctionQ[Head[expn]],
```

```
    Max[3,ExpnType[expn[[1]]],
```

```
  If[SpecialFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```



```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```